G.C.E. (Advanced Level)

Physics
Grade 13
Resource Book

Fields and Current Electricity
Units 5, 6, 7, 8

Department of Science
Faculty of Science and Technology
National Institute of Education
Maharagama
www.nie.lk

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Fields and Current Electricity - Unit 5, 6, 7, 8
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Message from the Director General

The National Institute of Education takes opportune steps from time to time for the development of quality in education. Preparation of supplementary resource books for respective subjects is one such initiative.

Supplementary resource books have been composed by a team of curriculum developers of the National Institute of Education, subject experts from the national universities and experienced teachers from the school system. Because these resource books have been written so that they are in line with the G. C. E. (A/L) new syllabus implemented in 2017, students can broaden their understanding of the subject matter by referring these books while teachers can refer them in order to plan more effective learning teaching activities.

I wish to express my sincere gratitude to the staff members of the National Institute of Education and external subject experts who made their academic contribution to make this material available to you.

Dr. Sunil Jayantha Nawarathna
Director General
National Institute of Education
Maharagama

Message from the Director

Since 2017, a rationalized curriculum, which is an updated version of the previous curriculum is in effect for the G.C.E (A/L) in the general education system of Sri Lanka. In this new curriculum cycle, revisions were made in the subject content, mode of delivery and curricular materials of the G.C.E. (A/L) Physics, Chemistry and Biology. Several alterations in the learning teaching sequence were also made. A new Teachers' Guide was introduced in place of the previous Teacher's Instruction Manual. In concurrence to that, certain changes in the learning teaching methodology, evaluation and assessment are expected. The newly introduced Teachers' Guide provides learning outcomes, a guideline for teachers to mould the learning events, assessment and evaluation.

When implementing the previous curricula, the use of internationally recognized standard textbooks published in English was imperative for the Advanced Level science subjects. Due to the contradictions of facts related to the subject matter between different textbooks and inclusion of the content beyond the limits of the local curriculum, the usage of those books was not convenient for both teachers and students. This book comes to you as an attempt to overcome that issue.

As this book is available in Sinhala, Tamil, and English, the book offers students an opportunity to refer the relevant subject content in their mother tongue as well as in English within the limits of the local curriculum. It also provides both students and teachers a source of reliable information expected by the curriculum instead of various information gathered from the other sources.

This book authored by subject experts from the universities and experienced subject teachers is presented to you followed by the approval of the Academic Affairs Board and the Council of the National Institute of Education. Thus, it can be recommended as material of a high standard.

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Content

Unit 5

Gravitational Fields

Unit 5 : Chapter One

Gravitational Force Field

1.1 Gravitational Fields

During the initial decades of the 13^{th} century, the scientist Keplar, having done investigations into the motion of celestial bodies has recorded, his observations about the elliptical motion and the periods of revolution of these bodies.

A young scientist Isac Newton who lived during the mid era of the same century carrying investigations on the approximate circular motion of the planets in the solar system, around the Sun revealed that a certain force of attraction made by the sun on them helps in their circular motions.

Furthermore, he confirmed that it is this same force of attraction that holds the moon in its orbit and also brings the apple on to the Earth leaving the tree.

On further research it was proved that in the universe, between any two particles having masses exist these mutual forces of attraction mutually and these forces were named as "gravitational forces".

Broad research was subsequently done by Newton on factors affecting this force of attraction and he, being a mathematician too, expressed his findings mathematically by the following law.

1.2 Newton's Law of Gravitation

The gravitational force of attraction between two particles of masses *M* and m situated at a distance *r* is given by,

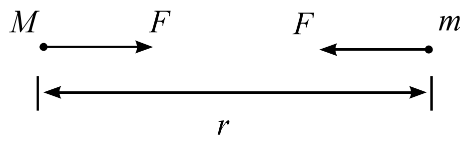


Figure 1.1

$$F = G\frac{Mm}{r^2}$$

where *G* is a universal constant known as Gravitational constant. ($G = 6.67 \times 10^{-11}$ N $m^2 kg^{-2}$)

The gravitational field has been introduced as a new concept for gravitational force. According to this concept, there exists a gravitational field around any mass. The gravitational force of attraction between two masses is considered as the result of the inter-action between the two fields when they intersect. Hence, a gravitational field is a force field.

1.3 Intensity of a Gravitational field

The strength of a gravitational field at any point in it is measured by the quantity known as the " Gravitational field intensity " at that point. The intensity of a gravitational field at such a point is the force acting on a unit mass placed at that point.

Hence, if F is the force acting on a mass m placed at a certain point in a gravitational field, the gravitational field intensity at the point is,

$$g = \frac{F}{m} \text{ N kg}^{-1}$$

The gravitational field due to an isolated mass can be mentioned as the simplest of all gravitational fields.

1.3.1 Gravitational field intensity at a point in the gravitational field due to an isolated mass

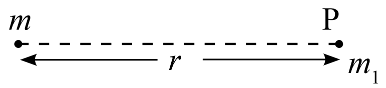


Figure 1.2

P is a point in the gravitational field due to the isolated mass m, situated at a distance r from the same mass. In order to determine the gravitational field at P let us place a mass element Δm at P and find the force on it.

According to Newton's law of gravitation,

The force mass m_1, $(F) = G \frac{m\, m_1}{r^2}$

Hence, gravitational field intensity $g = \frac{F}{m_1} = G\frac{m}{r^2}$

Gravitation is essentially a force of attraction. Hence the above intensity is a vector quantity directed towards the mass m.

The variation of gravitational intensity with the distance from the isolated point mass can be represented graphically as follows.

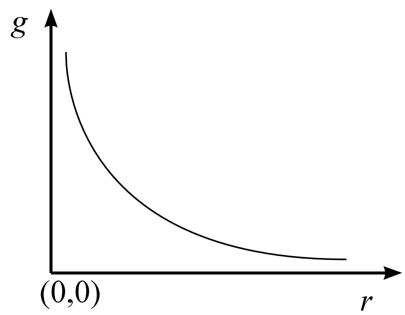


Figure 1.3

1.3.2 Gravitational Intensity around a spherical mass

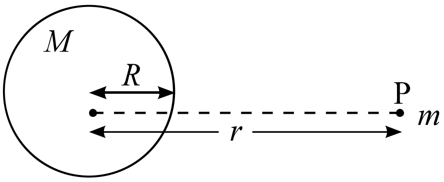


Figure 1.4

P is a point situated at a distance r ($r > R$) of a spherical body of mass M and radius R. In order to determine the gravitational intensity at P, let us place a mass element m at P and find the force on it.

Assuming the whole mass of the sphere is accumulated at its centre.

$$F = G\frac{Mm}{r^2}$$

$\therefore$ gravitational field intensity at P, (g) = $\dfrac{F}{m} = G\dfrac{M}{r^2}$

Similarly gravitational field intensity on the surface of the sphere, $g = \dfrac{GM}{R^2}$

1.4 Gravitational potential

Since any mass placed at a certain point in a gravitational field is subjected to a gravitational force, a certain amount of work is done by the field in bringing this mass from outside the field (infinity) to this point. While this work is accumulated as potential energy and the work done per unit mass in bringing it to this point is referred to as the " **gravitational potential**" at the point.

1.4.1 Gravitational potential at a point in the gravitational field due to an isolated mass

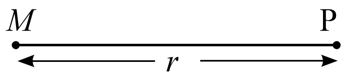


Figure 1.5

P is a point in the gravitational field due to mass M, and is situated at a distance r from the same mass. It is possible to derive mathematically that the work done in bringing unit mass from infinity up to the point P is, $W = -G\dfrac{M}{r}$

$\therefore$ gravitational potential at point P, = $V = -G\dfrac{M}{r}$

The negative sign above indicates that the relevant work is done by the field and not against the field. Also since work is a scalar, gravitational (field) potential too is a scalar quantity.

The following graph illustrates the variation of gravitational potential with the distance from a point mass.

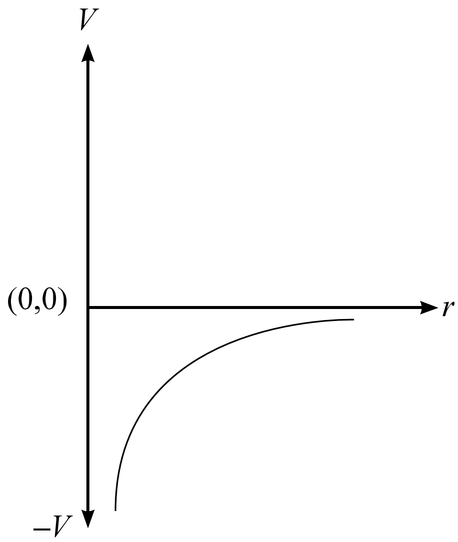


Figure 1.6

1.4.2 Potential energy of a mass placed in a gravitational field

If V is the potential at a point in a gravitational field, the potential energy of a unit mass placed at that point too is V.

$\therefore$ Potential energy of a mass placed at the point, $E = mV$

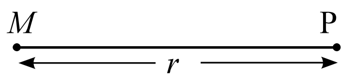


Figure 1.7

If the above point is situated in the gravitational field due to a mass M, at a distance r from M, then the gravitational potential at point P,

$$V = -G\frac{M}{r}$$

$\therefore$ The potential energy of a mass placed at the point P, $E = mV = -G\frac{Mm}{r}$

Unit 5 : Chapter Two

Earth's Gravitational Field

According to the mass of the Earth, a specific gravitational field has been created around it.

2.1 Gravitational field due to the Earth

Considering Earth as a spherical body of mass M and radius R, the gravitational force of a body of mass m placed on the surface of Earth,

$$F = G\frac{Mm}{R^2}$$

Also the force on a mass m placed on the surface of Earth is its weight.

ie: $F = mg$, where g = acceleration due to gravity

$$\therefore \quad mg = G\frac{Mm}{R^2}$$

Therefore, gravitational field intensity can be written as,

$$g = G\frac{M}{R^2}$$

Hence it is seen that the acceleration due to gravity and the gravitational field intensity on the surface of Earth take the same value.

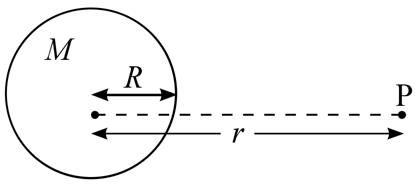


Figure 2.1

Consider a mass m placed at a point P at a distance r ($> R$) from the centre of the Earth.

The gravitational force on mass m, $F = G\frac{Mm}{r^2}$

$\therefore$ Gravitational field intensity, $g = \frac{F}{m} = G\frac{M}{r^2}$

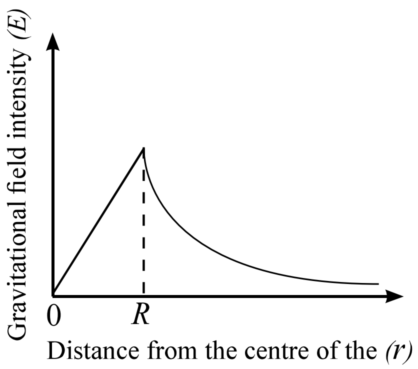


Figure 2.2

The variation of the above intensity at points from the centre of Earth to the surface and outside Earth's surface, with the distance from the centre of Earth can be shown graphically as given in Figure 2.2.

2.2 Potential energy at a height h in the gravitational field of the Earth

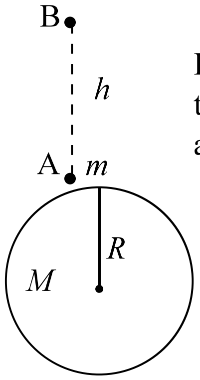


Figure 2.3

Figure 2.3 shows a body of mass '*m*' being raised from a point A close to the surface of Earth to a point B at a height *h*. Let *M* be the mass of Earth and *R* its radius.

Gravitational potential at point A $= -G\dfrac{M}{R}$

Potential energy possessed by the body at point A $= -G\dfrac{Mm}{R}$

Gravitational potential at point B $= \dfrac{-GM}{(R+h)}$

Potential energy possessed by the body at point B $= \dfrac{-GMm}{(R+h)}$

If *E* is the excess potential energy received by the body when raised from A to B,

$$E = \frac{-GMm}{(R+h)} - \left(\frac{-GMm}{R}\right)$$

$$E = \left[\frac{-GMm\,h}{(R+h)\,R}\right]$$

When $h << R$, $(R+h)\,R \simeq R^2$

$$\therefore \text{ Hence } E = \frac{GMm\,h}{R^2} \quad \text{——} (1)$$

If *g* is the gravitational acceleration,

$$g = \frac{GM}{R^2} \quad \text{——} (2)$$

From the above two equations it can be derived that $E = mg\,h$

Hence the gravitational potential energy possessed by a body relative to a zero energy level is given as $E = mgh$ where h is the height of the body above the zero level.

Note:
This expression can be used only for very small values of *h* compared with *R*.

2.3 Motion of a satellite around Earth.

Many satellites revolve around our Earth. Out of these satellite the only natural satellite is the Moon. The motion of the Moon as well as of all other artificial satellites is based on common principles.

For example, consider a satellite of mass m revolving around the Earth in an orbit of radius *r* with a speed *v*,

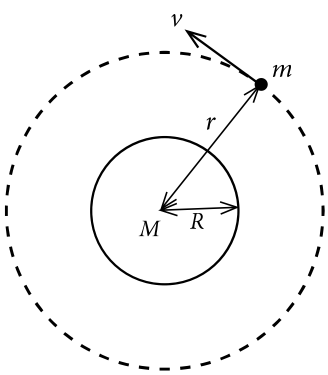


Figure 2.5

The centripetal force $\left(\frac{mv^2}{r}\right)$ required for this circular motion is provided by the gravitational force of attraction of the Earth.

If *M* is the mass of the Earth, then,
According to the equation *F =ma,*
[$\left(\frac{v^2}{r}\right)$ = centripetal force]

$$\left(G\frac{Mm}{r^2}\right) = \left(\frac{mv^2}{r}\right)$$

$$\therefore \quad v^2 = \frac{GM}{r}$$

Hence, speed of the satellite,

$$v = \sqrt{\frac{GM}{r}}$$

The periodic time of revolution of the satellite *(T)*,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

The quantities *G* and *M* are constants. Hence from the above expression it is seen that the speed of a satellite and its period of revolution depend only on the radius of its orbit.

When a satellite revolving close to the Earth is considered,
Radius of the orbit *(r)* ≈ Radius of the Earth *(R)*

Also if '*g*' is the acceleration due to gravity on the Earth, the weight of any body on it is equal to the gravitational force of attraction.

$$mg = G\frac{Mm}{R^2}$$

$$\therefore GM = gR^2$$

By substituting the above expressions,

$$\text{Speed, } v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

Periodic time of revolution, $T = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$

2.3.1 Calculation of the speed and periodic time of a satellite revolving close to Earth

Let us consider the radius of the Earth $R = 6.4 \times 10^6$ m and $g = 10$ m s^{-2}
By substituting to the equation $v = \sqrt{gR}$

Satellite speed $v = \sqrt{10 \times 6.4 \times 10^6} = \sqrt{64 \times 10^6} = 8 \times 10^3$ m s^{-1}

By substituting to the equation $T = 2\pi\sqrt{\frac{R}{g}}$,

Periodic time , $T = 2\pi\sqrt{\frac{6.4 \times 10^6}{10}} = 5028$ s

≈ 1 h 24 min

2.3.2 Energy of a revolving satellite

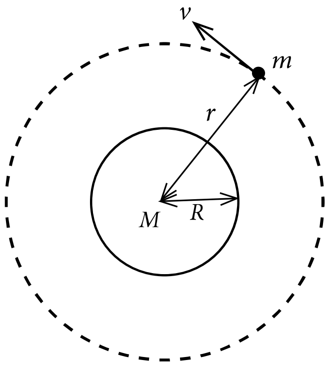


Figure 2.6

Consider a satellite of mass *'m'* moving in an orbit of radius *'r'* at a speed *'v'* around Earth of mass M radius R.

Then the gravitational potential at any point on the orbit, $V = -G\frac{M}{r}$

$\therefore$ Gravitational potential energy of the satellite moving along the orbit, $(E_1) = -G\frac{Mm}{r}$

Also, the kinetic energy of the satellite $(E_2) = \frac{1}{2}mv^2$.

But, the centripetal force = gravitational force

$$\frac{mv^2}{r} = G\frac{Mm}{r^2}$$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}G\frac{Mm}{r}$$

$$E_2 = \frac{1}{2}G\frac{Mm}{r}$$

$\therefore$ Total energy of the satellite, $E = E_1 + E_2 = -G\frac{Mm}{r} + \frac{1}{2}G\frac{Mm}{r}$

$$E = -\frac{1}{2}G\frac{Mm}{r}$$

The above expression is known as the energy equation of a satellite.

2.3.3 Geo - stationary satellites

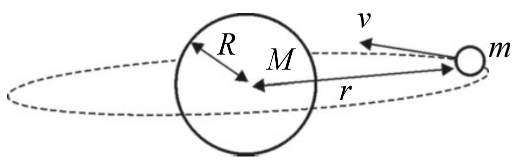


Figure 2.7

A satellite which stays relatively unmoved vertically above a precise point on Earth is called a geo – stationary satellite. The radius of the orbit of a geo – satellite is decided so that its period of revolution is 24 hours.

These satellites which have been launched to employ for communication purposes among various lands on Earth will have to essentially satisfy the following conditions regarding their motion and orbit.

1. Geo – stationary satellite should revolve in the same sense as the sense of rotation of Earth.
2. The angular velocity of revolution of a geo – stationary satellite should be equal to that of the rotation of the Earth. In other words the periodic time of revolution of a geo – stationary satellite should be 24 hours.
3. The orbit of a geo – stationary satellite should be in the same plane as the equator of the Earth.

Considering the periodic time of a satellite as $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Periodic time of a geo – stationary satellite is 24×60×60 = 86400 s.

$$2\pi\sqrt{\frac{r^3}{GM}} = 24 \times 60 \times 60 = 86400\ s$$

$$\therefore \quad 86400 = 2 \times \frac{22}{7}\sqrt{\frac{r^3}{6.7 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$$r = 42400 \times 10^3\ \text{m}$$

If earth's radius is 6400km, height of satellite's above the earth surface

$$= (42400 - 6400)\ \text{km}$$
$$= 36000\ \text{km}$$

2.3.4 Escape velocity

The minimum velocity with which a body has to be projected from the surface of Earth to over come Earth's gravitation and never return to Earth again is called the escape velocity of Earth.

Considering a body of mass *m* placed on the surface on Earth of mass *M* and radius *R*, its potential energy $E_1 = -G\frac{Mm}{R}$

If this body is projected with a velocity *v* from the surface of Earth, its total energy,

$$E = \text{Kinetic energy} + \text{Potential energy}$$

$$= \frac{1}{2}mv^2 + \left(-G\frac{Mm}{R}\right)$$

If v is the escape velocity, this total energy will be spent and become zero at an infinite distance. Then according to the law of conservation of energy,

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0$$

$$\frac{1}{2}mv^2 = G\frac{Mm}{R}$$

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2gr^2}{R}} = \sqrt{2gR}$$

where $G\frac{M}{R^2} = g$ = Gravitational Intensity on the Earth's surface

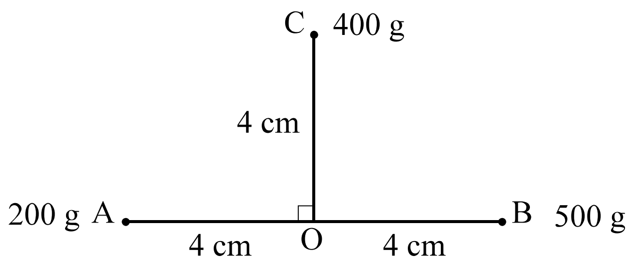
considering $g = 9.8\ \text{m s}^{-2}$ and the radius of the Earth $R = 6.4 \times 10^6$ m,

Escape velocity of the Earth

$$= \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$
$$= \sqrt{19.6 \times 6.4 \times 10^6}$$
$$= \sqrt{196 \times 64 \times 10^4}$$
$$= 14 \times 8 \times 10^2 = 112 \times 10^2\ \text{m s}^{-1}$$
$$= \underline{\underline{11.2\ \text{km s}^{-1}}}$$

Solved exercises

1.



At the two ends A and B of a straight line, AB of length 8 cm are placed masses of 200 and 500 g respectively

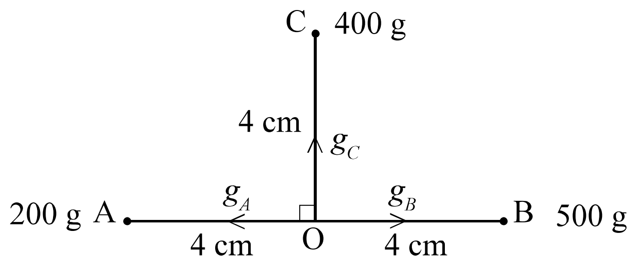
At the end C of the perpendicular bisector, OC of the line AB is placed a mass of 400 the length OC being 4 cm

Calculate,

(a) The resultant gravitational field intensity at the point O.

(b) The total gravitational potential at the point O.

(a)



Considering x Components,

$$\longrightarrow \quad g_1 = g_B - g_A \quad = G\frac{500\times10^{-3}}{(4\times10^{-2})^2} - G\frac{200\times10^{-3}}{(4\times10^{-2})^2}$$

$$= G\frac{300\times10^{-3}}{(4\times10^{-2})^2} = G\frac{3\times10^{3}}{16}$$

Considering y Components,

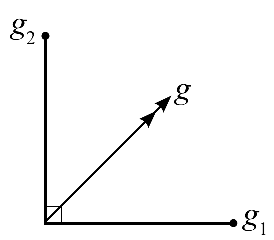
$$\uparrow \quad g_2 = g_C = G\frac{400\times10^{-3}}{(4\times10^{-2})^2} = G\frac{4\times10^{3}}{16}$$

Gravitational intensity at O,

$$g = \sqrt{g_1^{\,2} + g_2^{\,2}}$$

$$= G\frac{10^3}{16}\sqrt{4^2+3^2} = G\frac{5\times10^3}{16}$$

$$= \underline{\underline{3.125\times10^2\,G}}$$



(b) Gravitational potential at O,

$$V = \left[G\frac{200\times10^{-3}}{(4\times10^{-2})} + G\frac{500\times10^{-3}}{(4\times10^{-2})} + G\frac{400\times10^{-3}}{(4\times10^{-2})} \right]$$

$$= \frac{G\times10^{-1}}{4\times10^{-2}}\,(2+5+4) = -\,G\,\frac{110}{4}$$

$$= \underline{\underline{-\,27.5\ G\ \mathrm{J\ kg^{-1}}}}$$

Unit 6

Electrostatic Field

Unit 6: Chapter One

Electrostatic Force

1.1 Introduction

During the period of the 6th century B.C a Greek researcher named Thales found that various solid objects when rubbed with other dry materials possessed the properly of attracting light things such as little pieces of paper and hair etc. This property was first exhibited by a piece of amber when rubbed with silk

A number of centuries later, a British scientist William Gilbert and also an American scientist Benjamin Franklin had carried out systematic research on this property of attraction of objects rubbed by other materials and after scientific analysis of their findings the conclusions were summarized as follows.

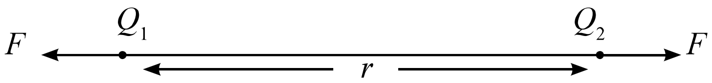
1. A body which acquires the property of attraction when rubbed by other materials is said to be electrically charged.
2. Certain bodies which are being charged by rubbing repel each other while some other bodies charged in the same way attract each other. Hence two types of charges which have properties opposite to each other exist and they are named as positive (+) charges and negative (-) charges.
3. Like charges (+, + and -, -) repel each other while unlike charges (+, -) attract each other.
4. In a neutral body both types of charges exist equally. Of these only negative (-) charges are mobile and they are also called " electrons "
5. When negative (-) charges are removed from a body it becomes a positively charged body due to the excess of positive charges remaining in it while the body receiving the negative charges becomes a negatively (-) charged body.

1.2 Force between two electrostatic charges

After confirmation of the fact that two like charges repel each other while two unlike charges attract each other, the French scientist Coulomb who lived in the 18th century researching on this mutual force between two charges forwarded the following law based on his results.

Coulombs' law

"The mutual force between two point electric charges is directly proportional to the product of the two charges and inversely proportionatel to the square of the distance between them".



Hence, the electrostatic field produced between Q_1 and Q_2 charges at a distance of r,

$$F \propto Q_1 Q_2 \text{ and } F \propto \frac{1}{r^2}$$

Therefore, $\Rightarrow F \propto \frac{Q_1 Q_2}{r^2}$

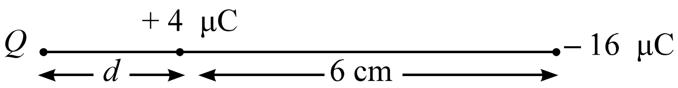
$$\Rightarrow F = k \times \frac{Q_1 Q_2}{r^2} \quad \text{———— ①}$$, where k is a constant

The force between two charges also depends on the medium between them. The property of this medium included in the constant k is known as "permittivity" of the medium. It is repersented by the symbol $''\varepsilon''$. For free space the symbol is $''\varepsilon_0''$ and for other media the common symbol is $''\varepsilon''$ which is obtained from the ratio $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ where ε_r is referred to as the relative permittivity of the medium.

However, the constant k above is represented as, $k = \frac{1}{4\pi\varepsilon}$. Accordingly the mutual force between two charges Q_1 and Q_2 placed at a distance $''r''$ from each other in free space can be given as, $F = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Q_1 Q_2}{r^2}$, $\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

Solved exercise :

Two charges +4 μC and -16 μC are fixed as two points 6 cm apart on a straight line. Another charge Q placed at a distance $''d''$ from the charge 4 μC on the same straight line but outside the charge, is in equilibrium. Find the distance d.



For the equilibrium of charge Q,

Force of repulsion on Q from charge +4 μC = Force of attraction on Q from charge -16 μC

$$\frac{1}{4\pi\varepsilon}\frac{4 \times 10^{-6} Q}{d^2} = \frac{1}{4\pi\varepsilon}\frac{16 \times 10^{-6} Q}{(6 + d)^2}$$

$$\frac{4}{d^2} = \frac{16}{(6 + d)^2}$$

$$\frac{2}{d} = \frac{4}{6 + d}$$

$$d = 6 \text{ cm}$$

1.3 Electrostatic fields

When a charge enters from outside into the region around any electric charge, the former charge would experience a force indicating that this region is a field of forces. Since these forces act on electrostatic charges, this field is called an **"electrostatic field".**

1.3.1 Electric field intensity

An electrostatic (or electric) field can be strong at a certain place, moderate at another and weak at yet another. This condition of strength of an electric field is measured by the quantity called **"electric field intensity"**.

The electric field intensity at a certain point in an electric field is the force acting on a unit positive (+) charge placed at that point. Also, field intensity is a vector quantity and its direction is represented by the direction of the force acting as this positive charge. Accordingly, if F is the force acting on a charge *"q"* placed at a certain point in an electric field, then, at that point,

Electric field intensity (E) = $\frac{F}{q}$ (N C^{-1}).

Hence, the force acting on a charge *"q"* placed at a point of intensity E is $F = Eq$.

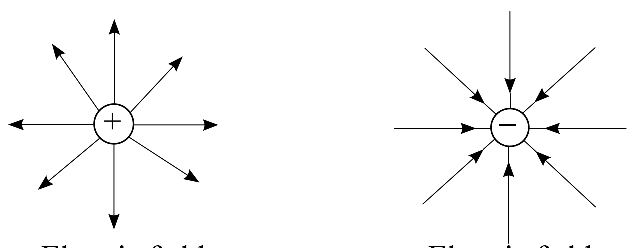
1.3.2 Representation of electric fields

Although an electric (electrostatic) field, is a force field it could be an empty space around an electric charge or a distribution of charges. However, when studying about this force field it becomes necessary to represent the paths along which these forces are acting. For this purpose, a type of hypothetical lines known as "electric lines of force" are used.

An electric line of force can be considered as a path along which a small positive charge would travel when placed in the field. The direction of the line of force is the direction in which the " positive" charge would move along the line. A large number of such lines of force would pass through a very small region in an electric field and no two lines of force would ever cross or even touch each other.

eg:

1. Electric field around a point electric charge

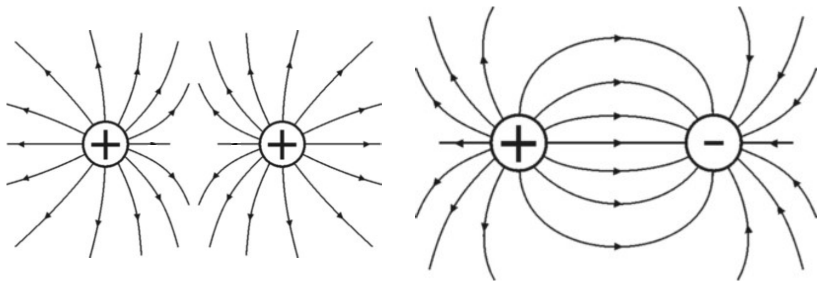


Electric field around a positive charge

Electric field around a negative charge

Figure 1.1

2. Electric field around a pair of electric charges



Field around a pair of like charges

Field around a pair of unlike charges

Figure 1.2

3. Electric field around two parallel charged plates

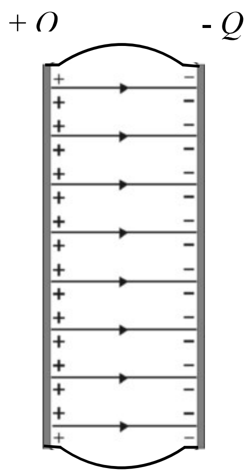


Figure 1.3

Electric field around two parallel plates charged with opposite charges.

1.3.4 Electric field intensity at a point in the electric field due to a point charge

The simplest of an electrostatic field is the field around an isolated point charge.

P is a point situated in the electric field due to the charge Q which is situated in a medium of permittivity ε, such that its distance from charge Q is r.

In order to determine the electric field intensity at P, let us place a small charge $+q$ at P and find the force on it.

According to Coulomb's law,

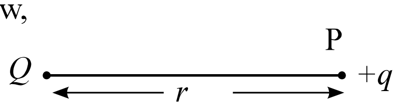


Figure 1.4

Force on charge $+q$, F $= \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2}$

$\therefore$ Electric field intensity at P, E $= \frac{F}{q} = \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2 q}$

$\therefore$ $E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r^2}$

If the electric field is a free space, $E = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Q}{r^2}$

The variation of this field intensity with the distance r from the charge Q can be illustrated graphically as follows.

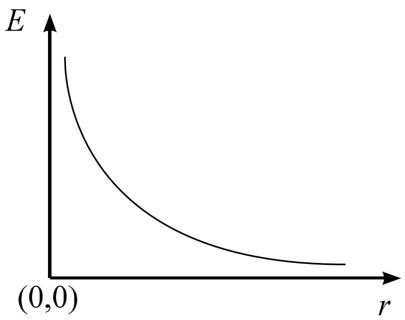


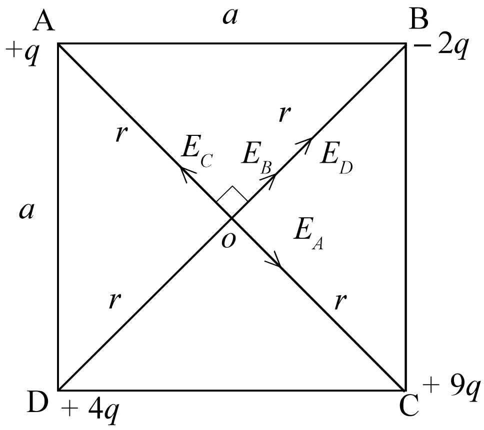
Figure 1.5

The field intensity at a point due to two or more point charges will be the resultant of all field intensities caused by all point charges separately at the points.

<u>Solved Problems</u>

Four charges $+q$, $-2q$, $+9q$ and $+4q$ each are placed at the four corners of a square ABCD of side 'a'. Calculate the electric field intensity at the central point O where the two diagonals of the square intersect.

If a = 2 cm and $q = 200\ \mu C$, what is the value of this intensity ?



$$r = \frac{\sqrt{2}\,a}{2}$$

If E_A, E_B, E_C and E_D are the electric field intensities of center O of the square due to charges at four corners A, B, C and D respectively,

$$E_A = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}, \qquad E_B = \frac{1}{4\pi\varepsilon}\frac{2q}{r^2},$$

$$E_C = \frac{1}{4\pi\varepsilon}\frac{9q}{r^2} \qquad E_D = \frac{1}{4\pi\varepsilon}\frac{4q}{r^2}$$

In order to determine the resultant intensity,

$$\nwarrow\ E_1 = E_C - E_A = \frac{1}{4\pi\varepsilon}\frac{8q}{r^2}$$

$$\nearrow E_2 = E_B + E_D = \frac{1}{4\pi\varepsilon}\frac{6q}{r^2}$$

The resultant intensity $= \sqrt{E_1^{\,2} + E_2^{\,2}} = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\sqrt{8^2 + 6^2}$

$$= \frac{1}{4\pi\varepsilon}\frac{q}{2a^2/4}\sqrt{100}$$

$$= \frac{1}{4\pi\varepsilon}\,.q.\,\frac{4\times 10}{2a^2}$$

$$= \left(\frac{1}{4\pi\varepsilon}\right)\frac{2\times 10}{a^2}\,q$$

When $\frac{1}{4\pi\varepsilon} = 9\times 10^9\ \text{N m}^2\,\text{C}^{-2}$, $q = 200\ \mu\text{C}$ and $a = 2$ cm,

$$E = \frac{9\times 10^9 \times 2\times 10\times 200\times 10^{-6}}{(2\times 10^2)^2} = \underline{\underline{9\times 10^{10}\ \text{N C}^{-1}}}$$

Unit 6: Chapter Two

Flux Model of an Electrostatic Field

In the study of electric fields, it was mentioned earlier about the usage of hypothetical electric lines of force (or flux) to illustrate the forces acting in electric fields. Using this concept as an opening a flux model for an electric field has been formed. However this flux model is only a concept used merely to comprehend the action of an electric field.

According to this flux model in a uniform electric field, the lines of force are parallel to each other and these lines are also known as electric lines of flux.

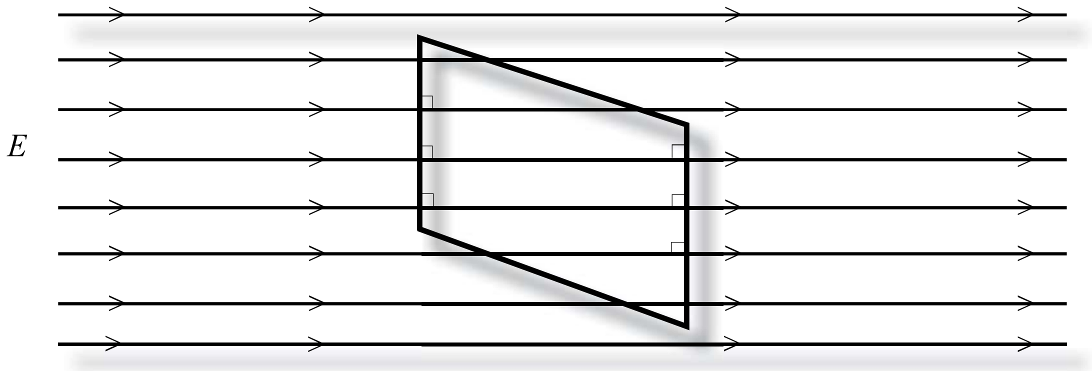


Figure 2.1

For the sake of convenience, it is considered that the electric field intensity is the number of electric flux lines crossing normally a unit area of the field.

Hence the total electric flux crossing normally an area A is given by $\phi = EA$

Then the electric field intensity $E = \dfrac{\phi}{A}$

Electric flux lines are continuous and are supposed to emerge from positive charges and end at negative charges.

According to the illustration of the electric flux lines spreading from a point charge, the gaps between lines widen with increasing distance from the charge.

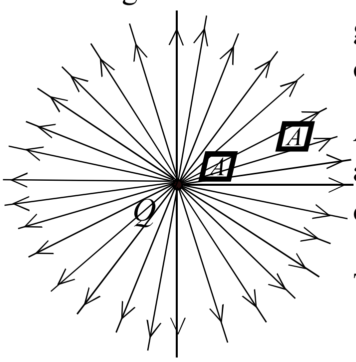


Figure 2.2

As a result the number of flux lines passing through a unit area decreases which means that the electric field intensity decreases with increasing distance from the charge.

The expressions $E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$ confirms the above fact.

2.1 Gauss Theorem

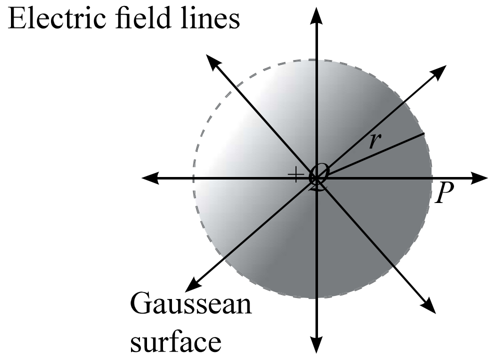


Figure 2.3

The electric field intensity at a point P, distance r from an electrostatic charge Q is given by,

$$E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r^2} \quad \text{——— (1)}$$

$$(1) \times 4\pi r^2 \Rightarrow (4\pi r^2)\,E = \frac{Q}{\varepsilon} \quad \text{——— (2)}$$

where ε is the electric permittivity of the medium

According to geometry, $4\pi r^2$ is the area of a spherical surface having the charge Q as centre and radius r. Then the expression on the left above represents some quantity which crosses the spherical surface surrounding the charge Q and according to the flux lines illustration it indicates the electric flux passing normally across that surface.

Hence this expression says that the total normal electric flux crossing a spherical surface surrounding the charge Q is equal to $\frac{Q}{\varepsilon}$.

It was proved by scientist Gauss that the above surface need not be spherical only and, that the above relation is applicable to closed surfaces of any shape. As a result of Gauss conclusion it came to be known as **"Gauss theorem"** which is stated as follows.

Gauss' Theorem

"The total normal electric flux crossing a closed surface of any shape surrounding a charge Q situated in a medium of permittivity $\varepsilon = \frac{Q}{\varepsilon}$.

Symbolically $\phi = \frac{Q}{\varepsilon}$

If the area of the closed surface is A and its electric field intensity is E, $\phi = EA$.

$$\therefore \quad EA = \frac{Q}{\varepsilon}$$

Applications of Gauss' theorem

Using Coulomb's law, it was derived that the electric field intensity at a point distant r from an isolated charge Q is $E = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Q}{r^2}$.

However can this expression be used to find the electric field intensity due to a collection or a distribution of charges? Gauss' theorem leads the way to determine the electric field intensity due to symmetrical distributions of charges.

For this purpose an imaginary closed surface of a suitable geometrical shape covering the charge distribution is considered. This imaginary surface is referred to as a **"Gaussean surface"**.

2.1.1 Electric field intensity at a point in the electric field around a point charge

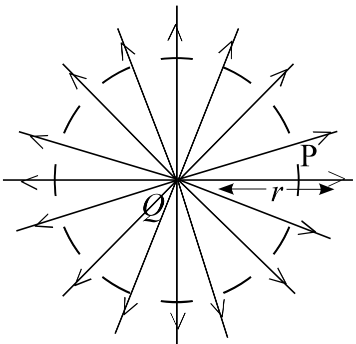


Figure 2.4

Point P is situated at a distance r from the point charge Q situated in a medium of permittivity ε. In order to determine the electric field intensity at the point P which is situated in the electric field due to charge Q, let us assume a spherical Gaussean surface having Q as its centre and radius r and passing through P (The electric flux originating from Q intersects this Gaussean surface normally).

If E is the electric field intensity at all points including P on the Gaussean surface, then by Gauss' theorem,

$$EA = \frac{Q}{\varepsilon}$$

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon}$$

$$E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r^2}$$

(This expression has been derived using Coulomb's law too)

2.1.2 Electric field intensity due to a charged hollow conducting sphere

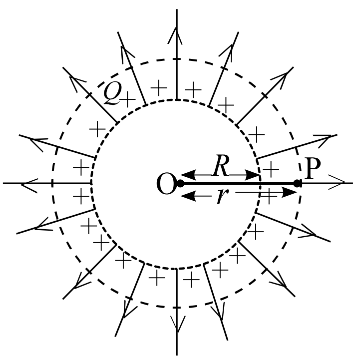


Figure 2.5

Consider a hollow conducting sphere of radius R which is given a charge Q (while this charge is spread uniformly over the outer surface of the sphere, no charge will exist inside it). Due to the uniformity of the curvature of the sphere, the electric flux lines will emerge normally along the surface of sphere.

P is a point at a distance r from centre of the sphere.

(i) Point P situated outside the sphere ($r > R$)

considering a spherical Gaussean surface of which the centre is the centre of the sphere and radius is r, so that the point P is on its surface.

According to Gauss' theorem, if E is the electric field intensity on the surface of the Gaussean surface,

$$EA = \frac{Q}{\varepsilon}$$

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon}$$

$$E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r^2}$$

(ii) Point P situated on the sphere ($r = R$)

In this situation, the outer surface of the sphere becomes the Gaussean surface.

$\therefore$ If E is the electric field intensity on it,

$$EA = \frac{Q}{\varepsilon}$$

$$E \times 4\pi R^2 = \frac{Q}{\varepsilon}$$

$$E = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{R^2}$$

(iii) Point ρ inside the sphere ($r < R$)

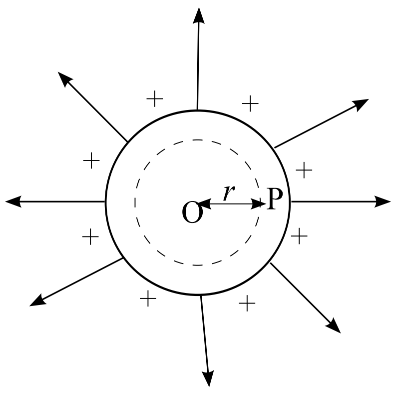


Figure 2.6

In this situation, the Gaussean surface exists inside the sphere.

Since there is no charge inside the sphere

$Q = 0$

Hence according to Gauss' theorem,

$E.\ 4\pi r^2 = 0$

$E = 0$

The following Figure 2.7 shows the electric field intensity variation with the distance from the centre of a conducting sphere.

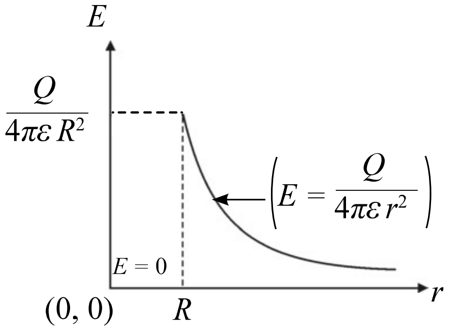


Figure 2.7

2.1.3 Electric field intensity at a point close to a uniformly charged conducting sheet of infinite extent

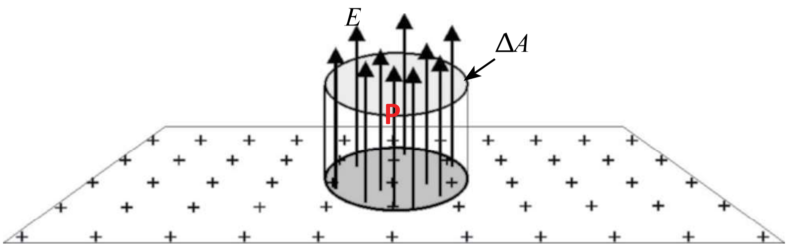


Figure 2.8

Let σ (cm^{-2}) be the surface density of charge on the charged surface.

Consider a small cylindrical Gaussean surface as shown in the Figure 2.8 above. Let the base of this cylinder of area ΔA be on the charged surface. Also let the point P where the intensity is required be on the top surface of the cylinder. Since the point P is very close to the charged surface flux lines emerging from charged surface cross the top surface of Gaussean surface normally.

Applying Gauss' theorem for these flux lines,

$$EA = \frac{Q}{\varepsilon}$$

$$E\,\Delta A = \frac{\sigma\,\Delta A}{\varepsilon}$$

$$E = \frac{\sigma}{\varepsilon}$$

2.1.4 Electric field intensity at a point situated at a distance *r* from a charged thin wire of infinite length

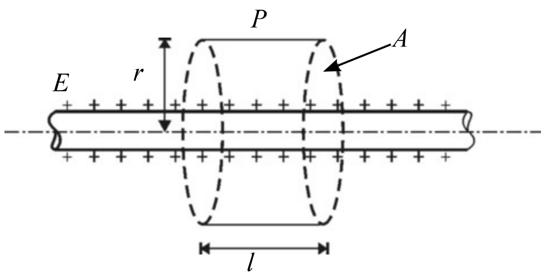


Figure 2.9

Point P is situated at a distance *r* from a charged long thin wire of which the linear density of charge is λ (cm^{-1}).

Consider a cylindered Gaussean surface of which the axis is the axis of the wire and radius is *r*. Also let Δl be the length of this surface. Point P will be on the curved surface of this Gaussean surface and electric flux lines emerging normally in all directions from the axis of the wire will cross the curved surface normally.

Then, the charges on the element Δl of the wire inside the Gaussean surface will be $Q = \lambda.\Delta l$. By considering the electric flux crossing the curved surface of the Gaussean surface if *E* is the electric field intensity on this surface, then by Gauss' theorem,

$$EA = \frac{Q}{\varepsilon}$$

$$E \times 2\pi r\, l = \frac{\lambda l}{\varepsilon}$$

$$E = \left(\frac{\lambda}{2\pi\varepsilon}\right) r$$

Unit 6: Chapter Three

Electrostatic Potential

3.1 Electric Potential

Due to the force acting on a charge placed at any point to an electric field, it has a capacity for doing work. Hence the charge possesses a certain potential energy. The charge obtains this energy due to the electric potential of the point in the electric field on which the charge is placed. If the charge is released now it will get displaced to a point which is at a lower potential. As a result, the potential energy it possesses would get spent.

On the other hand, if the charge is to be displaced from a point at lower potential to a point at higher potential, work has to be done on it against the field and the charge would gain potential energy.

The electric potential at a point in an electric field is measured by the work done in taking a unit positive charge from the point considered to be at zero potential up to that point. The most suitable point to be considered as that at zero potential is the one at an infinite distance away completely out of the region of the field.

Accordingly electric potential can be defined as follows.

"The electric potential at a point in an electric field is the work done taking a unit positive charge (+1 C) from infinity up to that point."

The unit of the electrostatic potential is J C^{-1}.

J C^{-1} can be given as V (volt) too.

3.2 Electric potential at a point in an electric field surrounding a point charge.

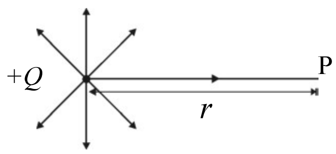


Figure 3.1

The point P is situated in the field around the point charge *Q*, at the distance *r* from the charge.

Force on a charge of + 1 C situated at a distance *r* from the charge *Q*,

$$F = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Q}{r^2}$$

Work done in taking this charge a distance Δx towards charge Q,

$$\Delta W = F \times (-\Delta x)$$
$$= \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{Q}{x^2}\ (-\Delta x)$$

Total work done in taking the charge of +1 C from $x = \infty$ to $x = r$,

$$W = \Sigma \Delta W$$
$$= \sum_{x=\infty}^{r} \frac{1}{4\pi\varepsilon}\frac{Q}{x^2}\ (-\Delta x)$$
$$= \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r}\ \text{(can be proved)}$$

$\therefore$ Electric potential at point P, $V = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r}$ (a scaler quantity)

Charge $+Q$ creates a positive potential while the charge $-Q$ creates a negative potential.

Variation of electric potential with the distance from a point charge can be represented graphically as follows.

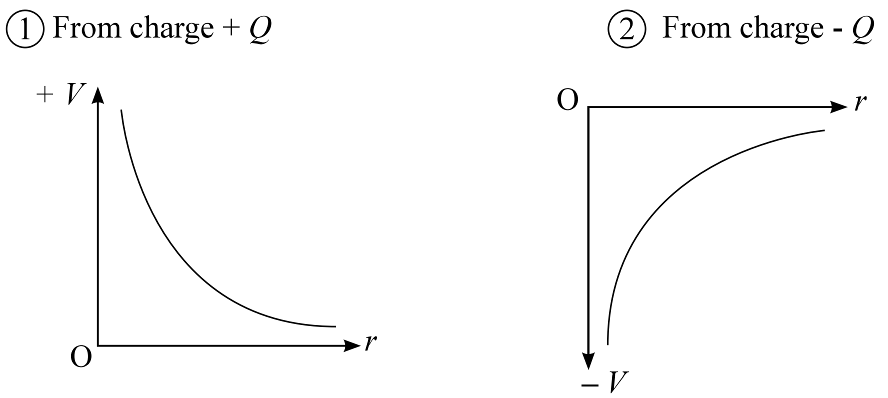


Figure 3.2

3.3 Potential energy of a charge placed in an electric field

The potential V at a point in an electric field means that the work done in taking a unit charge from infinity up to that point is V. Hence the work done in taking a charge q from infinity up to that point is $W = Vq$

This work is stored as potential energy of that charge.

$\therefore$ Potential energy of a charge q placed at a point of potential V in an electric field, $W = Vq$

3.4 Potential energy of a system of a pair of two charges

Figure 3.3

Point P is situated in the electric field due to the charge q_1 , at a distance r from the same charge.

$\therefore$ Electric potential at P, $V_p = \left(\frac{1}{4\pi\varepsilon}\right)\frac{q_1}{r}$

= Work done in taking a charge 1 C from infinity up to P

$\therefore$ Work done in taking a charge q_2 from infinity up to P, $W = \left(\frac{1}{4\pi\varepsilon}\right)\frac{q_1}{r}\ q_2$

This work gets accumulated in q_2 as potential energy and since this was made possible due to the contribution of charge q_1, this energy is considered as the potential energy of the pair q_1, q_2 of the charges.

$\therefore$ Potential energy of a pair of charges q_1, q_2 situated at a distance r apart,

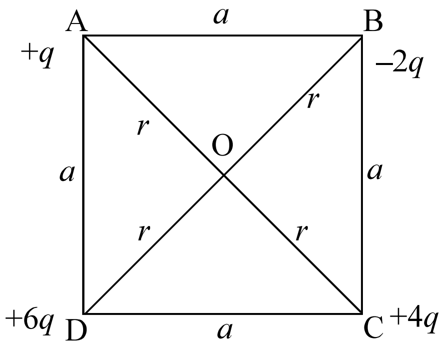
$$W = \left(\frac{1}{4\pi\varepsilon}\right)\frac{q_1 q_2}{r}$$

The total potential energy of a system consisting of a number of charges would be the sum of the potential energies of all positive pairs of charges in the system.

<u>Solved Problem</u>

1. Four charges + *q*, - *2q*, + *4q* and + *6q* are placed respectively on the corners of the square ABCD of side length '*a*'.

   If $a = 10$ cm and $q = 1.0$ μC, the electric potential at the mid point of the square is,



$$AO = BO = CO = DO = \sqrt{\frac{2a^2}{2}} = \frac{a}{\sqrt{2}}r$$

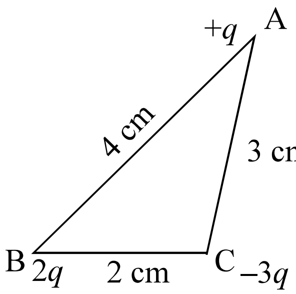
Since electric potential is a scalar quantity, the resultant potential at mid point O will be the algebraic sum of all potentials due to the charges of the four corners of the square.

$$V_O = \left(\frac{1}{4\pi\varepsilon_o}\right)\left(+\frac{q}{r} - \frac{2q}{r} + \frac{4q}{r} + \frac{6q}{r}\right)$$

$$= 9\times10^9\left(\frac{9q}{r}\right)$$

$$= \frac{9\times10^9\times 9\times 1.0\times 10^{-6}}{10\times10^{-2}/\sqrt{2}} = 81\times\sqrt{2}\times10^4 = \underline{\underline{11.5\times10^5\ \text{V}}}$$

2. On the three vertices of the triangle ABC of which AB = 4 cm , BC = 2 cm and AC = 3 cm are placed respectively three chargers + *q* , *+2q* and – *3q* each. If $q = 2$ μC find the total electrostatic potential energy of the system.



$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q\times2q}{4\times10^{2}} + \frac{2q\times-3q}{2\times10^{-2}} + \frac{-3q\times q}{3\times10^{-2}}\right)$$

$$= \frac{q^2}{4\pi\varepsilon_0}\times10^2\left(\frac{2}{4} - \frac{6}{2} - \frac{3}{3}\right)$$

$$= \frac{(2\times10^6)^2\times9\times10^9}{10^{-2}}\left(\frac{1}{2} - 3 - 1\right)$$

$$= 4\times9\times10^{-1}\times\frac{7}{2} = \underline{\underline{12.6\ \text{J}}}$$

3.5 Potential difference between two points in an electric field

The potential difference between two points in an electric field is the work done in taking a unit positive charge from one point to the other (This work is independent of the path along which the charge is carried).

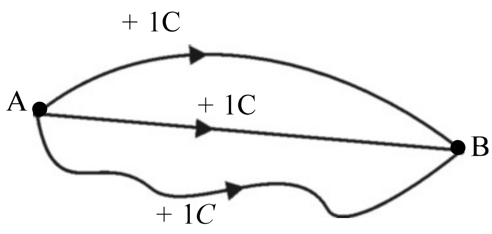
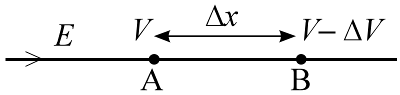


Figure 3.4

For example if V_A is the potential at a point A and V_B is the potential at a point B in an electric field ($V_A > V_B$) then,

the potential difference between A and B, $V_{AB} = V_A - V_B$

Suppose that the two points A and B are situated at a very small distance Δx apart along a line of force and that the electric field intensity remains constant at E along this range.



If V is the potential at A and ΔV the potential difference between A and B,

work done in taking a unit positive charge from A to B,

ΔW = Force × displacement = Charge of potential

$$E \times 1 \times \Delta x = (V - \Delta V) - V$$

$$E = \frac{-\Delta V}{\Delta x}$$ (-ve sign introduced to imply that work is done by the field)

The quantity $\frac{\Delta V}{\Delta x}$ is known as the potential gradient of the electric field. Hence according to the above expression, the electric field intensity at any point in an electric field is equal to the potential gradient at that point.

As the distance between A and B reaches the closest limit the expression is given is as

$$E = \frac{-dV}{dx}$$

Along a range of distance x in a uniform electric field, this relation can be expressed as

$$E = -\frac{V}{x}$$

Accordingly, the electric field intensity gets an additional unit as V m^{-1}.

The unit of measuring electrostatic potential is the 'volt' (V) and it is defined as follows. "If the work done in taking a unit charge (1 C) from some point in an electric field to another is one joule (1 J), then the potential difference between the two points is one volt.

The variation of electric potential with the distance measured from a specific point in an electric field can happen in various ways. When the potential is plotted against the distance, the gradient of the curve obtained represents the potential gradient along the field. Also it explains the variation of the electric field along the field.

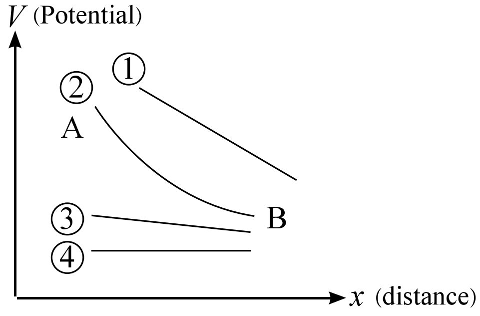


Figure 3.5

(1) A strong uniform field

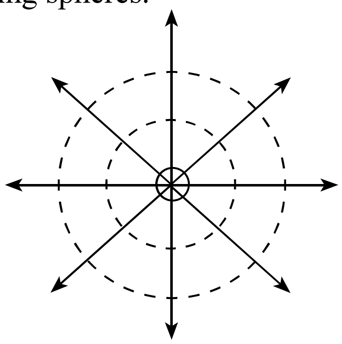
(2) Strong at the beginning, but gets weaker with increasing distance.

(3) A weak uniform field

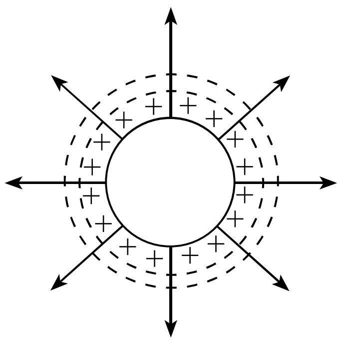
(4) Gradient is zero. Hence potential remains constant. No field exists in the given direction.

If the electric potential along a certain surface remains a constant, then that surface is called an "equipotential surface". Since an electric field does not exist along an equipotential surface no work is needed to carry a charge along such a surface. However, an electric field can act normal to an equipotential surface.

Such equipotential surfaces exist around point charges as well as around charged conducting spheres.



Equipotential surface around a positive point change
(a)



Equipotential surface around a positively charged conducting sphere
(b)

Figure 3.6

The unit for measuring potential difference is the "volt" while another unit called the "electron volt" is used to measure very small amounts of work and energy, specially such as those in the measurement of radiant energy. An electron volt is the work done in taking an electron across a potential difference of one volt.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.60 \times 10^{-19} \text{ J}$$

3.6 Electric potential due to a charged conducting sphere

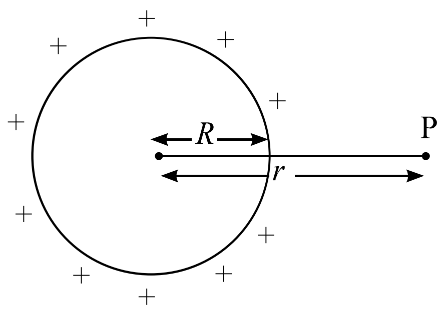


Figure 3.7

When a conducting sphere is given an electric charge, we already know that this will reside only on the outer surface of the sphere. Consider such a sphere of radius R and a point P situated at a distance r from its centre.

(1) When point P is situated outside the sphere ($r > R$)

It is known that in this case, with respect to point P, the charge Q on the sphere acts as if it were concentrated at the centre of the sphere.

Hence, when $r > R$ the potential at point P,

$$V = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{r}$$

(2) When point P is situated on the sphere ($r = R$)

The above argument for $r > R$ is valid up to the surface of the sphere.

Hence ,when $r = R$ the potential at point P,

$$V = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{R}$$

(3) When point P is inside the sphere ($r < R$).

Since there is no charge inside the sphere the field intensity inside too is zero.

$$E = 0$$

$\therefore$ Potential gradient inside the sphere $\frac{dV}{dr} = 0$

Thus starting from the surface of the sphere, the potential towards the inside remains unchanged and is constant.

$\therefore$ Potential at all points inside the sphere,

V = Potential at surface of sphere

$$V = \left(\frac{1}{4\pi\varepsilon}\right)\frac{Q}{R}$$

Variation of electric potential of a charged sphere with the distance from the centre is shown in Figure 3.8.

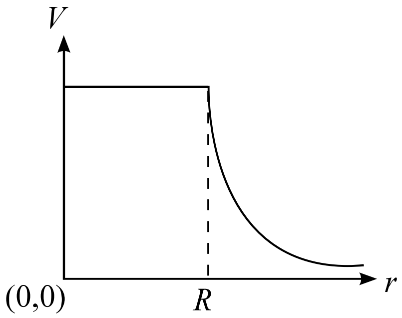


Figure 3.8

Unit 6: Chapter Four

Electric Capacitance

4.1 Electrostatic Induction

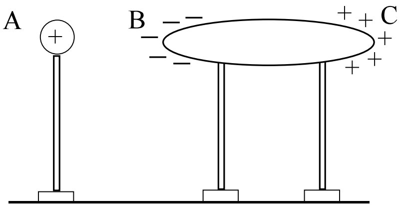


Figure 4.1

A is a positively charged body fixed on an insulating handle. BC is an uncharged body fixed on insulating stands.

When A is placed near BC as shown, what happens ?

The near end B of BC will acquire a negative (-) charge while the remote end C gets a positive (+) charge. If the sections B and C of BC can be separated, a negatively charged body B and a positively charged body C can be obtained.

This phenomenon of an uncharged body getting charged instantly when placed near a charged body is called **"electrostatic induction".**

It has been shown by scientist Michael Faraday that the magnitude of the two induced charges at B and C here are separately equal to the magnitude of the inducing charge in A.

4.2 Condensers (Capacitors)

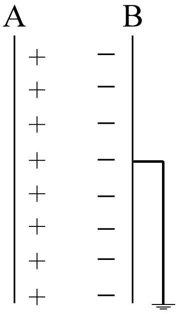


Figure 4.2

A and B are two conducting plates placed parallel to each other. The two plates are connected to the two terminals of an electric source such as a battery, to get charged, and later the battery is removed and the plate B is earthed.

Or else only A is positively charged. Then B is charged by induction and earthed to leave it negatively charged.

Now this system of two charged plates, A and B is a store of electric charges. This store of charges is called a condenser or a **capacitor.**

Hence, a capacitor is an electric device which stores static electric charges. As a current of electric charges from a battery is used for various activities (involving the generation of mechanical energy and thermal energy etc). Static electric charges stored in capacitors too are used for various purposes. For example, the capacitor is an essential component in electric equipment such as radios, TV s, computers and various other communication equipment.

Shown in the figure 4.2 is an illustration of the first capacitor, known as a **"parallel plate capacitor"**. The modern capacitor has changed in shape and reduced in size as to that of a drug capsule. However all components of these capacitors satisfy the conditions satisfied by those of a parallel plate capacitor. That is, each of these capacitors has an electric insulator pressed between two electric conductors.

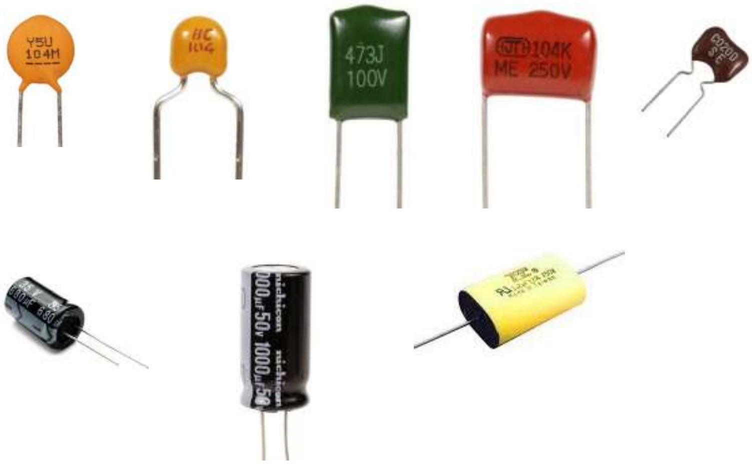


Figure 4.3 – Practical capacitors

The variable capacitors shown in Figure 4.3 serve useful purposes in those such as radios, TVs etc. By altering the capacitance of the capacitor, the amount of charge stored in it can be varied enabling such as location of channels etc.

The medium between the plates of those capacitors can be either air or any other insulating medium and this medium is known as the **'dielectric'**. In the modern capacitor, a sheet painted with paraffin wax is pressed between two thin metal sheets and rolled to serve this purpose.

4.3 Capacitance of a capacitor

The capacitance of a capacitor can be considered as a measure of its ability to store electric charges. It can be defined as follows.

$$\textit{Capacitance of a capacitor } (C) = \frac{\text{Charge on one plate } (Q)}{\text{Potential difference between the plates } (V)}$$

$$C = \frac{Q}{V}$$

Hence the capacitance of a capacitor is the ratio its charge subtending to the potential difference between its plates.

Unit : farad (F)

$$\text{farad } 1 = \frac{1 \text{ coulomb}}{1 \text{ volt}}$$

Practical unit : μF

$$1\ \mu F = 10^{-6}\ F$$

4.4 Charging and discharging of a capacitor

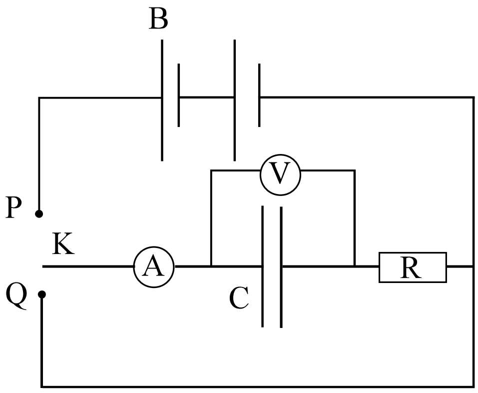


Figure 4.4

A circuit as shown in figure 4.4 can be used to charge and then to discharge a capacitor. C is the capacitor of about 100 μF which is to be charged. R is a high resistor in the range of mega ohms (MΩ). K is a two way switch and B is a direct current source of about 6 V. Microammeter A and voltmeter V while suits the voltage of source B are connected in the circuit as shown.

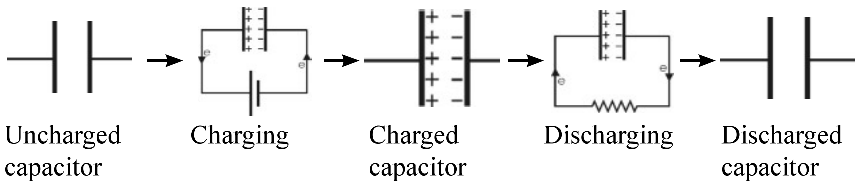


Figure 4.5

When the switch K is connected to the terminal P, the capacitor C begins to get charged. The reading of the voltmeter increases gradually while the microammeter after indicating a maximum reading gradually decreases to zero. Meanwhile the voltmeter showing maximum reading indicates that the capacitor is charged to the maximum. The reason why the microammeter now indicates zero is that when the capacitor is charged to the maximum, the circuit gets disconnected.

Now when the switch K is connected to terminal Q, the microammeter indicates a current in the direction opposite to that of the initial direction and this current too gradually reduces to zero. If there is no leakage, the voltmeter reading too which remained constant so far gradually reduces to zero indicating the discharge of the capacitor.

In the above charging process, electrons flow from the negative terminal of the electric source to the plate of the capacitor connected to it. At the same time, electrons flow from the other plate of the capacitor to the positive terminal of the source. So that one plate of the capacitor gets negatively charged and the other positively charged in this manner.

During the discharge, negative charges (electrons) flow from the negatively charged plate of the capacitor to the positive plate through the resistance R. This takes place until the positive charges of the positive plate are completely neutralized.

The variations of the voltmeter and the ammeter readings during charging and discharging are illustrated graphically below.

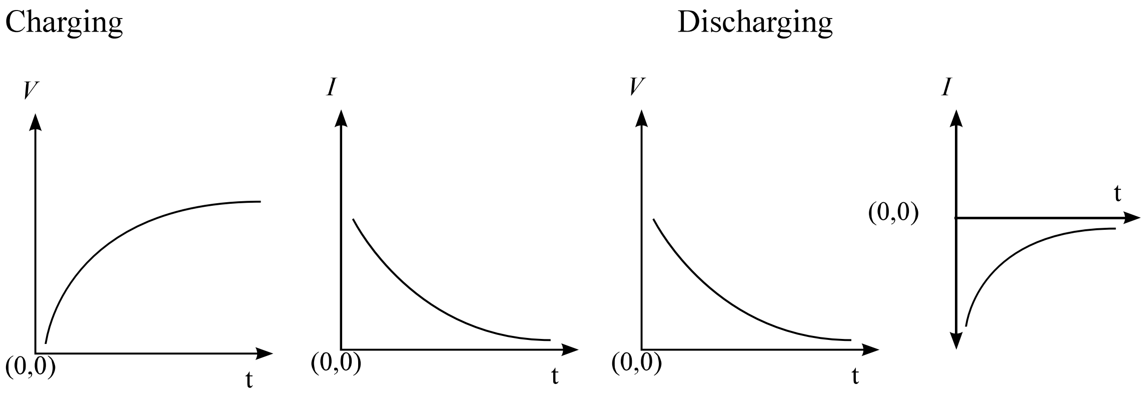


Figure 4.6

The graphs in Figure 4.6, clearly indicate that capacitor allows instant current to flow in both directions. That implies that the capacitor allows the alternate current to flow through it. Usually capacitors are used in AC circuits but not in DC circuits.

4.5 Parallel plate capacitors

The parallel plate capacitor can be considered as the first capacitor to be planned and constructed. The practical capacitors which are in use too function according to the same principle.

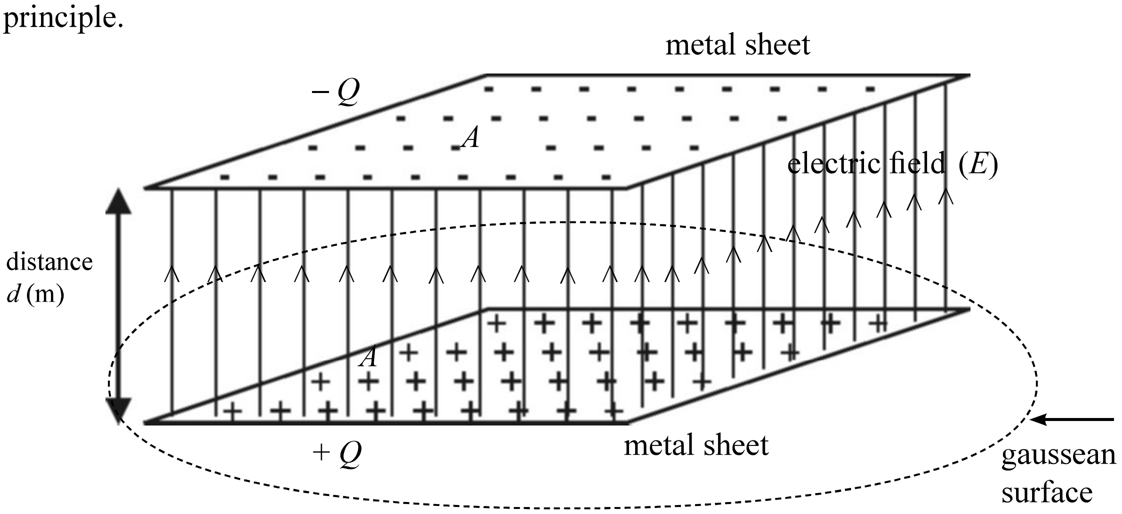


Figure 4.7

Consider a capacitor consisting of two parallel plates each of area. *A* placed at a distance *d* apart and having a charge *Q* spread uniformly over each one as shown.

It can be assumed that electric flux lines emitting from plate + *Q* pass as a parallel bunch to the plate - *Q* (Deviation of flux lines outwards at the edges of the plates can be neglected) Considering a Gaussean surface passing symmetrically between the plates normally to flux lines and covering one plate only as shown in Figure 4.7.

According to Gauss' theorem

$$AE = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{A\varepsilon_0}$$

But E = potential gradient = $\frac{V}{d}$

$$\therefore \quad V = Ed$$

$\therefore$ Capacitance of the capacitor $C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q}{\frac{Q}{A\varepsilon_0} \times d} = C = \frac{\varepsilon_0 A}{d}$

If another medium of relative permitting ε_r exists between the plates,

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Expressing $\varepsilon_r = k$ (dielectric constant), $C = \frac{K\varepsilon_0 A}{d}$

For free space $k = 1$ while for other media $k > 1$.

4.6 Systems of capacitors

4.6.1 Capacitors in series

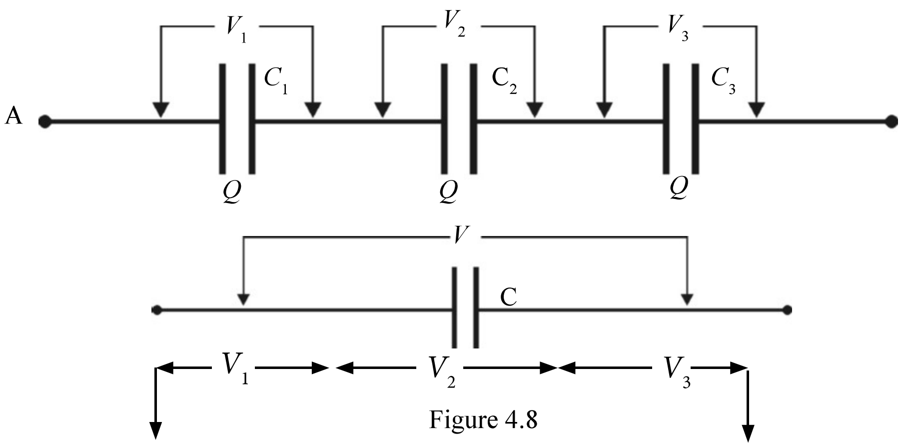


Figure 4.8

If a number of capacitors are connected so that they carry the same charge then, these capacitors are said to be in series.

The capacitance of the single capacitor which keeps the same charge and maintains the total potential difference across the whole system of capacitors is called the "Equivalent capacitance" of the system.

$$V_1 = \frac{Q}{C_1} \quad \text{———} (1)$$

$$V_2 = \frac{Q}{C_2} \quad \text{———} (2)$$

$$V_3 = \frac{Q}{C_3} \quad \text{———} (3)$$

① + ② + ③ , $V_1 + V_2 + V_3 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$ ——— ④

If C is the equivalent capacitance,

$$V_1 + V_2 + V_3 = \frac{Q}{C} \text{ ——— ⑤}$$

④ and ⑤ , $\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$$\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

4.6.2 Capacitors in parallel

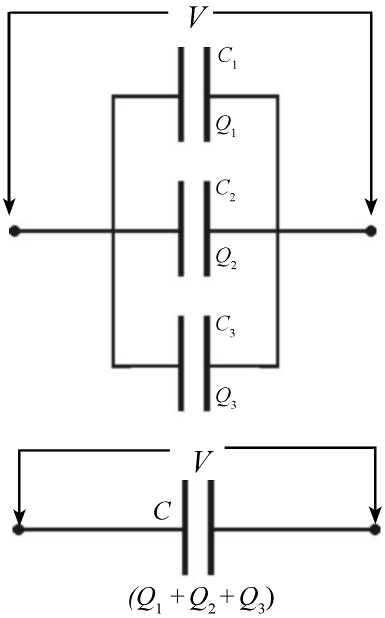


Figure 4.9

If a number of capacitors are connected so that they maintain the same potential difference across them, the capacitors are said to be connected parallel to each other.

The capacitance of the single capacitor which while keeping the total charge of the system maintains the same potential difference across it, is called the equivalent capacitance of the system.

$$Q_1 = C_1 V \text{ ——— ①}$$

$$Q_2 = C_2 V \text{ ——— ②}$$

$$Q_3 = C_2 V \text{ ——— ③}$$

① + ② + ③ $Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)\, V$ ——— ④

If C is the equivalent capacitance,

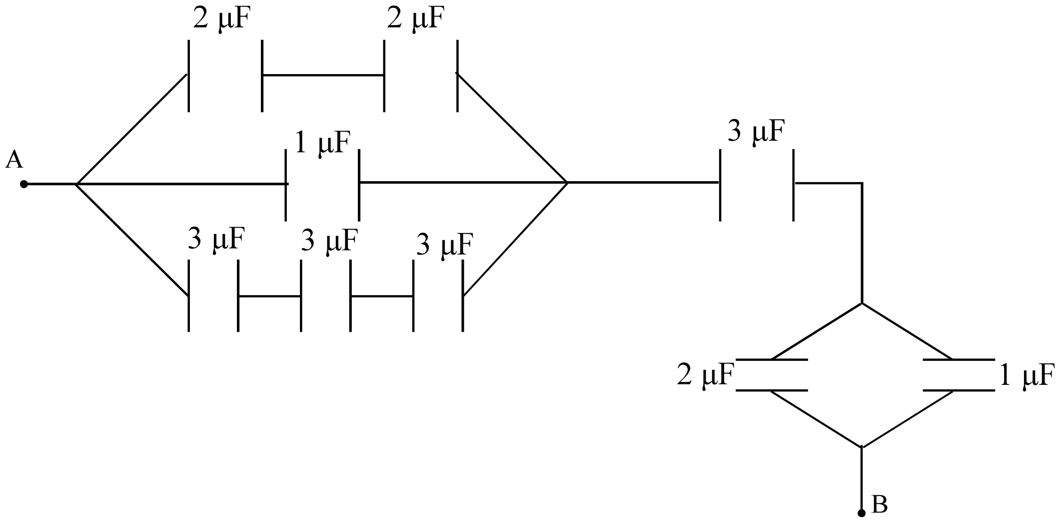
$$Q_1 + Q_2 + Q_3 = CV \text{ ——— ⑤}$$

From ④ and ⑤ , $CV = (C_1 + C_2 + C_3)\, V$

$$C = C_1 + C_2 + C_3$$

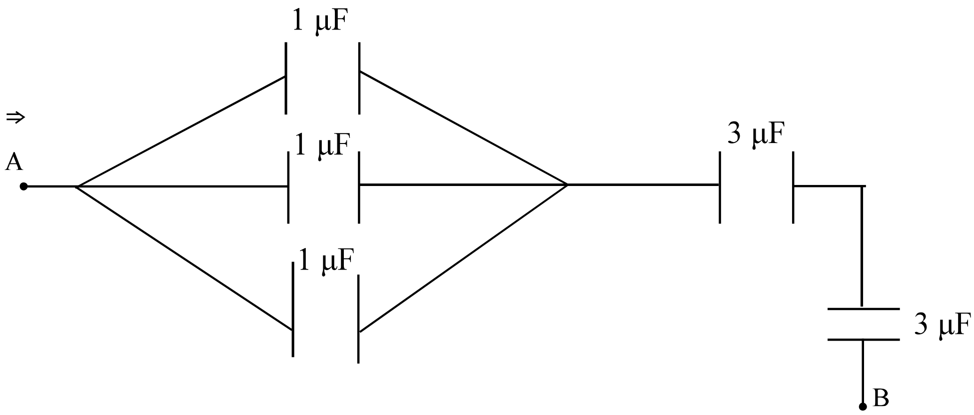
Solved Problem

Calculate the equivalent capacitance between A and B

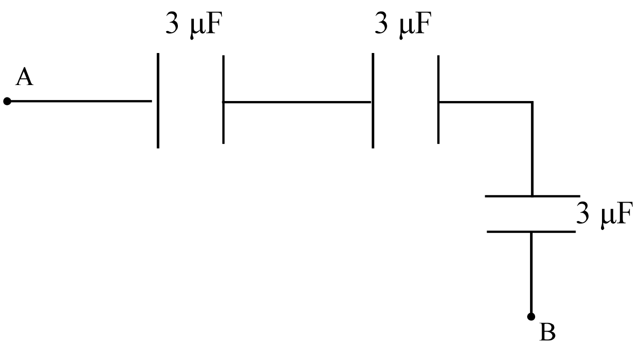


Answer

⇒



⇒



$$\Rightarrow \frac{1}{C} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

Equivalent capacitance $C = 1\mu F$

4.7 Capacitance of a conducting sphere

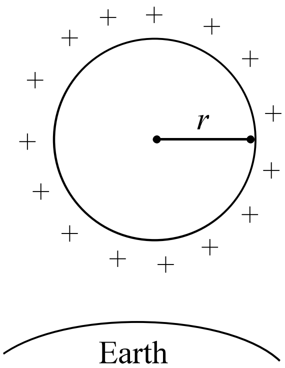


Figure 4.10

A conducting sphere of radius r is charged with a charge $+Q$ (whether the sphere is solid or hollow, the charge would be distributed uniformly over the outer surface only). In order to consider this sphere as a capacitor, let us assume that the Earth is the earthed component of the capacitor.

Then the potential of the charged sphere, $V_1 = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r}$

potential of the Earth, $V_2 = 0$

$\therefore$ Capacitance of the spherical capacitor $C = \frac{Q}{V} = \frac{Q}{V_1 - V_2}$

$$C = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r} - 0} = 4\pi\varepsilon_0 r$$

$$C = 4\pi\varepsilon_0 r$$

4.8 Energy stored in a charged capacitor

When a charged capacitor is discharged, the emission of a spark can be observed between its plates. This clearly shows that a charged capacitor contains energy and this energy gets converted to other forms such as heat during the discharge.

According to $Q = CV$, in a capacitor $Q \propto V$. Hence the graph of V against Q is a straight line and the variation of V when a small charge of Δq each is carried from one plate to another until it builds up to Q is illustrated by the graph.

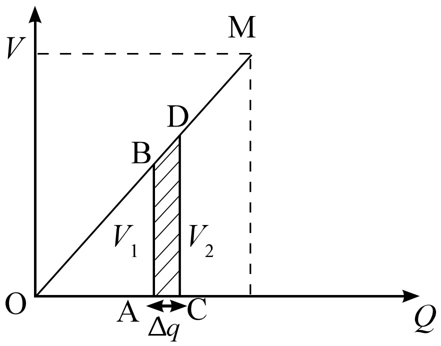


Figure 4.11

In the situation when the potential is V_1, if ΔW is the work done when a small charge Δq is carried from the first plate to the other,

$$\Delta W = \text{potential} \times \text{charge}$$
$$= V \times \Delta q$$
$$= \left(\frac{V_1 + V_2}{2}\right)\Delta q$$
$$= \left(\frac{AB + CD}{2}\right)AC$$
$$= \text{Area of trapezium ABCD}$$

When the charge Q is built up in this manner by small charges of Δq, the potential becomes V and the total work done will be represented by the sum of the areas of small strips such as ABCD and this area will be equal to the area of the triangle OLM.

Energy stored in the capacitor

W = total work done

$W = \Sigma\Delta W$

$W = OLM$

$W = \frac{1}{2}\,OL.LM$

$W = \frac{1}{2}\,QV$

Substitute $Q = CV$ and $V = \frac{Q}{C}$, $W = \frac{1}{2}\,QV = \frac{1}{2}\,CV^2 = \frac{1}{2}\,\frac{Q^2}{C}$

Solved problem

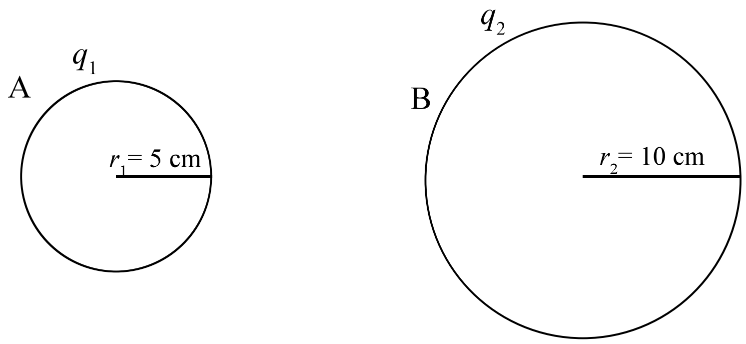
Two conducting sphere of radius $r_1 = 5$ cm and $r_2 = 10$ cm are charged as $q_1 = +70$ μC and $q_2 = +20$ μC respectively. When these two spheres are connected by a short piece of wire of negligible capacitance, find

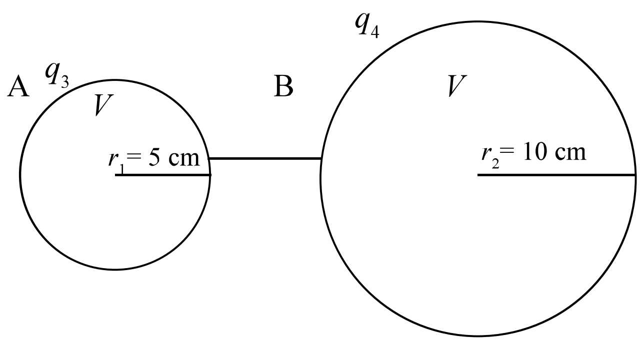
(i) the amount of charge remaining in each sphere.
(ii) the loss of energy that takes place.

Answer

Capacitance of sphere A $q_2 = +20$ μC
Capacitance of sphere B $q_1 = +70$ μC

When the two spheres are connected by a conductor they reach the same potential V and let the charges that remain in the two spheres be q_3 and q_4 respectively.





(i) Then, $V = \frac{q_3}{C_1} = \frac{q_4}{C_2}$

$$\Rightarrow \frac{q_5}{q_4} = \frac{C_1}{C_2} = \frac{r_1}{r_2} = \frac{5}{10} = \frac{1}{2}$$

Also, $q_3 + q_4 = q_1 + q_2$

$$q_3 + q_4 = 70 + 20 = 90$$

$$\Rightarrow \frac{q_3}{q_4} = \frac{1}{2} \text{ ——— (1)}$$

$$q_3 + q_4 = 90 \text{ ——— (2)}$$

Solving, $q_3 = 30\ \mu C$ $q_4 = 60\ \mu C$

(ii) Initial energy $E_1 = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$

Final energy $E_2 = \frac{1}{2}\frac{q_3^2}{C_1} + \frac{1}{2}\frac{q_4^2}{C_2}$

$\therefore$ Loss of energy $E = E_1 - E_2$

$$= \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2} - \left(\frac{1}{2}\frac{q_3^2}{C_1} + \frac{1}{2}\frac{q_4^2}{C_2}\right)$$

$$= \frac{1}{2}\left\{\frac{q_1^2 - q_3^2}{C_1} - \frac{q_4^2 - q_2^2}{C_2}\right\}$$

$$= \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\left\{\frac{(70\times10^{-6})^2 - (30\times10^{-6})^2}{5\times10^{-2}}\right\} - \frac{(60\times10^{-6})^2 - (20\times10^{-6})^2}{10\times10^{-2}}$$

$$= \underline{\underline{216\text{ J}}}$$

4.9 Charge distribution over a conducting surface

It is already known to us that when a conducting body is charged the charge will reside only on the outer surface of the body, whatever the shape of the body is. However the charge distribution or the way it is spread over the surface depends on the curvature or the amount it is curved. Hence a uniform charge distribution cannot be expected over a conducting surface with a varying curvature.

The charge distribution over a surface is measured by the surface density of the charge which is the amount of charge on unit area of the surface. It can be shown experimentally that when the curvature of a surface changes its surface density of charge too changes. It is such that when the curvature of a surface increases the surface density of charge on it increases while the curvature decreases the surface density decreases.

Accordingly, on a plane surface where the curvature is minimum the surface density of charge is minimum while when the curvature gradually increases and becomes spherical an increased uniform surface density of charge would exit.

The maximum surface density would exit on a pointed surface.

The distribution of charges on a conducting surface in the shape of a lotus bud can be shown as follows.

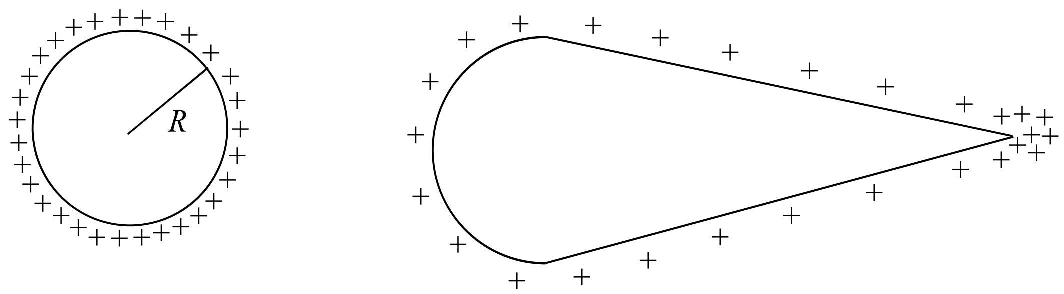


Figure 4.12

A uniform distribution of charges exists only around a conducting sphere. Due to high charge density on a pointed conducting surface, the electric field intensity around it becomes very high. As a result the insulating property of the air around the point falls and the molecules in the air get ionized. Then those surrounding air molecules which have acquired a charge equal to that of the charge on the pointed surface get emitted as a stream. If this phenomenon takes place in the dark the air around the point appears as a bright covering which is known as a "corona discharge".

Unit 7

Magnetic Field

Unit 7: Chapter One

Magnetic Force

1.1 Introduction

Evidence exists that magnetism was a property which had been identified by man since thousands of years ago. The first practical experience in this aspect can be cited as the invention of the compass needle by the Chinese and the subsequent identification that the Earth behaves as a weak magnet. This property possessed by Earth was named as natural magnetism.

As the compass needle always sets itself along the North – South line of the Earth, the end of the needle that points towards the North was named the North pole of the needle while the other end pointing towards the south of Earth was termed the South pole of the needle. The compass needle can be considered as the first artificial magnet and with the progress of science, the construction of artificial magnets such as bar magnets, horse shoe magnets etc had begun. It was also observed that during scientific experiments a compass needle which was moved around a current carrying a conductor was continually changing its direction.

Magnetic forces are also found to exist around the Earth and around artificial magnets in addition to around currents carrying conductors and hence these regions were named as "magnetic fields ". According to modern concepts, the forces which act in magnetic fields are a result of the interaction between two magnetic fields which intersect with each other.

1.2 Force acting on a current conducting wire placed in a magnetic field

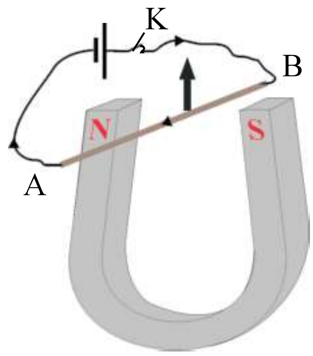


Figure 1.1

The conducting wire AB connected in series with a cell and a switch (K) is placed in the space between the strong poles of a horse – shoe magnet. When the switch K is closed the conductor (AB) will be thrown aside Why ? When the current starts flowing through the conductor AB a magnetic field is created around it. When this magnetic field intersects with the magnetic field between the poles of the horse – shoe magnet an interaction takes

place between the fields and as a result, a force acts on the conductor which throws the conductor out.
The strength of a magnetic field is measured by the quantity called "magnetic flux density. By considering the force on an element of the current carrying conductor above placed in the magnetic field of the horse – shoe magnet, an expression to define " magnetic flux density " can be obtained.

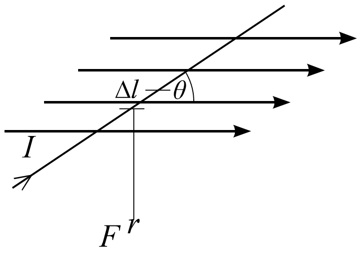


Figure 1.2

Suppose that the conductor is inclined at an angle θ to the magnetic field. It has been experimentally proved that the force on the element Δ*l*,

$F \propto I$
$F \propto \Delta l$
$F \propto \sin\theta$

$\Rightarrow F \propto I\Delta l \sin\theta$
$F = BI\Delta l \sin\theta$
where B is the constant of proportionality

$$B = \frac{F}{I\Delta l \sin\theta}$$

$$when\ \theta = 90°, \quad B = \frac{F}{I\Delta l}$$

The quantity *B* above represents the "magnetic flux density" at Δl and using the above expression, the "magnetic flux density" can be defined as follows. In this definition the quantity $I.\ \Delta l$ is introduced as a " **current element** ".

1.3 Magnetic flux density

The force acting on a unit current element of a current carrying conductor placed normally to a magnetic field is the magnetic flux density of the field at that point.

Unit : N A^{-1} m^{-1} or T (Tesla)

Force on a length *l* of a current carrying conductor placed at an angle θ to a magnetic field of flux density *B* can be expressed as,

$$F = IBl \sin\theta$$

When $\theta = 90°$ then $F = BIl$
This is the maximum force relevant to the given current.

When $\theta = 0°$ then $F = 0$
That is when the conductor is parallel to the magnetic field, no force acts on it even if a current passes through the conductor.

Magnetic flux models are made use of in the study of magnetic fields and their effects.

Example :

1. Magnetic field between the poles of a horse – shoe magnet

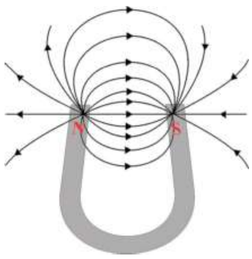


Figure 1.3

2. Magnetic field between the poles of a bar magnet

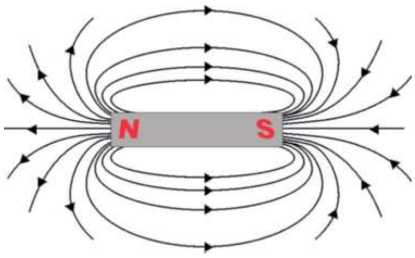


Figure 1.4

Magnetic lines of force (or flux lines) start from the North pole of the magnet and terminate at the south pole of the magnet.

3. Magnetic field around a straight conductor carrying a current

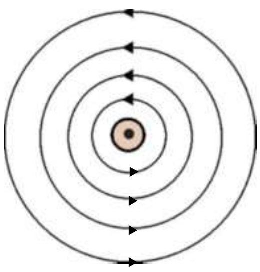


Figure 1.5

The direction of the concentric circular lines of force (flux) can be decided by Maxwell's right handed corkscrew rule.

4. Magnetic field around a circular coil carrying a current

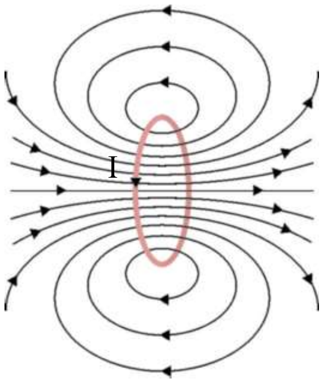


Figure 1.6

5. Magnetic field around a solenoid carrying a current

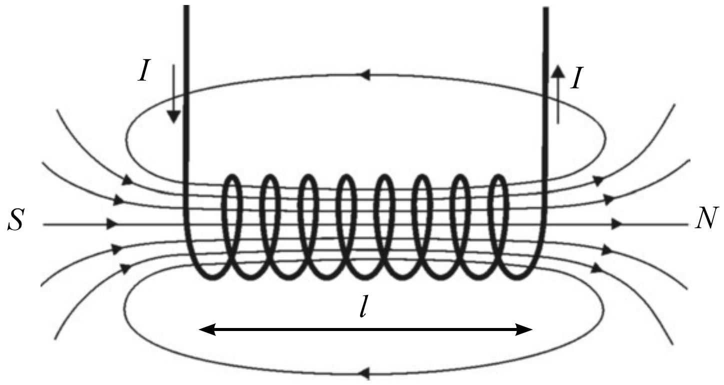


Figure 1.7

Deduction of the direction of the force on a current carrying conductor placed in magnetic field.

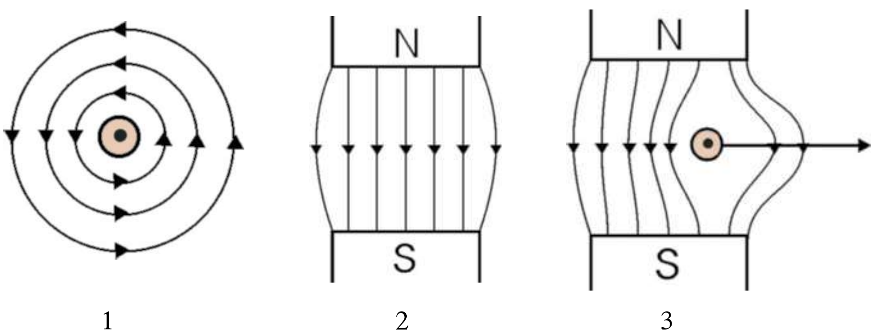


Figure 1.8

(1) Magnetic field around a long straight wire carrying a current.
(2) Magnetic field between the poles of a strong horse – shoe magnet.
(3) The composite field formed when the current carrying conductor is placed between the poles of the magnet.

According to the structure of the composite field, the two fields have added with each other to form a stronger field on the left side of the conductor while on the right side of the conductor the two fields oppose each other resulting a weaker field. The excess pressure on the left side of the conductor builds up a force (F) on the conductor towards the right and if the conductor is free to move it will move towards the right.

Described above is an energy conversion, electrical energy being converted into mechanical energy. Starting from the electric motor most machinery functioning from electricity are based on this principle. It is important to know that in this conversion the presence of a magnetic field is essential.

In the above energy conversion, there are three quantities normal to each other. They are the electric current, the magnetic field and the final result of the mechanical force or the motion. In order to decide the direction of the resulting force or motion, the following rule can the used.

1.4 Fleming's left hand rule

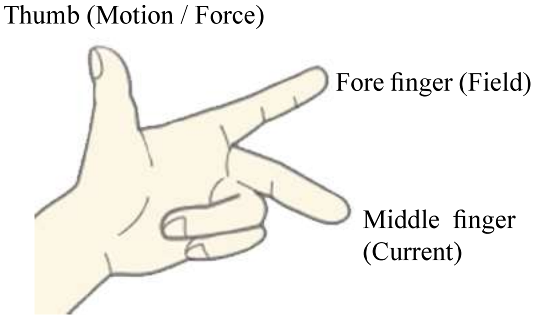


Figure 1.9

If the fore finger, the middle finger and the thumb of the left hand are stretched normally to each other with the fore finger indicating the direction of the magnetic field and the middle finger indicating the direction of the electric current through the conductor across it then the thumb indicates the direction of the force on the conductor or of its motion.

1.5 Force on an electric charge moving in a magnetic field

No force acts on an electric charge which is at rest in a magnetic field. However if the charge is moving it is equivalent to an electric current and hence a force would act on it.

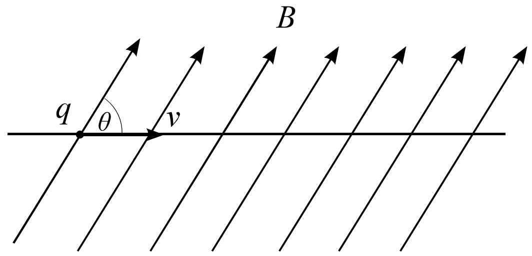


Figure 1.10

Consider a charge q moving with a velocity v in a direction inclined at an angle θ to a uniform magnetic field of flux density B.

If this charge moves a distance l in a time t,

$$l = vt$$

If the moving charge is considered to be equivalent to a current I flowing along a conductor of length l placed along the direction of the motion of the charge, then the force acting on it,

$$F = BIl \sin \theta$$

$$= \frac{Bq}{t} Vt \sin \theta$$

$$F = BqV \sin \theta$$

If $\theta = 90°$ then $F = BqV$

If the charge moves parallel to the field,

$$F = 0$$

1.6 Hall effect

Suppose a current carrying conductor is placed in a magnetic field as shown in the Figure 1.11. Then a force will act downwards on the conductor.

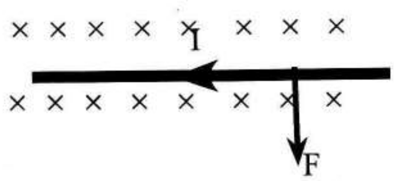


Figure 1.11

The force on a conductor is due to a downward forces on the electrons.

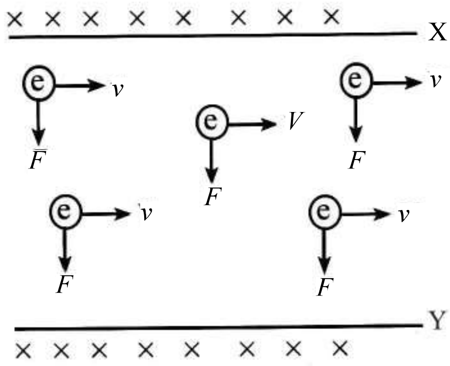


Figure 1.12

Consider a rectangular block of a conducting material across which passes a uniform magnetic field of flux density B as shown in the Figure 1.13. Suppose a current I passes through the block in a direction normal to the direction of the field B. Then the charge carriers of the current, the electrons would drift in the same path in the direction opposite to the conventional direction of the current I.

Applying Fleming' s left hand rule according to the conventional direction of I, it becomes clear that forces act on the charge carriers, the electrons, vertically down words (↓), as a result of which the electrons deviate downwards and begin to collect on the lower surface Y of the block. This brings the lower surface (Y) to a negative potential relative to the upper surface X.

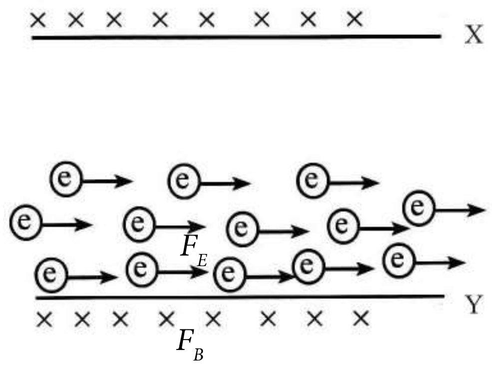


Figure 1.13

The deviation of the electrons to the surface Y stops when the force F_E which builds up with the developing electric field (E) along with the increasing potential difference between surfaces X and Y equals the force F_B which is formed due to the effect of the magnetic field B on the electrons

At this stage the potential difference between the two surfaces X and Y becomes a maximum and this maximum potential difference is known as "Hall voltage".

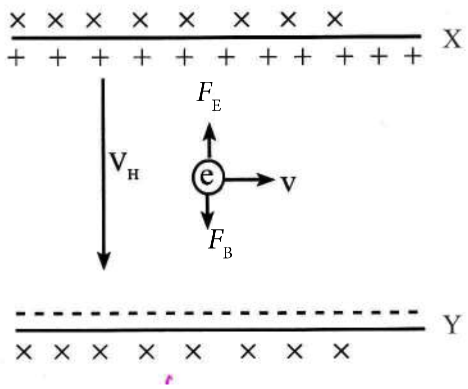


Figure 1.14

This phenomenon is known as Hall effect and was discovered by the scientist Edwin Hall in 1879 during his research activities.

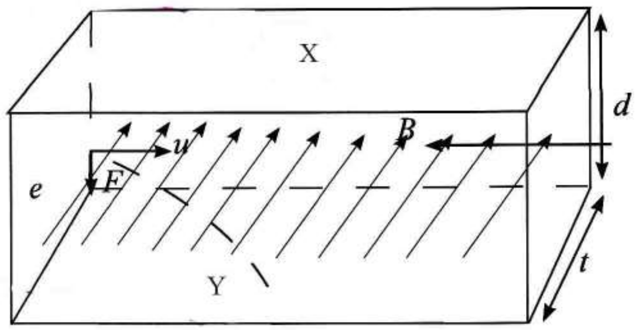


Figure 1.15

In order to deduce an expression for Hall voltage let d be the height of the conducting block and 't ' its thickness. Then the intensity of the electric field formed due to the potential difference created,

$$E = \text{Potential gradient } = \frac{V_H}{d}, \text{ where } V_H \text{ is the Hall voltage.}$$

Hence, the force acting vertically upwards on the negative charge of an electron

$$\uparrow F_e = Ee = \frac{V_H}{d}\ e$$

The downward deviation of the electrons stop when this force balances the downward force $\downarrow F_B = Bev$ acting on the electron.

$$\uparrow F_E = \downarrow F_B$$

$$\frac{V_H}{d}\ e = Bev$$

$$V_H = Bvd$$

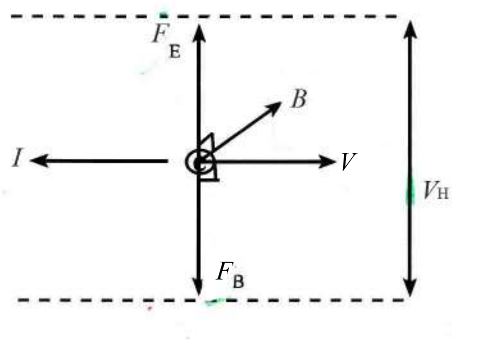


Figure 1.16

Also the relationship between the current I and the drift velocity

$$I = Avne$$

where A is the area of cross section of the block and n is the number of charge carriers (electrons) per unit volume of the block.

$$\therefore\ v = \frac{I}{Ane}$$

By substituting for v in the expression for V_H,

$$V_H = B.\frac{I}{Ane}.\ d = B.\ \frac{I}{dtne}.\ d$$

$$= \frac{BI}{net}$$

1.7 Applications of Hall effect

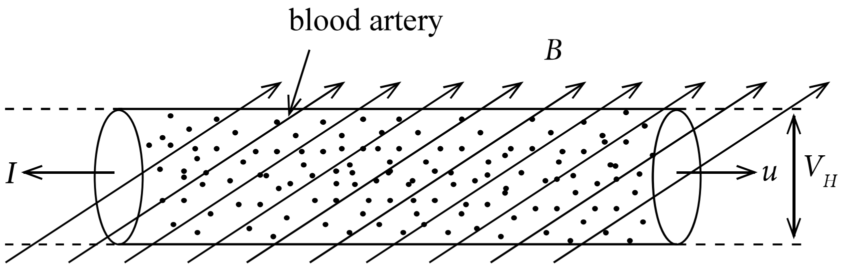
1. Hall effect is made use of to identify the charge carriers in semiconductors. That is whether a certain semiconductor is of p type or of n type. A semiconductor can be subjected to the Hall effect test and the direction in which the Hall voltage is building

up decided This enables to decide whether its charge carriers are negative electrons or positive holes. Hence it can be concluded whether the semiconductor is of n type or of p type.

2. Another important use of the Hall effect is the determination of the flux density of a magnetic field.

3. In the diagnosis of heart ailments doctors use an instrument called the " electromagnetic flowmeter " to examine the flow speed of blood through arteries. The action of this instrument is based on Hall effect.

Example



As shown in the above figure, a magnetic field of flux density *B* is activated normally across a blood artery which is being tested by an electromagnetic flow meter. Since Na^+ and $C1^-$ ions in the blood are transported along with the blood a Hall voltage builds up normally to the magnetic field across the artery. The value of the Hall voltage indicated by the flow meter is V_H. Assuming that the flow speed of the ions is equal to that of the blood, derive an expression for the flow speed of blood.
In the state of equilibrium.

Magnetic force (F_B) = Electric force (F_E)

If *u* is the flow speed of blood

$$Bqu = qE$$

$$u = \frac{E}{B}$$

But $$E = \frac{V_H}{d}$$

$\therefore$ $$u = \frac{V_H}{dB}$$

Unit 7: Chapter two

Magnetic Force Field

2.1 Introduction

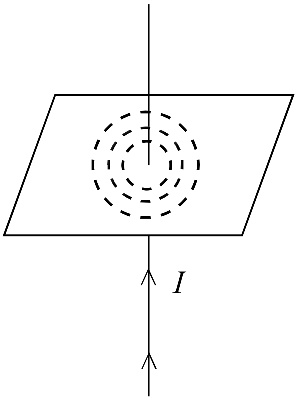


Figure 2.1

In order to confirm the presence of a magnetic field around a current carrying conductor the set up shown in Figure 2.1 can be used. A straight vertical conducting wire is passed through a horizontal sheet of cardboard by piercing it. Tiny amount of iron powder is spread on the cardboard around the wire. An electric current is now passed through the wire and at the same time the cardboard is tapped.

Then the iron powder will be found to set around the wire along concentric circular paths indicating the presence of a magnetic field and also its structure. The direction of the lines of force of this magnetic field can be decided as mentioned earlier, by Fleming's right handed cork – screw rule.

2.2 Biot – Savart's Law

This law leads to the obtaining of an expression for the magnetic flux density at a point in the magnetic field formed around a current carrying conductor.

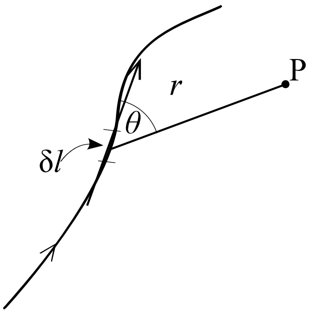


Figure 2.2

A current I flows through a conductor as shown in figure 2.2 and P is a point at a distance r from any element δl of the conductor. The straight line drawn from P to this element makes an angle θ with the element in the direction of the current.

It has been shown that due to the current element $I.\delta l$ the magnetic flux density at P,

$$\left.\begin{aligned} \delta B &\propto I \\ \delta B &\propto \delta l \\ \delta B &\propto \sin\theta \\ \delta B &\propto \frac{1}{r^2} \end{aligned}\right\}$$

$$\therefore \quad \delta B \propto \frac{I\,\delta l \sin\theta}{r^2}$$

$$\delta B = K\frac{I.\delta l \sin\theta}{r^2}$$, where K is a constant

The constant K is represented as $K = \frac{\mu_o}{4\pi}$, where μ_o is called the magnetic permeability of free space.

$\left(\frac{\mu_0}{4\pi} = 10^{-7}\text{Hm}^{-1}\right)$

$$\therefore \ \delta B = \left(\frac{\mu_0}{4\pi}\right) \frac{I.\delta l \sin\theta}{\text{r}^2}$$

Biot – Savart's Law is represented by the above result and it expresses the magnetic flux density at a point situated at a distance *r* from a current element *I* δ*l* of a current carrying conductor.

2.3 Maxwell's clockwise corkscrew rule

When a right handed corkscrew is passed in the direction of an electric current passing through a conductor, the sense of rotation of the screw represents the direction of the magnetic field formed around the conductor.

2.4 Derivations from Biot – Savart's Law

1.Magnetic flux density at a point around a very long straight conductor carrying a current.

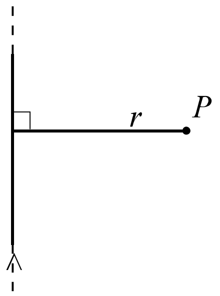


Figure 2.3

A very long conductor carries a current *I*. The perpendicular distance from an external point P up to the wire is *r*. Then the magnetic flux density at P due to the current passing through the whole conducting wire can be derived by using mathematical integration for Biot Savart's expression which leads to the expression for flux density at P as

$$B = \left(\frac{\mu_0}{4\pi}\right) \frac{2I}{r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(2). Magnetic flux density at the centre of a flat circular coil of wire carrying a current.

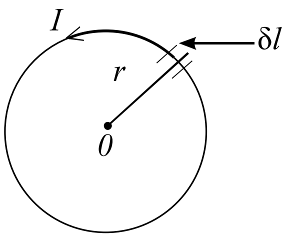


Figure 2.4

A flat circular coil of *N* turns and radius *r* carries a current *I*. According to Biot- Savart's law, the magnetic flux density at the centre of the coil due to any current element (*I*.δ*l*) of the coil is

$$\delta B = \left(\frac{\mu_0}{4\pi}\right) \frac{I.\delta l \sin 90^\circ}{r^2}$$

Since the magnetic flux at the centre of the coil due to all current elements of the coil pass normally to the plane of the coil (either upwards or downwards) , the resultant magnetic flux density at the centre of the coil,

$$B = \Sigma\, \delta B = \Sigma \left(\frac{\mu_0}{4\pi}\right) \frac{I.\delta l}{r^2}$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{I}{r^2} \Sigma\, \delta l$$

$$= \left(\frac{\mu_0}{4\pi}\right) \frac{I}{r^2}\, 2\pi r N$$

$$= \frac{\mu_0 N I}{2r}$$

(3). The magnetic flux density along the axis of a long solenoid carrying a current.

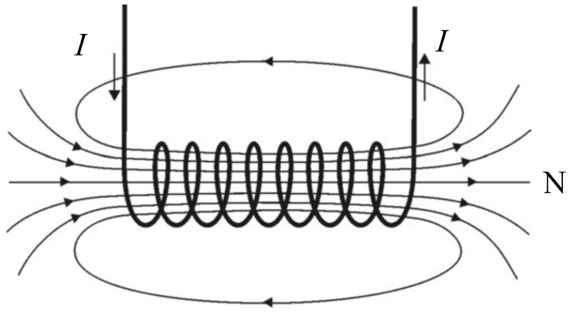


Figure 2.5

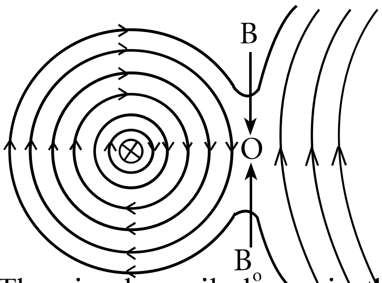
A long solenoid containing '*n*' turns of wire per unit length carries a current *I.* Using Biot-savart's law it can be proved that the magnetic flux density along the axis can be given as,

$$B = \mu_0 n I$$

Solved problems

1. A current flows down a long vertical conducting wire. A neutral point is located at a distance of 4 cm from the wire. If the horizontal component of the Earth's magnetic flux density is $B_0 = 4 \times 10^{-5}$ T, find the value of the current ($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})

At the neutral point,
Flux density due to current = component of Earth's field



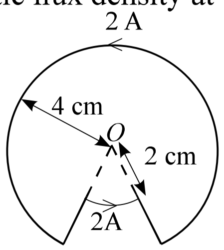
$$B = B_0$$

$$\left(\frac{\mu_0}{4\pi}\right) \frac{2I}{r} = 4 \times 10^{-5}$$

$$\frac{10^{-7} \times 2I}{4 \times 10^{-2}} = 4 \times 10^{-5}$$

$$I = 8 \text{ A}$$

2. The circular coil shown in the following figure carries a current of 2 A. Find the magnetic flux density at the centre of the coil.



$$B_0 = \left[\frac{\mu_0 I}{2 \times 4 \times 10^{-2}} \times 2\pi \times \frac{3}{4}\right] + \left[\frac{\mu_0 I}{2 \times 2 \times 10^{-2}} \times 2\pi \times \frac{1}{4}\right]$$

$$= \frac{\mu_0 I \times 2\pi}{2 \times 2 \times 10^{-2} \times 4} \left(\frac{3}{2} + 1\right)$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 2\pi}{2 \times 2 \times 10^{-2} \times 4} \times \frac{5}{2}$$

$$= (2.5\pi^2)\, 10^{-5} \text{ T}$$

2.5 Force between two parallel conductors carrying currents

Two parallel conductors are carrying currents in the same direction. Consider the composite magnetic field around the two conductors.

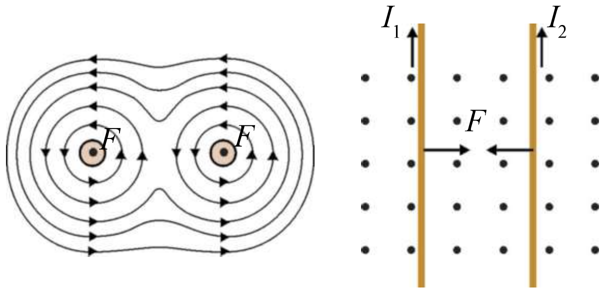


Figure 2.6

According to the above lines of force diagram, a weak magnetic field exists between the two conductors while a strong field exists around the two conductors. This results in an attraction between the two conductors due to thrusts from outside.

Consider the composite magnetic field around the two conductors when they are carrying currents in opposite directions.

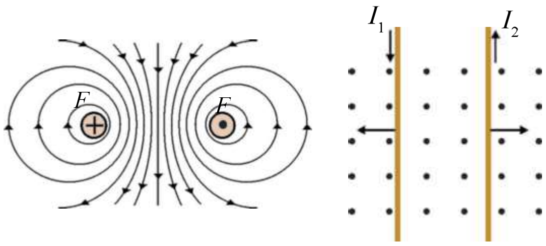


Figure 2.7

According to the above lines of force diagram, a strong field exists between the two wires while a weak field exists around the wires. The result is that forces act outwards on the two wires and they repel each other.

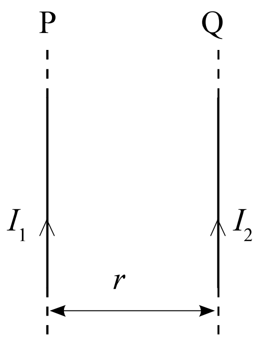


Figure 2.8

Consider two long parallel wires *P* and *Q* carrying currents I_1 and I_2 respectively in the same direction.

The current I_1 along 'P' creates a magnetic field around it and cross '*Q*' and the magnetic flux density of this field crossing the conductor '*Q*',

$$B_P = \left(\frac{\mu_0 I}{2\pi r}\right)$$

The force on a length l of the conductor Q,

$$F_Q = B_P I_2 l$$

$$= \left(\frac{\mu_0}{2\pi}\right) \frac{I_1}{r} I_2 l$$

Similarly, the magnetic flux density of the field on P due to the current I_2 in Q,

$$B_Q = \left(\frac{\mu_0}{2\pi}\right) \frac{I_2}{r}$$

The force on a length l of P carrying the current I_1 due to the effect of the above field,

$$F_P = B_Q I_1 l = \left(\frac{\mu_0}{2\pi}\right) \frac{I_2}{r} I_1 l$$

Hence the force on a common length l of both conductors due to the currents flowing in them is,

$$F = \left(\frac{\mu_0}{2\pi}\right) \frac{I_1 I_2}{r} l$$

Also, the force on a unit length $\frac{F}{l} = \left(\frac{\mu_0}{2\pi}\right) \frac{I_1 I_2}{r}$

The above expression is also used to define the unit of current, the ampere (A)

When, $I_1 = I_2 = 1\text{A}$, $r = 1$ m, $\frac{F}{l} = \left(\frac{\mu_0}{2\pi}\right) \frac{I^2}{1} = \left(\frac{\mu_0}{4\pi}\right) \frac{2I^2}{1}$

$$= \left(\frac{\mu_0}{2\pi}\right) \frac{I^2}{1} = 10^{-7} \times 2$$

$$= 2 \times 10^{-7} \text{ N m}^{-1}$$

Hence, when the same current is passed through two infinitely long parallel conductors of negligible cross – section placed 1 m apart, if mutual forces of 2×10^{-7} N m^{-1} act on each conductor, then the current passing through each conductor is 1 A.

Unit 7: Chapter Three

Torque Acting on a Current Loop

3.1 Effect on a current carrying rectangular coil of wire placed in a uniform magnetic field

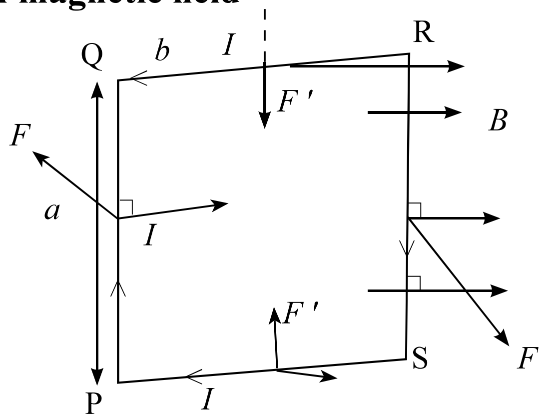


Figure 3.1

A rectangular coil of height ' a ' and breadth 'b' consisting of N turns is suspended in a plane inclined at an angle θ to a uniform magnetic field of flux density B so that it is capable of rotation about a vertical axis through its centre.

When a current (I) is allowed to pass through this coil, the four arms of the coil experience forces due to the magnetic field crossing them and the directions of these forces are decided by Fleming's left hand rule as follows.

(1). Forces F, F each act on arms PQ and RS in directions normal to the field B and also opposite to each other to constitute a couple (or a torque)

$$F = NBIa \sin 90^\circ = NBIa$$

The couple on the coil causes it to rotate about the axis of suspension.

(2). Forces F, F each act on sides QR and ST at their mid points in directions opposite to each other. However since these forces act along the same straight line they cancel out each other.

Considering the couple of forces acting on sides PQ and RS, its torque,

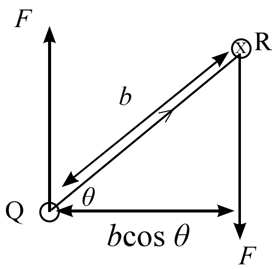


Figure 3.2

$$M = Fb \cos\theta$$
$$= NBI\,a\ b\cos\theta$$
$$= NBI\,A\ \cos\theta$$ where A is the area of the coil

when $\theta = 0^0$, $\cos(0) = 1$ hence $M = NBIA$

This is the maximum torque acting on the coil and acts when the coil is in a plane parallel to the field.

When $\theta = 90^0$, $\cos \theta^0 = 0$ $M = 0$

Hence when the coil is in a plane normal to the field, the torque reaches its minimum value zero.

Using this torque acting on a rectangular coil carrying a current, the moving coil galvanometer is constructed. The moving coil galvanometer is the one instrument modified and used as the ammeter to measure electric currents, as the voltmeter to measure electric potential differences and as the ohmmeter to measure electric resistances. This galvanometer makes use of an exceptional magnetic field called a " radial field". In this field, radial magnetic lines of force are arranged to form between cylindrical poles of a magnet.

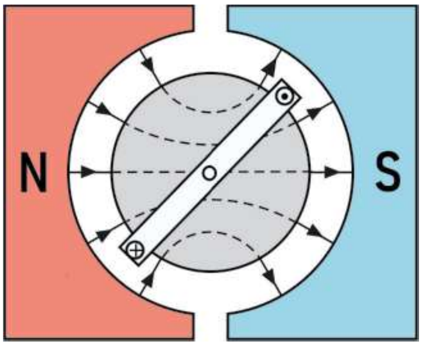


Figure 3.3

3.2 Moving coil galvanometer

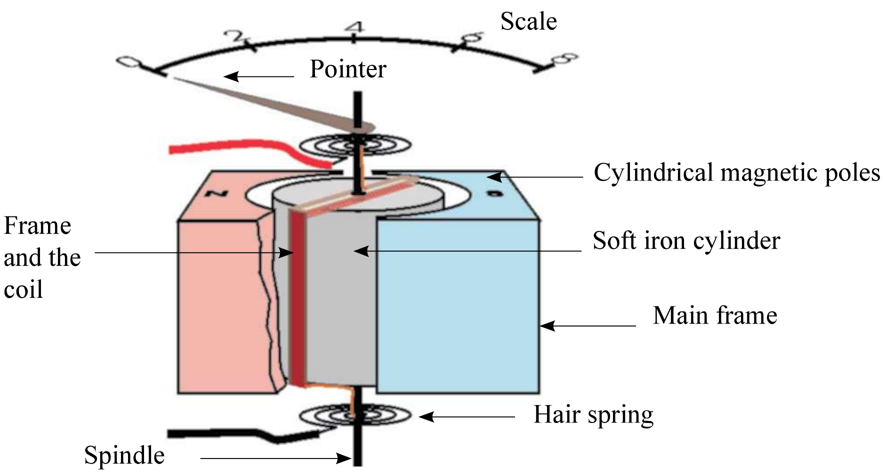


Figure 3.4

The moving coil galvanometer consists of a rectangular coil of thin insulated copper wire suspended between the cylindrical poles of a strong horse shoe magnet. The two suspension wires attached to the coil from above and below are fixed to two gem bearings and current enters and leaves the coil through two hair springs connected to it. The coil is wound round a light aluminium frame and the damping effect on it by the field controls the oscillations of the coil once it is deflected by the current.

The coil wound round the frame is free to rotate around a soft iron cylinder fixed in its middle. But the hair springs limit its rotation.

The current to be measured is allowed to flow in and out through the coil. The two forces acting on the vertical arms of the coil form a couple (torque) makes the coil rotate about the suspension. After the coil has rotated through a certain angle θ, the moment of this couple is balanced by the moment of the restoring couple of the hair springs to stop the rotation. Although the coil has rotated in the field it still remains in the plane of the field

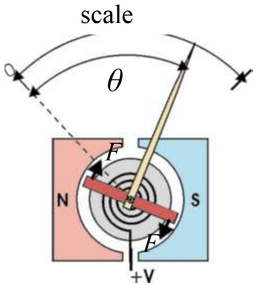


Figure 3.5

For the equilibrium of the coil ,

$F.\ b = C.\ \theta$ where C is the torsion constant of the spring

$N.BIa.b = C\ \theta$

$N.BIA = C\ \theta$ $\quad A = a.b$ = area of coil

$$I = \left(\frac{C}{NBA}\right).\ \theta$$

Since the quantities C, N, B, A are constants for a given galvanometer, $I \propto \theta$

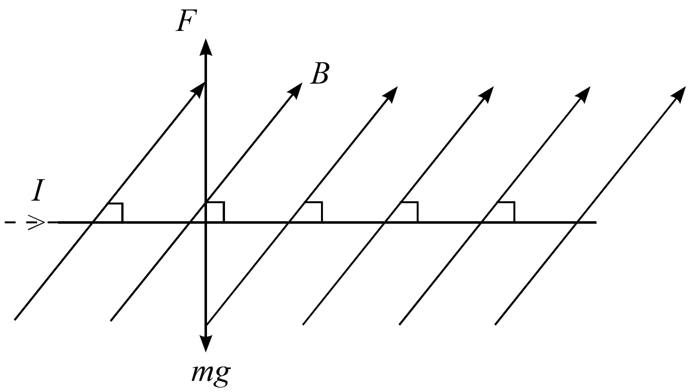
Accordingly the current measured is proportional to the deflection of the coil and hence a linear scale calibrated in accordance can the used. The tip of a pointer attached to the axis of the coil moving along the scale indicates the current directly.

Sensitivity of moving the coil galvanometer $= \dfrac{\theta}{I} = \dfrac{NBA}{C}$

In order to increase the sensitivity of the galvanometer it can be suggested that the area of the coil should be increased, a more intense magnetic field has to be used and the torque has to the decreased using finer hair springs.

Solved problem

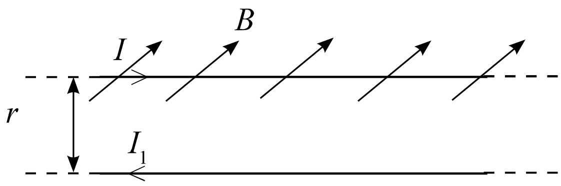
A straight conducting wire carrying a current of 25 A is to the suspended horizontally in space by means of a magnetic field acting normally across it. If the mass per unit length of the wire is 0.02 g m^{-1} what is the flux density of this magnetic field.



If 'l' is the length of the wire, in order to suspend it in space,

$$F = B\,I\,l = mg$$
$$B.\ 25.\ l = (0.02 \times 10^{-3}) \times 100\, l \times 10$$
$$B = \left(\frac{0.02}{25}\right) = 8 \times 10^{-4}\ \text{T}$$

In order to create this field if another straight current carrying current is placed parallel to the above wire 2 cm vertically below it what should be the current through this wire ?



$$\frac{\mu_0}{4\pi}\,\frac{2I_2}{r} = B$$
$$10^{-7} \times \frac{2I_2}{2 \times 10^{-2}} = 8 \times 10^{-4}$$
$$I_1 = 80\ \text{A}$$

3.3 Direct current motor

The motor can be regarded as the most valuable instrument used for the conversion of electric energy to mechanical (Kinetic) energy. All machinery functioning by electricity are based on the principle of this instrument.

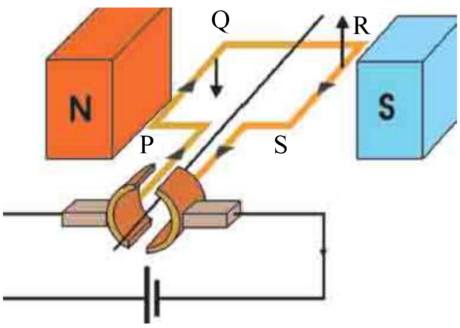


Figure 3.6

The direct current motor, as shown in the figure 3.6 consists of a rectangular coil (PQRS) of wire installed between the curved poles of a strong horse shoe magnet. This coil is known as the "armature" and across a device called the "commutator" is connected to a direct current source. This commutator as shown consists of a single ring of two halves.

When the current from the direct current source passes through the commutator to the armature and as the current flows through the armature forces act on the two arms of the armature, the directions of which are decided by Fleming's left hand rule. Accordingly a couple of forces is set up on the armature which makes it to rotate.

At the end of each half of a turn the positions of the arms of the armature interchange their positions in the rotating space and at the same instant, according to the plan of the commutator the current gets reversed in the armature. Due to this, the sense of the couple and also of rotation of the armature continues without interruption.

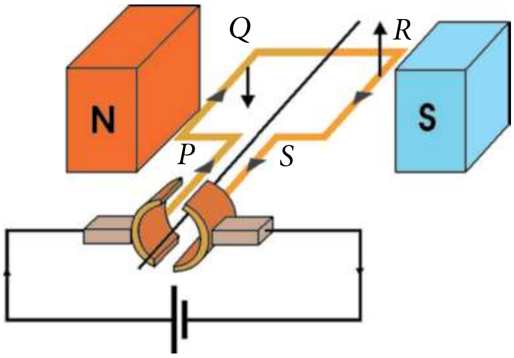


Figure 3.7

Unit 8

Current Electricity

Unit 8: Chapter one

Basic Principles of Current Electricity

1.1 Introduction

Materials can the sub – divided into three categories depending on their ability to conduct electricity. That is as conductors, semi–conductors and insulators. Free electrons are considered as the carriers which conduct electricity in conductors.

Due to the weakness of the bonds in the atoms of conducting materials, they break up releasing many electrons, as a result of which they are found in abundance in those materials. As an example in a cubic centimetre of copper which is considered as a strong conductor of electricity, about 8×10^{22} of free electrons exist at normal temperature.

In semi – conducting materials the existence of free electrons is not significant. For example in a semi conductor like silicon, a cubic centimetre contains a comparatively low amount such as 2×10^{10} of free electrons at normal temperature.

In insulators, the existence of free electrons is almost none. In insulator, such as quartz, the existence of at least one free electron per cubic centimetre at room temperature cannot be confirmed.

Considering conductors again, the free electrons in them are in random motion with no specific directions.

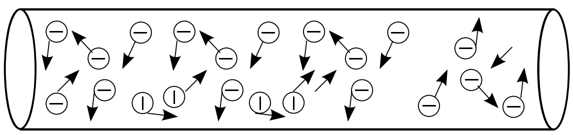


Figure 1.1 Random motion of electrons

When an electric source such as a battery is connected across the ends of such a conductor, an electric field (E) is formed across it.

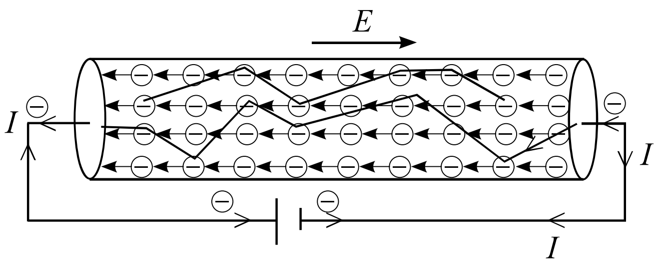


Figure 1.2

The result is that forces begin to act on negatively charged electrons in the direction opposite to that of the electric field (E) causing a flow of electrons in that direction. This stream of electrons flowing along the closed circuit consisting the source represents an electric current. However the conventional direction of the electric current is taken as the direction of drift of positive electricity which is opposite to that of electrons.

Although electrons are the charge carriers in an electric current, electric charges are measured by the unit coulomb (C). (An electron possesses only an amount of -1.60×10^{-19} C)

1.1 Electric current

An electric current is measured by the rate at which charges flow across a cross-section of the conductor through which it is flowing.

$$I = \frac{Q}{t}$$

Accordingly the unit of measuring, the electric charge is coulombs per second (C s^{-1}). It is this unit which is being called as the "ampere" (A) in general usage.

1.2 Drift velocity

The velocity of the electrons of an electric current flowing through a conductor is called the **"drift velocity"** of the electrons.

Let v be the mean drift velocity of the electrons of an electric current I flowing through a conductor of which the area of cross – section is A

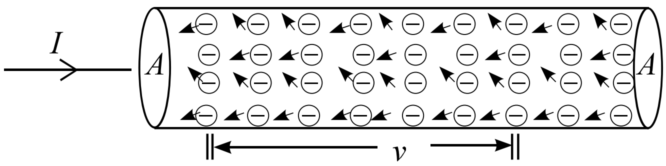


Figure 1.3

Then, if n is the number of electrons contained in a unit volume of the conductor, 'e' is the charge of electrons.

Considering a portion of length v of the conductor,
Number of electrons contained in a volume vA of the conductor = vAn
Charge contained in the volume $vA = vAne$
Since this is the charge crossing the cross – section A
per second, current $I = \frac{Q}{t} = \frac{vAne}{1} = vAne$

$$\boxed{I = vAne}$$

1.3 Current density

The current flowing across unit area of the cross – section of a current carrying conductor is called the current density of the conductor.

$\therefore$ when a conductor of uniform area of cross – section A' is carrying a current I,
The current density of the conductor $J = \frac{I}{A'}$ (unit : A m^{-2})

Since $I = vA'ne,\ J = \frac{vA'ne}{A'} = vne$

The flow of a current through a conductor takes place due to the maintenance of an electric potential (energy) difference between its two ends. Also this current (I) varies with the potential difference (V) to a set pattern. Scientist Ohm has explained this variation of electric current with the potential difference as follows.

1.4 Ohm's law

"The current flowing in a conductor is directly proportional to the potential difference between its ends provided temperature and its other physical factors remain constant".

$V \propto I$

$V = IR$, where R is a constant.

The constant R represents the obstruction to the flow of current through the conductor and is hence known as "electric resistance".

The unit of measuring electric resistance is "ohm" (Ω).

(1) When the length of a conductor increases its resistance increases.
(2) When the area of cross–section of a conductor increases its resistance decreases.

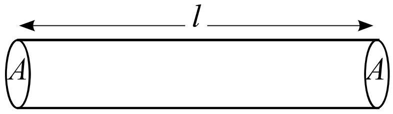


Figure 1.4

$R \propto l$ —— (1)

$R \propto \frac{1}{A}$ —— (2)

$\Rightarrow R \propto \frac{l}{A}$

$R = \rho \frac{l}{A}$, where ρ is a constant.

The constant ρ represents the contribution from the material of the conductor to its resistance and is known as the "resistivity" of the material.

Resistivity $\rho = \frac{RA}{l}$ (unit : Ω m)

The resistivity of alloys such as constantan is high where as that of pure metals is much low.

Solved Problem

(1) A current of 5 A flows through an aluminium wire of area of cross–section 4×10^{-6} m^2 which 6.08×10^{28} charge carriers (or electrons) exist in a cubic metre of aluminium the charge in an electron is 1.60×10^{-19} C. Calculate the mean drift velocity of the electrons through the aluminium wire.

$$I = vAne$$

$$v = \frac{I}{Ane} = \frac{5}{4 \times 10^{-6} \times 6.08 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= \frac{5 \times 10^{-3}}{6.4 \times 6.08}$$

$$= 1.28 \times 10^{-4} \text{ m s}^{-1}$$

This answer shows that in a flow of an electric current, the motion of electrons take place very slowly.

(2) The resistivity of the metal copper is $\rho = 1.7 \times 10^{-8}\ \Omega$ m. In order to make a uniform cylindrical resistor of resistance 1 Ω from 2 g of copper, what should be the length of the resistor ? The density of copper can be taken as 9000 kg m^{-3}.

volume of the theresistor $V = \frac{2 \times 10^{-3}}{9000} = Al \longrightarrow ①$ where A is the area of the cross section

Resistance $R = 1 = \rho \frac{l}{A}$

$$\therefore \quad A = \rho l = 1.7 \times 10^{-8}\, l$$

From ①, $\frac{2 \times 10^{-3}}{9000} = 1.7 \times 10^{-8}\, l \times l$

$$l^2 = \frac{2 \times 10^{-3}}{9000 \times 1.7 \times 10^{-8}} = \frac{200}{15.3}$$

$$l^2 = 3.6 \text{ m}$$

1.5 Variation of resistance with temperatures

When a current flows through a conductor, there is a stream of electrons which during their motion collides with the ions in the conductor. These collisions obstruct the movements or drifting of electrons in the conductor. It is this obstruction which appears as the resistance of the conductor. When the temperature increases the vibrations of the ions become more rapid resulting in increase of resistance of a conductor with change of temperature. This change of resistance of a conductor with change of temperature is represented by the following formula.

$R_\theta = R_0 (1 + \alpha\, \theta)$, where α is a constant,
R_o = Resistance of the conductor at θ ^{o}C
R_θ = Resistance of the conductor at 0 ^{o}C
α = Temparature coefficient of resistance
θ = Rise of temperature from 0 ^{o}C
Constant $\propto$ is known as the temperature coefficient of resistance

Temperature coefficient of resistance (α) $= \dfrac{R_\theta - R_0}{R_0\theta}$

This quantity which is the fractional increase of the resistance of a material for a rise of temperature of 1^oC is a property specific for the given material.

The resistivity of the conducting material too increases with increasing temperature according to the same formula.

Hence $\rho_\theta = \rho_0 (1 + \alpha\,\theta)$

Where ρ_θ = Resistivity of the material at temperature $\theta\ ^oC$

ρ_0 = Resistivity of the material 0 ^oC

$\propto$ and θ will be the same quantities

What happens to the resistance of an insulator with the rise of temperature? Since the existence of free electrons in an insulator is negligibly small, there is no compression of electrons. Hence with the increase of temperature electrons get released in insulators thereby increasing conductivity and decreasing resistivity.

Thus with the increase of temperature, the resistance of conductors increases but the resistance of insulators decreases.

1.6 Superconductivity

As the resistance of conductors increases with the rise of temperature, the resistance of conductors decreases with the fall of temperature. In certain conductors, when the temperature falls below a certain specific temperature, their resistance becomes exactly zero and such conductors are known as **"super conductors"**. The temperature at which the resistance becomes zero in this manner is called the **"critical temperature".** When a conductor passes the critical temperature and becomes a super conductor any magnetic flux that was present in it would vanish. Also in a loop made out of a super conductor, a current would flow continuously without any power supply.

Most of the super conducting materials are metals. Examples are lead and aluminium. Mercury is also considered as a super conductor. Many other elements in the periodic table belongs to this category. It has been shown that these elements become super conductors when the temperature falls below their critical values.

There are super conductors which do not exist in nature, but can be formed artificially. Neobium Titanium alloy is an example of such a super - conductors. It becomes a perfect super – conductor when the temperature falls below the temperature of 9 K.

The common properties of super conductors are,

1. When the temperatures falls below the critical value, their resistance becomes zero.
2. After the resistance becomes zero, no magnetic flux exists in them.
3. Superconductors do not exhibit any thermo electric properties (such as a thermo couple).

Uses of super conductors

1. Super conductors are used with super speed railway lines for the passage of trains without touching the lines. Absence of friction facilities the movement of trains more efficiently.
2. Super conductors are used for the production of strong magnetic fields required for the operation of MRI machines.
3. In order to maintain the stability of electric grids and to raise the efficiency of electric generators, super conductor cables are used in place of normal cables.
4. In the electric transmission network of large cites, super conducting cables are used to minimize space.

1.7 Ohmic conductors

It has been shown that metallic conductors take prominence of place in obeying Ohm's law. Conductors such as metals which strongly obey Ohm's law are called **"Ohmic conductors"**.

If a current is flowing through an ohmic conductor under a certain potential difference and the same potential difference is now reversed, the same current will flow through the conductor in the opposite direction.

Even an electrolyte such as a copper sulphate solution with copper electrodes is found to obey Ohm's law satisfactrily and hence can be considered as an ohmic conductor.

The $I - V$ graph of an ohmic conductor is as follows.

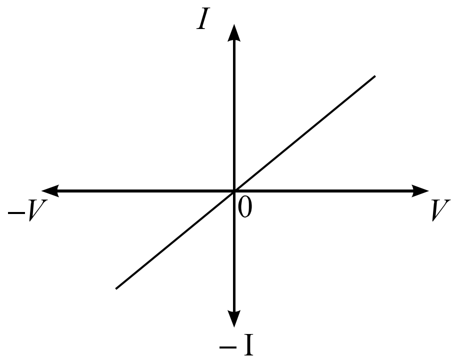


Figure 1.5

Those conductors which do not obey Ohm's law are called non-ohmic conductors. The junction diode which is used an electronic circuits can be cited as an example. When forward biased it conducts a current but does not do so when reverse biased.

I-V graph of a junction diode

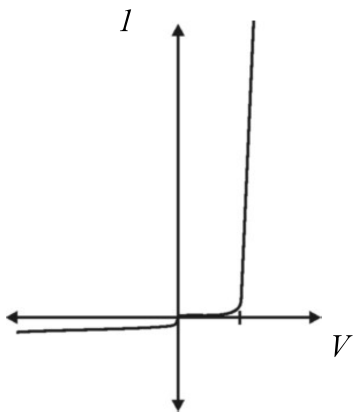


Figure 1.6

Solved problem 1

The resistance of a tungsten filament is 7.0 Ω at 20 ^{0}C. In which temperature will its resistance be 14.0 Ω. The temperature efficient of resistance of tungsten is $4.5 \times 10^{-3}\ {}^{0}C^{-1}$

Considering 0°C as the initial temperature

$$R_\theta = R_0 (1 + \alpha\,\theta)$$

at 20 ^{o}C, $7 = R_0 (1 + \alpha.20) \longrightarrow ①$

at θ ^{o}C, $14 = R_0 (1 + \alpha.\theta) \longrightarrow ②$

② ÷ ① gives

$$2 = \frac{1 + \alpha.\theta}{1 + \alpha.20}$$

$$2 = \frac{1 + 4.5 \times 10^{-3}\,\theta}{1 + 4.5 \times 10^{-3} \times 20}$$

$$2 \times 1.09 = 1 + 4.5 \times 10^{-3}\,\theta$$
$$2.18 - 1 = 4.5 \times 10^{-3}\,\theta$$
$$\theta = \frac{1.18}{0.0045} = 262\ {}^{o}C$$

1.8 Potential divider

When a certain fraction only of an input voltage is required to be used for another electrical activity, the circuit set up called the **"potential divider"** is used.

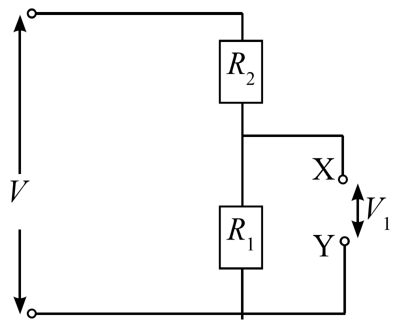


Figure 1.7

As shown in the above figure for this purpose, two resistors R_1 and R_2 are connected in series with the input voltage V By this the voltage V is divided in the ratio $R_1 : R_2$. The output (V_1) voltage can be obtained from the resistance R_1 or from the resistance R_2 as required. For example, the output voltage from the resistance

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V$$

Also, by using a potential divider consisting of a single variable resistor with a sliding key, any output voltage varying from zero to V can be obtained.

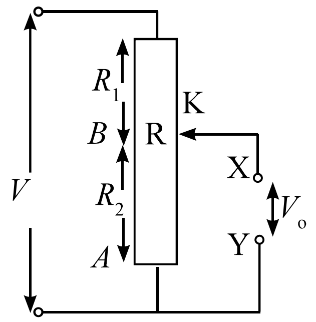
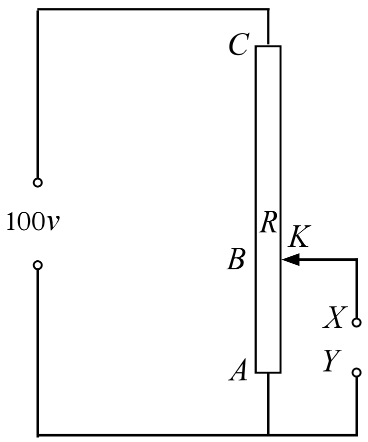


Figure 1.8

An output voltage which can be controlled in this manner can be used to control the supply voltage of a load such as an electric lamp. The load is connected between terminals X and Y, and the required input voltage is obtained by adjusting the sliding key K along the resistor R. However for V_o the expression given above cannot be used, since the resistance of the load which is connected between the terminals X and Y also belongs to the circuit now.

Solved problem 2

A resistor of resistance 1kΩ (R), is connected to a source of 100 V and a potential divider is set up capable of obtaining any potential difference from OV to 100 V using a sliding key.



(i) What should be the resistance R_1 between points A and B in order to obtain a potential difference of 20 V between terminals X and Y?

(ii) If an electric appliance which needs a voltage of 20 V for its operation is connected between terminals X and Y, what should be the resistance between A and B to be adjusted by the sliding key? The resistance of the appliance is 400 Ω.

(i) Using $V_0 = \left(\frac{R_1}{R_l + R_2}\right) V_1$

$$20 = \left(\frac{R_1}{1000}\right) 100$$

$$R_1 = 200\ \Omega$$

(ii) If R' is the equivalent resistance between A and B when the electric appliance is connected between x and y,

$$\frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{400} = \frac{400 + R_1}{400\ R_1}$$

$$R^1 = \frac{400\ R_1}{400 + R_1}$$

Now, $V_o = \frac{R_1}{1000} \times 100$

$$20 = \left(\frac{R_1}{1000}\right) 100$$

$$200 = \frac{400\ R_1}{400 + R_1}$$

$$R_1 = 400\ \Omega$$

Systems of resistors

In complicated electric circuits it becomes necessary sometimes to employ a number of resistors in them. There are two main systems that are being used in this manner.

1. Resisters in series

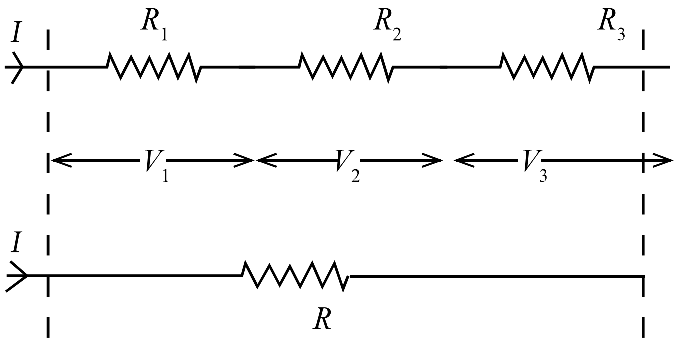


Figure 1.9

If a number of resistors are connected so that the same current flows through them, then it is a series connection. The resistance of the single resistor which carries the same current but is maintained across it, the total potential difference of the system is called the equivalent resistance of the system.

$$V_1 = IR_1 \longrightarrow (1)$$
$$V_2 = IR_2 \longrightarrow (2)$$
$$V_3 = IR_2 \longrightarrow (3)$$

(1) + (2) + (3) $\quad V_1 + V_2 + V_3 = I\,(R_1 + R_2 + R_3) \longrightarrow (4)$

If R is the equivalent resistance,

Then, $V_1 + V_2 + V_3 = I\,R \longrightarrow (5)$

From (4) and (5) ,

$$IR = I(R_1 + R_2 + R_3)$$
$$R = R_1 + R_2 + R_3$$

2. Resistors in parallel

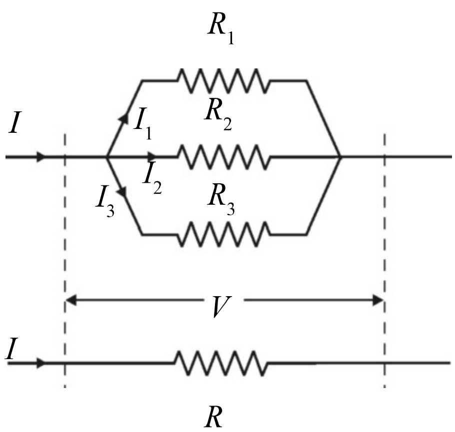


Figure 1.10

If a number of resistors are connected so that they maintain the same potential difference, then it is a parallel connection. The resistance of the single resistance which carries the total current passing through the system but maintaining the same potential difference across it is called the equivalent resistance of the system.

$$I_1 = \frac{1}{R_1} \longrightarrow (1)$$
$$I_2 = \frac{1}{R_2} \longrightarrow (2)$$
$$I_3 = \frac{1}{R_3} \longrightarrow (3)$$

① + ② + ③ gives

$$I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\Rightarrow \qquad I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \longrightarrow ④$$

If R is the equivalent resistance

$$I = \frac{V}{R} \longrightarrow ⑤$$

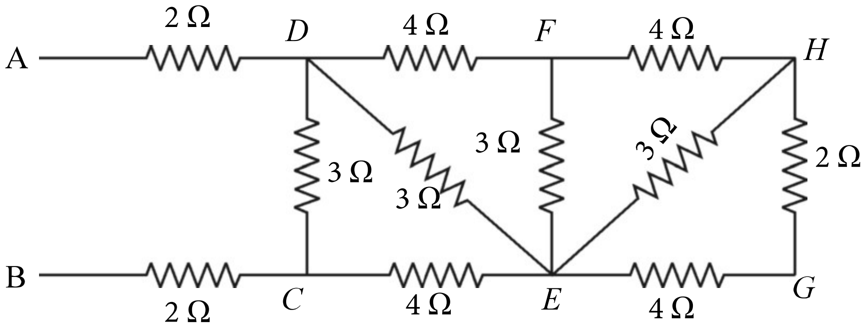
from ④ and ⑤ ,

$$\frac{V}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{I}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Solved Problem

Find the equivalent resistance between the terminals of the following network.



$R_{EGH} = 4 + 2 = 6\ \Omega$

$R_{EH} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} = 2\ \Omega$

$R_{EHF} = 2 + 4 = 6\ \Omega$

$R_{EF} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} = 2\ \Omega$

$R_{EFD} = 4 + 2 = 6\ \Omega$

$R_{ED} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} = 2\ \Omega$

$R_{CED} = 4 + 2 = 6\ \Omega$

$R_{CD} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} = 2\ \Omega$

$R_{AB} = 2 + 2 + 2 = \underline{\underline{6\ \Omega}}$

Unit 8: Chapter Two

Electric Energy and Power

2.1 Generation of energy from electricity

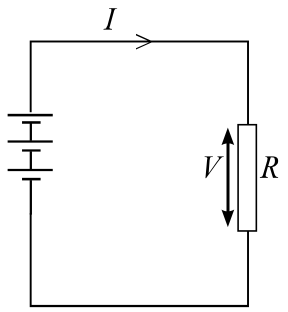


Figure 2.1

Consider a simple circuit in which a continuous current is allowed to flow through a load. This load could be an electric motor or an electric drill. When the current flows through each appliance the relevant energy gets liberated. If V is the potential difference across the load and Q is the amount of electricity flowing through each load in time the amount of energy liberated,

$$W = QV$$

But the current flowing $I = \frac{Q}{t}$

$$Q = It$$

$\therefore$ $W = VIt \ (J)$

Rate of generation of energy $P = \frac{W}{t} = \frac{VIt}{t}$

$$P = VI \quad \text{J s}^{-1}\,(\text{W})$$

The unit for measuring energy is named after the name of the scientist James Prescott Joule who performed systematic investigations on the above generation of energy while the unit of measuring power is named after the name of the researcher James Watt who was an inventor in the same field.

Heat is mainly liberated when a current passes through a resistor and this is known as **"joule heating".**

The above expressions are also valid for appliances used to obtain heat and the resistors used in such appliances are called **"passive resistors"**. Passive resistors are those resistors in which the electrical energy flowing through them gets completely converted into heat.

Across such a resistor, $V = IR \longrightarrow (1)$

$W = VIt \longrightarrow (2)$

$P = VI \longrightarrow (3)$

Substituting $V = IR$ in (3), $P = I^2 R$

$I = \frac{V}{R}$ in (3), $P = \frac{V^2}{R}$

The above expressions can also be used to all appliances used to obtain heat such as electric lamps used to obtain light and electric motors used to obtain mechanical (kinetic) energy.

2.2 Consumption of electricity

A special unit is being used to measure the electrical energy consumed by the public and this unit is known as the 'kilowatt hour' (kW h).

$$1 \text{ kW h} = 1000 \text{ (J s}^{-1}) \times 3600 \text{ (s)} = 36 \times 10^5 \text{ J}$$
$$\therefore \ 1 \text{ kW h} = 3.6 \times 10^6 \text{ J}$$

Solved problem

A heating element is to be formed by a metallic thread of cross sectional area of 0.05 mm^2 and resistivity 1.2×10^{-1} Ω m for an electric appliance of specifications 240 V, 400 W. Calculate the length of the thread required for the element to achieve these specifications. If this electric appliance is used in a house three times a day at 8 minutes each time, find the expense for a month for using it at Rs.8.00 per unit (1 kW h)

Solution
If R is the resistance of the heating element,

$$\text{From } P = \frac{V^2}{R}, \quad R = \frac{V^2}{\rho} = \frac{240^2}{400} = 144\ \Omega$$

$$\text{From } R = P\frac{l}{A}, \quad l = \frac{RA}{\rho} = \frac{144 \times 0.05 \times 10^{-6}}{1.2 \times 10^{-6}}$$
$$= 6.0 \text{ m}$$

Amount of electric energy spent per month, $= P \times t$
$= 400 \times 8 \times 60 \times 3 \times 30$ J

Amount of electric units spent per month, $= \dfrac{400 \times 8 \times 60 \times 3 \times 30}{3.6 \times 10^6}$
$= 4.8$ KWh

Expense $= 4.8 \times 8$
$=$ Rs.38.40

2.3 The simple cell

A potential difference has to be applied across any electrical appliance, if it is to be activated. A source of electricity is essential to supply this potential difference. The simple cell can be considered as the first introduction as a source of electricity.

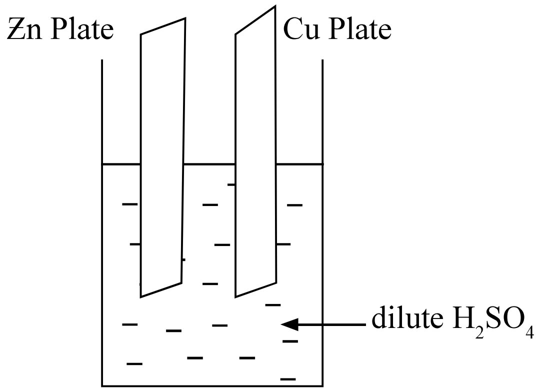


Figure 2.2

The simple cell consists of a copper plate and a zinc plate immersed in a dilute solution of Sulphuric acid (Figure 2.2). Since Zinc exists above copper metal in the activity series, Zn^{2+} ions formed due to ejection of electrons from zinc atoms pass into the solution and the electrons gather on the zinc plate which gives it a negative potential. Relative to the zinc plate, the copper plate acquires a positive potential and as a result a potential difference is built up between the zinc plate and the copper plate. The magnitude of this potential difference depends on the chemical composition of the two metals and the solution.

Due to the potential difference produced between the two terminals of the cell, when the terminals are connected externally by a conductor a flow of electrons takes place from the zinc plate to the copper plate and the direction of the conventional current is considered to be in the opposite sense.

Since the simple cell sends an electric current through an external circuit, it can be considered as an electric source situation Where no electric current is flowing the potential difference between the two terminals is known as the electro-motive force.

As seen above the electro-motive force in the simple cell occurs due to a chemical reactivity taking place in it , similar reactions take place in any other electric source to make electrons gather in one terminal and produce an electromotive force between the two terminals .

Hence when an electric source sends an electric current through an external circuit , the conversions of one form of energy into electrical energy takes place.

Dynamo ⟶ Mechanical energy ⟶ Electrical Energy

Chemical cell ⟶ Chemical energy ⟶ Electrical Energy

Solar cell ⟶ Light ⟶ Electrical Energy

Unit 8: Chapter Three

Electromotive Force

The electromotive force of a cell is defined as the amount of form of energy existing in the cell which gets converted into electrical energy when a unit electric charge passes through a circuit which includes the cell.

Units of electromotive Force is J C^{-1} and 1 J C^{-1} = 1 V

A resistance acts against the flow of the current inside the cell and it is called the internal resistance of the cell. This resistance exists in series with the resistance of the external circuit.

Figure 3.1 illustrates an occasion where a current *I* flows along a source of electromotive force *E*.

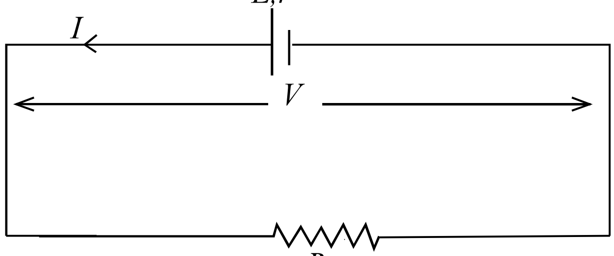


Figure 3.3.1

Amount of electrical energy discharged during the flow of a unit amount of charge = E

∴ Amount of energy dis charged during the flow of a charge Q = EQ

= EIt

∴ Rate of discharge of energy during the flow of a current I = EI

Applying the principle of conservation energy for the energy transformation in the circuit,

Rate of discharge of electrical enegy from the cell = Rate of liberation of heat due to external resistance + Rate of liberation of heat due to internal resistance

$$EI = I^2R + I^2r$$

$$E = IR + Ir \quad (1)$$

Across the external resistance it can be written as, $V = IR$.

$\therefore$ From equation ①, $E = V + Ir$

$V = E - Ir$

Given above is an expression for the potential difference between the terminals of a source when a current is flowing through the source. This is referred to as the "Terminal potential difference".

In the open circuit $V=E$. That is, when no current is flowing along an external circuit, the terminal potential difference of a source is equal to its electromotive force.

Also, if the internal resistance (r) of the source is zero $V=E$. This reveals that if the internal resistance of a source is zero, whatever the current that flows through it, its terminal potential difference equals its electromotive force. Such a source is considered as an ideal source.

Combinations of electric sources
Cells in series

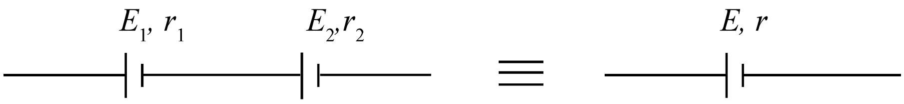


Figure 3.2

If a number of cells are connected so that they carry the same current then the cells are said to be in series. The equivalent electromotive force of this combination is the algebraic sum of the individual electromotive forces.

$$E = E_1 + E_2$$

The equivalent internal resistance of the combination is the sum of the individual resistances

$$r = r_1 + r_2$$

Identical Cells in Parrallel

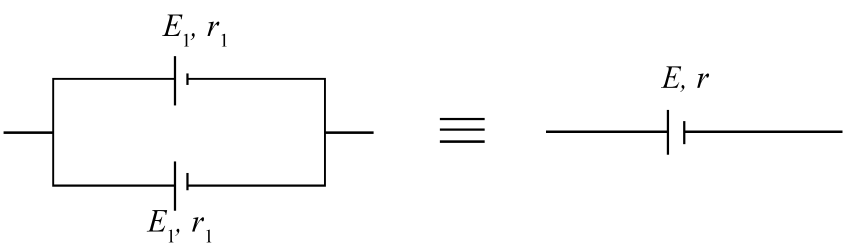


Figure 3.3

The equivalent electromotive force and the equivalent internal resistance of a combination like this is given by

$$E = E_1 \text{ and } r = \frac{r_1}{n} \text{ where } n \text{ is the number cells.}$$

3.4 Maximum power discharge of a circuit

The above curve below (Figure 3.4) shows the variation of the power of a source discharged with the resistance (R) of the load. According to this curve, the output power becomes a maximum when $R = r$. That is when the load resistance equals the internal resistance of the cell.

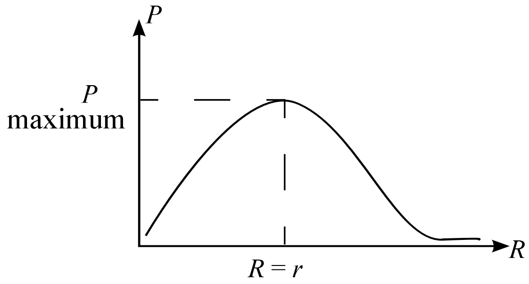


Figure 3.4

Unit 8: Chapter Four

Electrical Circuits : Kirchhoff's Laws

3.1 Kirchhoff 's laws

When an electric circuit is being set up certain accessories get included in it. Accessories such as the electric source (cell), the ammeter to measure the current, the voltmeter to measure the potential difference, the variable resistor (rheostat) to vary the current by varying the resistance, and the switch etc. are denoted by various symbols in the circuit diagram. Kirchhoff 's laws indicate how the currents flowing in various parts of a circuit are being controlled.

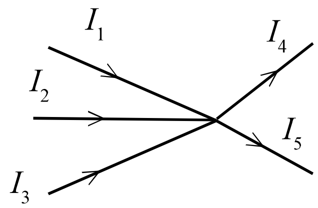
3.1.1 Kirchhoff 's first law

A current conservation exists around in an electric circuit. This means that the electric charges flow without getting collected at any point or a junction of a circuit. Accordingly the total charge entering a junction of a circuit should be equal to the total charge leaving the junction. Kirchhoff 's first law states this fact as follows.

"The algebraic sum of the currents flowing across a junction of a circuit is zero".

$$\Sigma I = 0$$

Example :



$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

$$\Rightarrow I_1 + I_2 + I_3 = I_4 + I_5$$

Figure 3.1

3.1.2 Kirchhoff 's second law

In addition to the conservation of charges, the energy too gets activated in a circuit. The electromotive force (E) of a cell, which is the source of electricity in a circuit can be considered as the energy released when the cell gives out a unit charge. Accordingly when a charge q is given out whithin time, the rate at which energy is released is, $= E\frac{q}{t} = EI.$

If this energy is released as the heat in the only resistance (R) of the circuit, the rate of release of energy $= I^2R.$

Then, According to the law energy conservation $EI = I^2R$

$$\therefore \; E = IR$$

If there are a number of resistors and a number of cells in the circuit, the above relation becomes, $\Sigma E = \Sigma IR$.

Kirchhoff 's second law is stated according to the above expression as follows.

"The algebraic sum of the electromotive forces of a closed circuit is equal to the algebraic sum, of the products of the currents flowing along all branches of the circuit and their respective resistances".

Example :

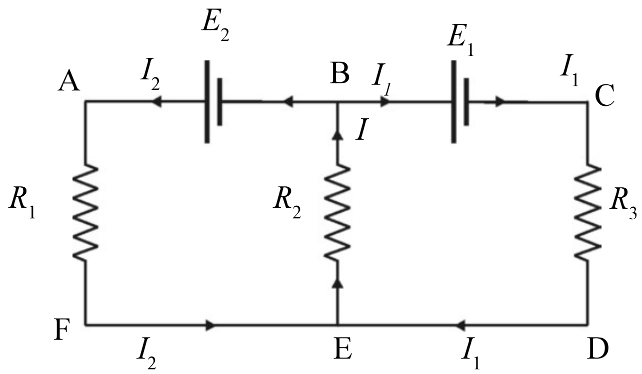


Figure 3.2

In the above circuit, with the two cells E_1 and E_2 which are connected in apposition, two resistances R_1 and R_2 are connected in series. If I is the current through the resistance in the middle then according to Kirchhoff 's first law.

$$I_1 + I_2 - I = 0$$

$$I_1 + I_2 = I \longrightarrow (1)$$

Considering three closed circuits and applying Kirchhoff 's second law to circuits,
For ACDEFA circuit,

$$- E_2 + E_1 = I_1R_3 - I_2R_1 \longrightarrow (2)$$

For ABEFA circuit,

$$E_2 = I_2R_1 + IR_2 \longrightarrow (3)$$

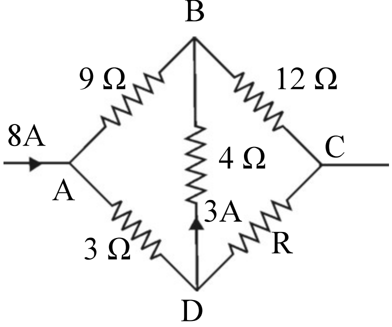
For BCDEB circuit,

$$E_1 = I_1R_3 + IR_2 \longrightarrow (4)$$

The current flowing through the different branches of the circuit can the obtained from the above equations.

<u>Solved Problem</u>

In the network below only certain resistances and currents are indicated.



Calculate the following,
1. Current through BC
2. Value of the resistance R
3. Potential drop between A and C
4. Equivalent resistance of the network between A and C

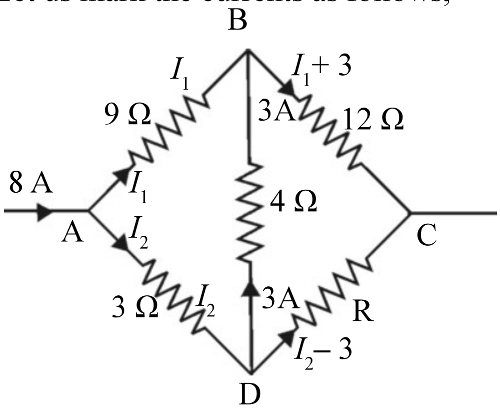
Solution

There are three closed circuit in the network. No electric source (a cell) exists in any of these circuits.

$\therefore$ For all circuits $\Sigma E = 0$

$$\Rightarrow \quad \Sigma IR = 0$$

Let us mark the currents as follows,



(1) Applying Kirchhoff 's first law to the junction A,

$$I_1 + I_2 = 8$$

$$I_1 = 8 - I_2$$

Applying Kirchhoff 's second law to the circuit ABDA,

$$I_1 \times 9 - 3 \times 4 - I_2 \times 3 = 0$$

$$9\,(8 - I_2) - 12 - 3\ I_2 = 0$$

$$12 I_2 = 60$$

$$I_2 = 5 \text{ A}$$

$I_2 = 8 - 5 = 3$ A $\therefore$ Current through BC = $I_1 + 3 = 3 + 3 = \underline{\underline{6 \text{ A}}}$

(2) For the circuit BCDB,

$$(I_1 + 3)\ 12 - (I_2 - 3)\ R + 3 \times 4 = 0$$

$$(3 + 3)\ 12 - (5 - 3)\ R + 12 = 0$$

$$72 - 2R + 12 = 0$$

$$2R = 84$$

$$R = \underline{\underline{42\ \Omega}}$$

(3) Potential difference between A and C,

$$V_{AC} = V_{AD} + V_{DC}$$

$$= I_2 \times 3 + (I_2 - 3)\ R$$

$$= 5 \times 3 + (5 - 3)\ 42$$

$$= 15 + 84 = \underline{\underline{99 \text{ V}}}$$

(4) If R is the equivalent resistance of the network between A and C,

$$V = IR$$

$$99 = 8\ R \qquad R = \frac{99}{8} = \underline{\underline{12\frac{3}{8}\ \Omega}}$$

Unit 8: Chapter Five

Measurement of Electric Current and Potential Difference

5.1 Usage of Ammeters and Voltmeters

The instrument which is been used as the ammeter to measure electric currents is the moving coil galvanometer which has been calibrated accordingly. This is due to the fact that it is the electric current which is responsible for the working of this instrument. The ammeter which is used mainly for this purpose is also modified and used as voltmeters to measure potential differences and also as ohmmeters to measure electric resistances.

However ammeters are basically constructed as milli-ammeters capable of measuring small currents in the range of milli- amperes (mA). The milli- ammeter formed in this manner is then modified to measure currents in the range of amperes using the following device.

5.1.1 Modification of a milli- ammeter to an ammeter

Consider a milli- ammeter of internal resistance *r*. Let *i* be the maximum current (or full scale deflection) which can be measured by this milli ammeter.

Suppose this milli- ammeter is to be converted to an ammeter which can measure up to a larger current *I.* Then in order to pass the excess current (I - i), which cannot be allowed to pass through the milli-ammeter, a parallel resistor is connected across it.

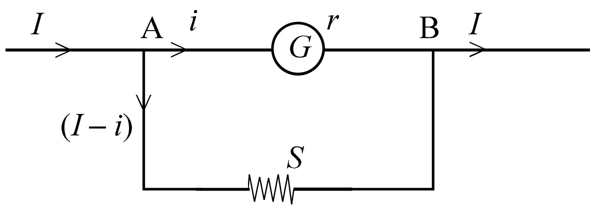


Figure 5.1

To calculate the value of this parallel resistor which is known as a shunt.

$$V_{AB} = (I - i)\,S = ir$$

$$S = \left(\frac{i}{I - i}\right)\; r$$

5.1.2 Conversion of a milli - voltmeter to a voltmeter

The milli - ammeter constructed from the moving coil galvanometer is now calibrated in units of measuring potential differences to obtain the potential difference.

However since the potential difference is proportional to the current, What is obtained by converting (or re- calibrating) a milli-ammeter is a mili-voltmeter capable of measuring potential differences in the range of milli-volts.

In order to modify this milli-voltmeter to a voltmeter capable of measuring potential differences in the range of volts (V), the following device is used.

For example, from a milli-ammeter of resistance *r* and full-scale deflection *i* , the maximum potential difference across it would be *ir* which would be its full deflection in mV as a "milli-voltmeter". Suppose that it has become necessary to convert this milli-voltmeter to a voltmeter which can be used to measure potential differences in the range of volts (V). For this purpose, a resistor of resistance *R* is connected to in series with the milli-voltmeter.

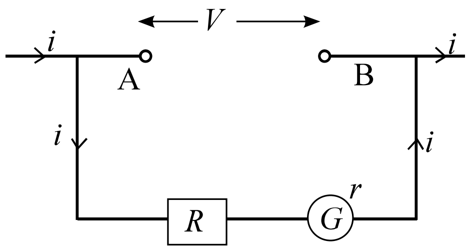


Figure **5.2**

In order to calculate this series resistance,

$$V_{AB} = i\,(R + r) = V$$

$$R = \frac{V}{i} - r$$

This series resistance is called a "multiplier". The milli - voltmeter along with the multiplier is now being converted to a voltmeter which can measure up to potential differences in the range of volts.

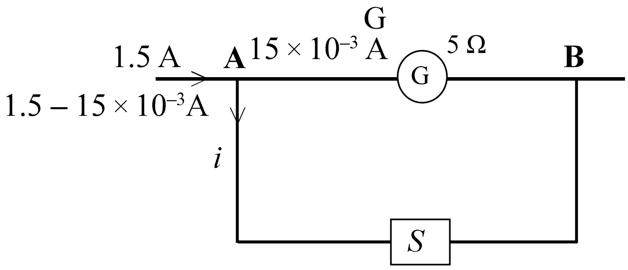
Solved problem

A moving coil galvanometer which is used as a milli - ammeter has a resistance of 5 ohms, while its full scale deflection is 15 milli amperes.
Explain how you would modify this instrument to,

a) an ammeter which can measure up to 1.5 amperes
b) a voltmeter which can measure up to 3.0 volts

Solution

(a)



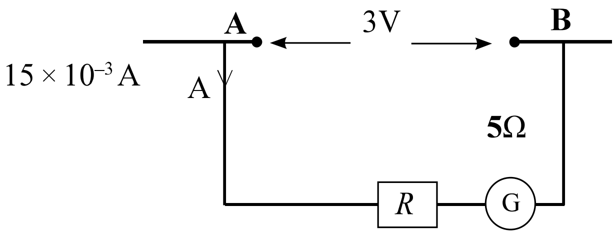
$$V_{AB} = (1.5 - 15 \times 10^{-3})\,S = 15 \times 10^{-3} \times 5$$

$$1485 \times 10^{-3} S = 15 \times 10^{-3} \times 5$$

$$S = \frac{15 \times 5}{1485} = \frac{5}{99}\ \Omega$$

A shunt of resistance $\frac{5}{99}$ Ω should be connected as shown above.

(b)



$$V_{AB} = (R + 5)\, 15 \times 10^{-3} = 3$$

$$R = \frac{3 \times 10^{3}}{15} - 5$$

$$= 195\ \Omega$$

A multiplier of resistance 195 Ω should the connected as shown above.

5.2 The Ohm meter

A moving coil galvanometer is also used to measure electric resistances by means of a special arrangement and it is called an ohmmeter. Since the resistor which is to be measured may be an independent one not connected in a circuit, the ohmmeter is accompanied by a built in circuit consisting of a small battery, a variable resistor and two terminals to connect the resistor to be measured.

In addition, since the resistance is inversely proportional to the current, the linear scale used in the galvanometer to measure currents cannot be used to measure resistances for which it has to be calibrated especially to make it an ohmmeter.

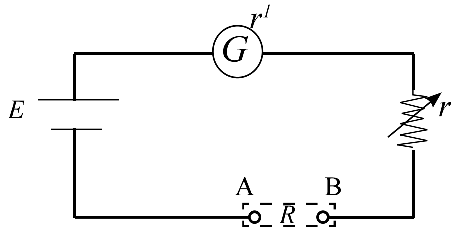


Figure 5.3

In the ohmmeter circuit shown in the Figure 5.3. let E be the E.M.F. of the battery, r the resistance of the variable resistor and r^1 the resistance of the galvanometer (to be used as ohmmeter).
A and B are the two terminals between which the resistor to be measured is connected.

Calibration

(1) The terminals are short – circuited by connecting a suitable conductor ($R = 0$) between A, B and the variable resistor r is adjusted until the pointer of the instrument indicates full scale deflection. Calibrate this point of indication as 0 Ω. The resistance r is now kept fixed.

(2) Standard resistances of values such as R = 10, 20, 50 Ω etc. are now connected between terminals A and B, and the points of indication of the pointer on the scale are calibrated with corresponding values.

(3) The point on the scale relevant to zero current is calibrated as infinity (∞).

In the above, circuit if I is the current corresponding to a certain resistance R which is being measured, then using Kirchloff's law,

$$E = I(R + r^1 + r)$$

$$\frac{E}{I} = R + r^1 + r$$

$$R = \frac{E}{I} (r^1 + r)$$

According to the above expression,

(1) When R increases I decreases and hence the deflection shown on the scale decreases.

(2) A linear relationship does not exist between R and deflection and hence the scale is not linear.

5.3 The multimeter

The multimeter is an instrument formed by adapting a moving coil galvanometer to serve a dual purpose of measuring an electric current as well as an electric potential difference. In addition an ohmmeter circuit too is included in the multimeter.

Example :

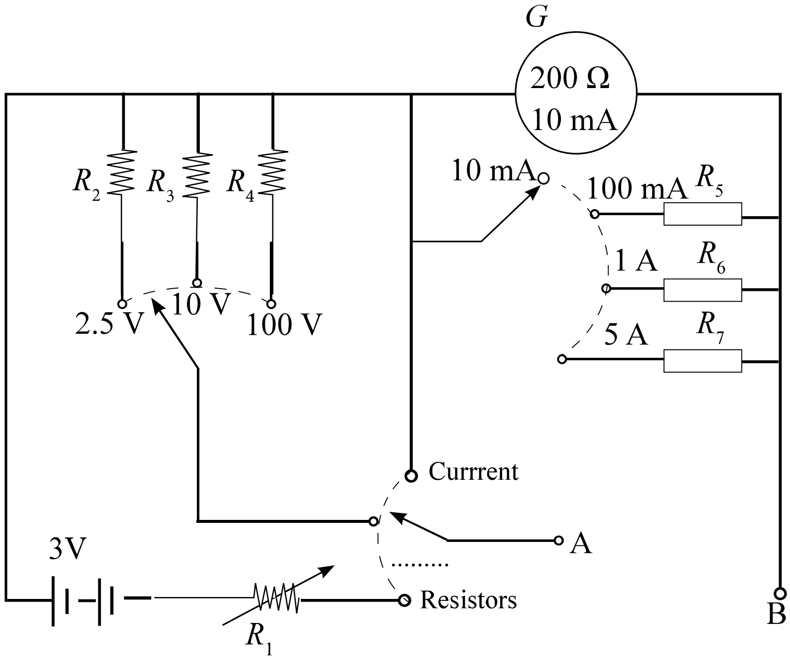


Figure 5.4

The multimeter shown above is constructed using a galvanometer of resistance 200 Ω and full scale deflection 10 mA. A and B are the two terminals of the instrument. R_1 is the resistance of the ohmmeter which is being kept fixed. R_2, R_3, R_4 are the multipliers used when measuring to the corresponding potential differences. R_5, R_6, and R_7 are the shunts when measuring up to the corresponding currents.

The values of these resistances can be calculated as follows.

(1) To calculate the ohmmeter resistance,

$$3 = (R_1 + 200)\ 10 \times 10^{-3}$$
$$300 = R_1 + 200$$
$$R_1 = 100\ \Omega$$

(2) To calculate the values of the multipliers

$$2.5 = (R_2 + 200)\ \frac{10}{10^3}$$

$$250 - 200 = R_2$$
$$R_2 = 50\ \Omega$$

(3) $$10 = (R_3 + 200)\ \frac{10}{10^3}$$

$$1000 = R_3 + 200$$
$$R_3 = 800\ \Omega$$

(4) $100 = (R_4 + 200)\ \frac{10}{10^3}$

$10000 = R_4 + 200$

$R_4 = 9800\ \Omega$

To calculate the values of the shunts,

(5) $(100 - 10)\ 10^{-3}\ R_5 = 200 \times 10 \times 10^{-3}$

$R_5 = \frac{2000}{90} = 22.2\ \Omega$

(6) $(1000 - 10)\ R_6 = 200 \times 10$

$R_6 = \frac{2000}{990} = 2.02\ \Omega$

(7) $(5000 - 10)\ R_7 = 200 \times 10$

$R_7 = \frac{2000}{4990} = 0.40\ \Omega$

The values obtained above show that high resistances are suitable for multipliers while low resistances are suitable for shunts. Another fact revealed here is that although the same instrument is used, the ammeters should essentially have low resistances while the voltmeters should essentially have high resistances.

5.4 Wheatstone's Bridge

The Wheatstone's Bridge circuit provides a principle for comparing two resistances or for determination of an unknown resistance. The circuit of Wheatstone's Bridge consists of a network of four resistors as shown below.

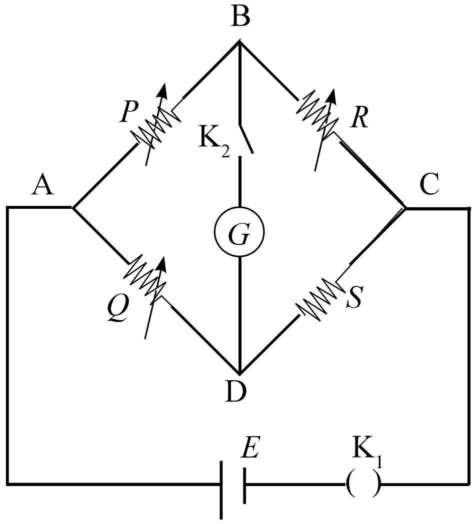


Figure 5.5

In the above circuit, *P, Q* and *R* are three known but adjustable resistors. *S* is an unknown resistance. Between junctions *A* and *C* are connected a cell (*E*) with a key K_1 while between junctions *B* and *D* are connected a centre zero galvanometer (*G*) and a key K_2. One or more of the variable resistors are adjusted until the galvanometer indicates zero deflection (As experimental steps the key K_1 is closed first and is followed by closing the key K_2). This situation of zero deflection is referred to as a "balanced" state. In this balanced state, no current passes through the branch BD of the circuit and hence the potentials of the junctions B and D are equal.

$$V_B = V_D$$

$$\therefore \quad V_{AB} = V_{AD} \text{ and } V_{BC} = V_{DC}$$

$$\Rightarrow \frac{V_{AB}}{V_{BC}} = \frac{V_{AD}}{V_{DC}} \longrightarrow (1)$$

If I_1 and I_2 are the currents flowing through branches ABC and ADC respectively,

From (1) $$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

$$S = \frac{QR}{P}$$

5.5 Application of Wheatstone's Bridge principle

5.5.1 Metre Bridge

The Metre Bridge is an instrument constructed to use the Wheatstone's Bridge principle practically to determine resistances. As shown in the diagram below, the metre bridge consists of a uniform resistance wire of length about one metre stretched on a plank. The two ends of the wire are attached to thick copper strips fixed on the plank and across two gaps between these copper strips are connected two resistors. The value of one of these resistor (Q) is to the found. Between the two ends of the wire are also connected a battery and a switch, while to the mid point of the copper strip between the two gaps is connected a sliding key K through a centre zero galvanometer (G).

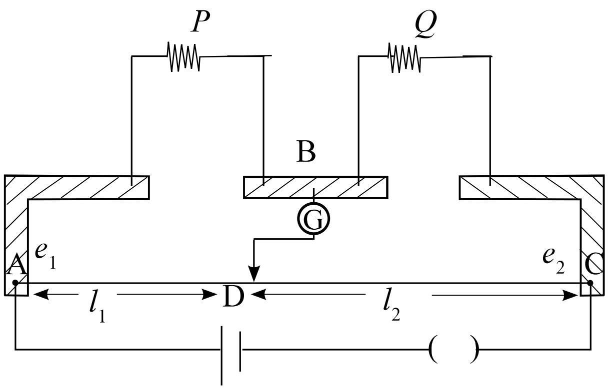


Figure 5.6

In order to find the unknown resistance, the sliding key K is moved from the left end of the wire along it smoothly until the balance point (D) at which the galvanometer indicate **zero deflection** is found.

Applying Wheatstone's Bridge principle to the balanced bridge,

$$\frac{P}{Q} = \frac{R_{AD}}{R_{DC}} = \frac{K.AD}{K.DC} = \frac{AD}{DC}$$ (where K is the resistance per unit length of the wire)

If AD = l_1 and DC = l_2

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

For a more accurate measurement, it is necessary to consider two small lengths (e_1, e_2) at the two ends of the wire which stretch beyond the metre scale. These lengths are known as "end corrections".

Hence, $\frac{P}{Q} = \frac{(l_1 + e_1)}{(l_2 + e_2)}$

The value of Q can be calculated from the above expression.

Mentioned below are some important facts to be considered when using the metre bridge.

(1) When searching for the balance point, care should be taken to protect the uniformity of the wire by not dragging the sliding key along the wire. Instead the key should be placed smoothly at points along the length of the wire.

(2) However, in order to minimize any errors due to non – uniformly of the wire, after taking the first readings on the balance point, the two resistances P and Q are interchanged in their respective gaps, a new balance point is found and two more balance lengths are taken. The mean of the two results thus obtained is taken as the correct result.

(3) In order to determine the values of the end corrections two resistances of known values are used for P and Q and a separate experiment is to be done.

(4) By obtaining the balance point close to the midpoint of the wire, the results would be most accurate, since then small values of the end corrections would be insignificant.

Application of the metre bridge

Determination of the temperature coefficient of resistance of a conducting material.

As mentioned before, the resistance of a conducting material changes with temperature according to the expression,

$$R_\theta = R_0 (1 + \alpha \theta)$$

By finding the resistance (R_θ) at various temperatures (θ) , the temperature coefficient of resistance (α) of a conductor can be found graphically using the above expression. For this purpose, the conductor to be experimented is taken in the form of a small coil, immersed in a vessel of water and connected to the gap of a metre bridge where resistances to be found are connected, as shown below.

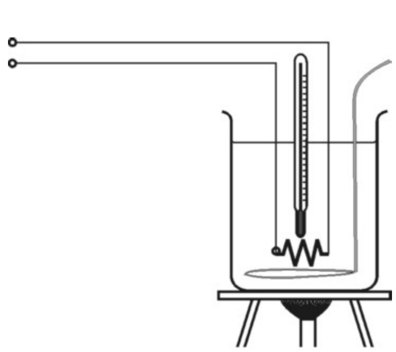


Figure 5.7

The vessel of water is heated and at various temperatures (θ) the metre bridge is balanced. The balance lengths are found and the resistance (R_θ) of the coil is calculated.
The graph of resistance (R_θ) against temperature (θ) is then plotted. One should be able to calculate the value of the temperature coefficient (α) from the gradient and the intercept of this graph.

Solved Problems

Calculate the equivalent resistance between points A and C of the following resistance network.

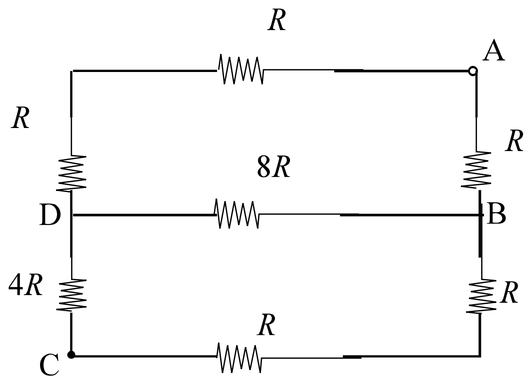


Figure 5.8

Considering the network between A and C,

$$\frac{AB}{BC} = \frac{R}{2R} = \frac{1}{2}$$

$$\frac{AD}{DC} = \frac{2R}{4R} = \frac{1}{2}$$

$$\therefore \frac{AB}{BC} = \frac{AD}{DC}$$

$\therefore$ Wheatstone Bridge principle is satisfied between A and C. Hence no current flows between B and D. Considering the circuit after removing the resistance $8R$ between B and D, if R_1 is the resistance between B and D,

$$\frac{1}{R_1} = \frac{1}{6R} + \frac{1}{3R} = \frac{3}{6R} = \frac{1}{2R}$$

$$\therefore R_1 = 2R$$

5.5.2 The Potentiometer

It cannot be taken for granted that the voltmeter adapted from the moving coil galvanometer provides very accurate readings when measuring potential differences. This is since the galvanometer absorbs for its working a part of the current responsible for the potential difference which is being measured.

The Potentiometer is an electrical arrangement constructed to determine potential differences more accurately using a balancing method. The Potentiometer as shown in the diagram below consists of a uniform resistance wire about one metre long stretched on an insulating board. A source such as an accumulator which supplies a uniform current and a switch is connected in series with the wire to form a complete circuit.

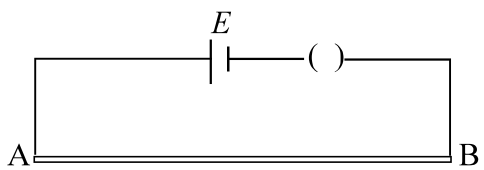


Figure 5.9

When the current is allowed to flow through the circuit a potential difference exists between the ends of the wire. Any other external potential differences (eg. EMF of a cell) which is either equal or less than this potential difference can be balanced on a certain length of the wire.

By balancing this way it can be used to find the value of this unknown external potential difference. For this, the Potentiometer wire has to calibrated as a scale for measuring potential difference. That is the potential difference per unit length or the potential gradient along the wire has to be determined.

Calibration of a potentiometer

For this purpose, a standard cell such as a Lechlanche cell or a lead accumulator or some Nickel Cadmium or Weston – Cadmium cells have to be used. The selected standard cell is connected to the potentiometer as follows.

1. The positive terminal of the standard cell is connected to the same end of the potentiometer wire to which the positive terminal of the potentiometer cell (accumulator) is connected.
2. The negative terminal of the standard cell is connected through a centre zero galvanometer to a sliding key.

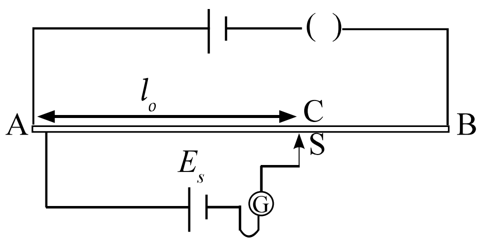


Figure 5.10

The sliding key S is then moved smoothly along the potentiometer wire and the balance point is found. The balance length l_0 is measured. Since no current flows through the standard cell now, its e.m.f. (E_s) now equals the potential difference across the length l_s of the potentiometer wire.

If K is the potential gradient (or the potential difference per unit length) of the wire,

$$V_{AC} = k\, l_s = E_s$$

$$k = \frac{E_s}{l_s}$$

After determining the potential gradient (K) in this manner the potentiometer wire becomes a scale for determining potential differences and e.m.f s.

For example, the potentiometer can now be used to determine the e.m.f. of a cell by one simple step. That is by removing the standard cell and replacing it with the cell under question and finding the balance length corresponding to the e.m.f. of the cell.
If l is the balance length, then the e.m.f. of the cell $E = kl$.

Applications of the Potentiometer

1. Comparison of e.m.f.s of two cells

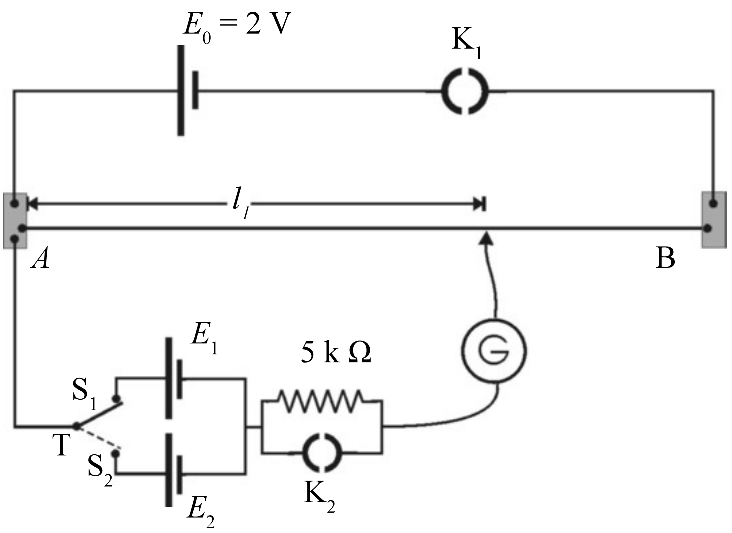


Figure 5.11

The circuit consisting of the two cells to be compared is connected to the potentiometer as shown above and,

1. The switch S_1 is closed first with S_2 open and the balance length l_1 corresponding to the cell E_1 is found.
   Then, $E_1 = kl_1 \longrightarrow (1)$

2. The switch S_2 is closed next with S_1 open and the balance length l_2 for the cell E_1 is found.
   Then, $E_2 = kl_2 \longrightarrow (2)$

① ÷ ②, $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

The e.m.f.s of two cells can be compared using the potentiometer as shown above. In the above experiment, if for a certain cell the deflection in the galvanometer is always to one side of the zero thereby not giving a balance point, this could be due to one of the following reasons.

1. The e.m.f. of the cell to be balanced is greater than the potential difference across the whole potentiometer wire.
2. The opposite terminals of the cell and of the potentiometer accumulator are connected to the same end A of the potentiometer wire .
3. The potentiometer circuit is disconnected at some point.
4. The cell may be discharged to such an extent that no measurable balance length is attained.

Note

1. Since no current passes through the cell in the balanced position, its internal resistance has no effect on its e.m.f..
2. When the e.m.f. of a cell is measured by a voltmeter the resistance of the voltmeter lowers the potential difference between the terminals of the cell and hence registers a value which is less than the true e.m.f. of the cell. But since this does not happen in the balancing method of the potentiometer it provides a more accurate value.
3. Longer the balance length (l) lesser the error in measuring it. In addition, the end correction of the wire gets less significant.
4. In order to locate the exact balance point, a highly sensitive galvanometer should be used. Also in order to safeguard it from high currents, a safety resistor should be connected as shown below.

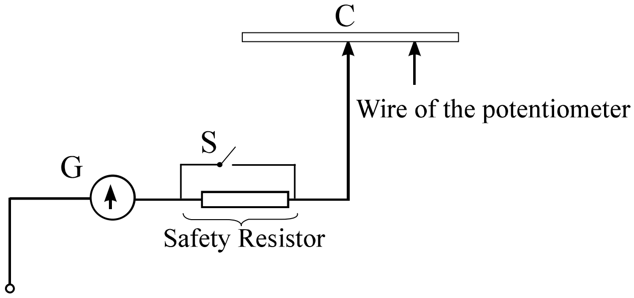


Figure 5.12

At first, the switch S is kept open and an approximate balance point is found. Next, the switch S is closed and the exact balance point is located.

Determination of the internal resistance of a cell using the potentiometer

Another important application of the potentiometer is the determination of the interval resistance of a cell using it.

For this purpose, the cell under question is connected in series with a resistance box in a closed circuit and then connected to the potentiometer to balance the potential difference between the terminals of the cell.

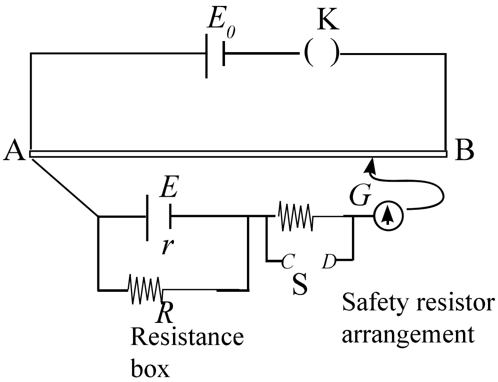


Figure 5.13

After the circuit is set up, the first step is to keep the cell in open circuit and find the balance length L. (The infinity plug of the resistance box can be pulled out for this purpose).
Then if E is the e.m.f.of the cell,

Then, $E = kL$

Next, a known resistance (R) is offered from the resistance box and the balance length l is obtained. Since the cell is now in closed circuit what is being balanced is a terminal potential difference (V) which is less than the e.m.f.(E) of the cell.

Then, $V = kl$

$$\therefore \frac{E}{V} = \frac{L}{l} \longrightarrow (1)$$

If r is the internal resistance of the cell, applying Kirchhoff's law for the closed circuit,

$$E = I(R + r)$$

Also using Ohm's law across the resistance R,

$$V = IR$$

$$\therefore \frac{E}{V} = \frac{R + r}{R} \longrightarrow (2)$$

From (1) and (2) $\frac{R + r}{R} = \frac{L}{l}$

$$1 + \frac{r}{R} = \frac{L}{l}$$

$$\frac{1}{l} = \left(\frac{r}{L}\right)\frac{1}{R} + \frac{1}{L}$$

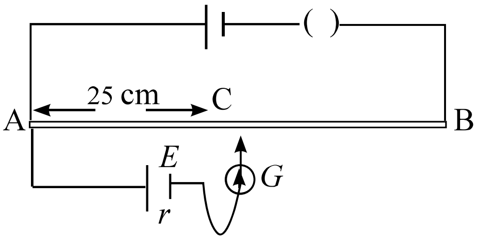
Using different known values for R from the box, the corresponding values of l are found $1/l$ is plotted against $1/R$ and the gradient and the intercept of the graph are obtained from which the value of the internal resistance of the cell can be calculated.

Solved Problems

When the e.m.f. of a cell is balanced on a potentiometer the balance length obtained is 25.0cm when a resistance of 20 Ω is connected across the terminals of the cell and the potential difference between the terminals of the cell is balanced on the potentiometer the balance length obtained is 20.0 cm. Finally, when the e.m.f. of the cell is measured by a calibrated voltmeter of resistance 40 Ω the value obtained is 1.60 V.

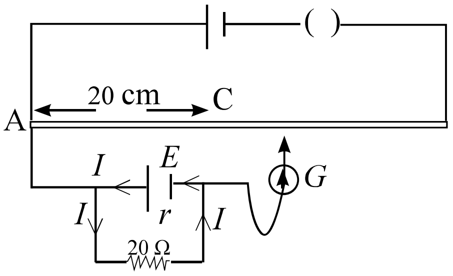
Find (i) the internal resistance of the cell
(ii) the e.m.f. of the cell

(i)



From ①, $E = kl$

$E = k\,.25 \longrightarrow$ ①



From ②, $V = kl$

$V = k\,.20 \longrightarrow$ ②

and $E = I\,(20 + r)$

$V = I.20$

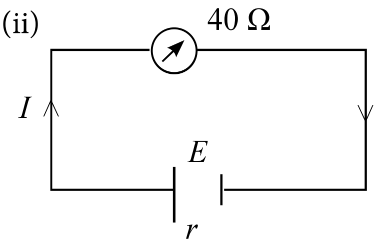
$$\frac{E}{V} = \frac{20 + r}{20}$$

$$\frac{k\,.25}{k\,.20} = 1 + \frac{r}{20}$$

$$\frac{1}{4} = \frac{r}{20}$$

$$r = \underline{\underline{5\ \Omega}}$$

(ii)



From ③, $E = I\,(40 + r)$

$V = I.40$

$$\frac{E}{V} = \frac{40 + r}{40}$$

$$\frac{E}{1.60} = \frac{40 + 5}{40}$$

$$E = \frac{45}{40} \times 1.60$$

$$E = \underline{\underline{1.80\ \text{V}}}$$

Unit 8: Chapter Six

Electromagnetic Induction

6.1 Magnetic Flux

The magnetic flux across a certain space in a magnetic field is a measurement of the number of lines of force across that space.
As shown in Figure 6.1 (a), the magnetic flux across an area A normal to the magnetic flux density B at a place is given as $\phi = AB$.

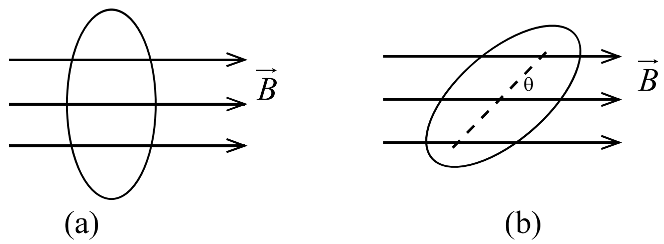


Figure 6.1

As shown in figure 6.1 (b), if the lines of force pass inclined to the area concerned , then the magnetic flux across an area A is given as $\phi = AB \sin\theta$.

Units of magnetic flux is Wb and 1 T = 1 Wb m^{-2}

6.1.1 Magnetic flux bond number

The net magnetic flux across a conducting coil is expressed as the magnetic flux $N\phi$, where N is the number of turns of the coil.

6.2 Electromagnetic Induction

It is already known to us that a magnetic field is produced by an electric current and you are also aware of the reverse effect, where an electric current or an electromotive force is produced by a varying magnetic field, as discovered after a long research by scientist Michael Faraday (1822). This phenomenon is known as "Electromagnetic induction".

Electromagnetic induction can be briefly introduced as a phenomenon where an electromotive force is produced between the ends of a conducting coil or any other conductor when the magnetic flux intersecting is changing.

The different ways of changing magnetic flux along the conducting coil are mentioned below by Figure 6.2, Figure 6.3 and Figure 6.4.

1. Changing the distance between bar magnet and the conducting coil.

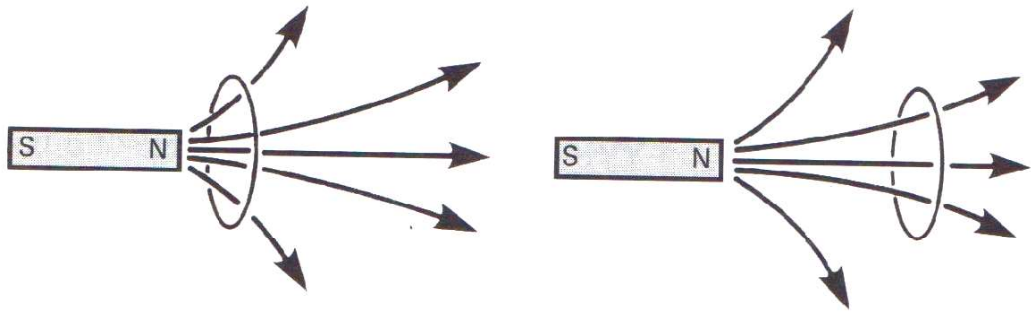


Figure 6.2

2. Rotating the conducting coil near the bar magnet

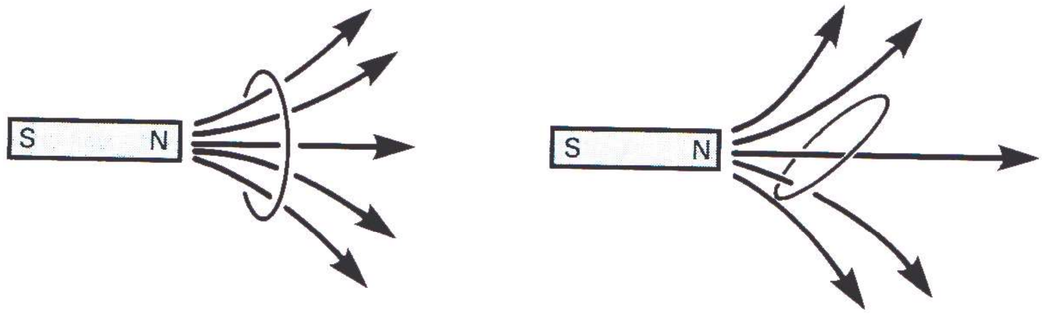


Figure 6.3

3.Changing the current of a solenoidal coil near the conducting coil

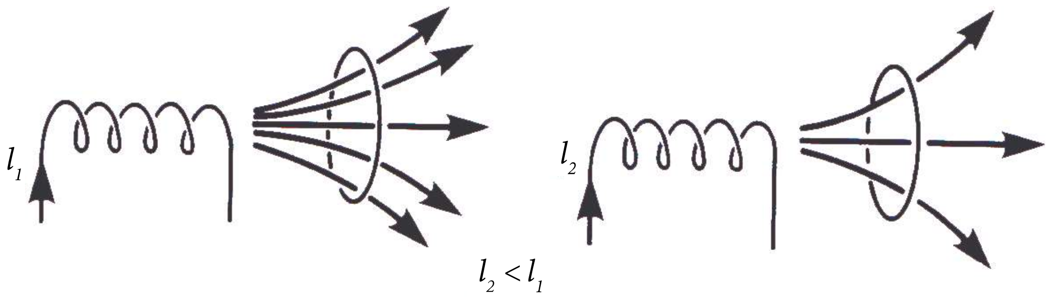


Figure 6.4

Despite all these, by changing the surface area of the coil and the number of turns of the coil magnetic flux the bond number can be changed.

6.3 Laws of electromagnetic induction

6.3.1 Faraday's law

The magnitude of the electromotive force induced in a conducting coil or a circuit at any instant is directly proportional to the rate of change of the magnetic flux or the flux linkage number intersecting it at that instant.

Hence, $E \propto \frac{\Delta\phi}{\Delta t}$ or $E \propto \frac{d(N\phi)}{dt}$

$E = K \frac{\Delta\phi}{\Delta t}$ where K is a constant

In SI unit $K = 1 \therefore E = \frac{\Delta\phi}{\Delta t} = \frac{d\phi}{dt} = \frac{d(N\phi)}{dt}$

6.3.2 Lenz's law

The direction of the induced electromotive force is such that its current flows in a direction opposing the change of flux causing it.

Figure 6.5

When the South pole approaches the coil, the flux across the coil increases. The electromotive force is induced in the coil to repel the approaching South pole. Thereby inducing a clockwise current at the closer end of the coil.

When the south pole leaves the coil, the flux linked with the coil decreases. The electromitive force is induced in the coil to attract the leaving south pole. Thereby inducing an anticlockwise current at the closer and of the coil.

The following expression is formed by combining both Faraday's and Lenz's laws,

$$E = -\frac{d(N\phi)}{dt}$$

Here E - Induced electromotive force (V)

Where $\frac{d(N\phi)}{dt}$ is the rate of change of flux linkage numbers (Wb m^{-2})

6.4 Electromotive force induced in a conductive rod moving across a magnetic field intersecting it

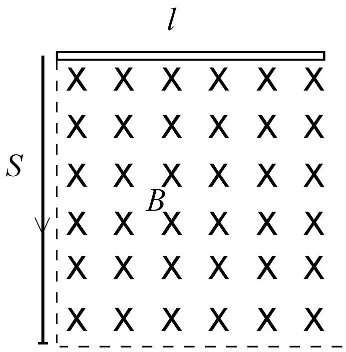


Figure 6.6

Suppose that a conducting rod of length l is moving across a uniform magnetic field of flux density 'B'. If the rod moves a distance 's' normally to the field in a time t,

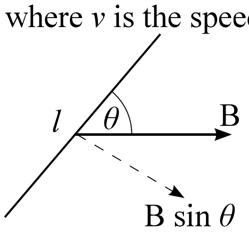
The area intersected by the rod across the field $A = ls$

$\therefore$ Magnetic flux intersected by the rod in time t $\Delta \phi = BA = Bls$

$\therefore$ Electromotive force induced between the ends of the rod

$$E = \frac{\Delta\phi}{\Delta t} = \frac{Bls}{t} = Blv$$

where v is the speed of the rod.



If the rod moves in a direction inclined at an angle θ to the field, the normal component of the field will affect,

$E = B \sin \theta . lv$

$= Blv \sin \theta$

$\therefore$ The induced electromagnetic force in a conductive rod, $E = Blv \sin \theta$.

6.4.1 The direction of electromotive force induced in a conducting rod moving in a magnetic field

We know that the electromotive force induced in a straight conductor of length l moving in speed v in a direction perpendicular to a magnetic field is given by $E = B\, l\, v$

The direction of the correct caused by this induced electromotive force can be deduced by Len'z law but the direction of this current can be easily obtained by using fleming's right hand rule.

Fleming's right hand rule

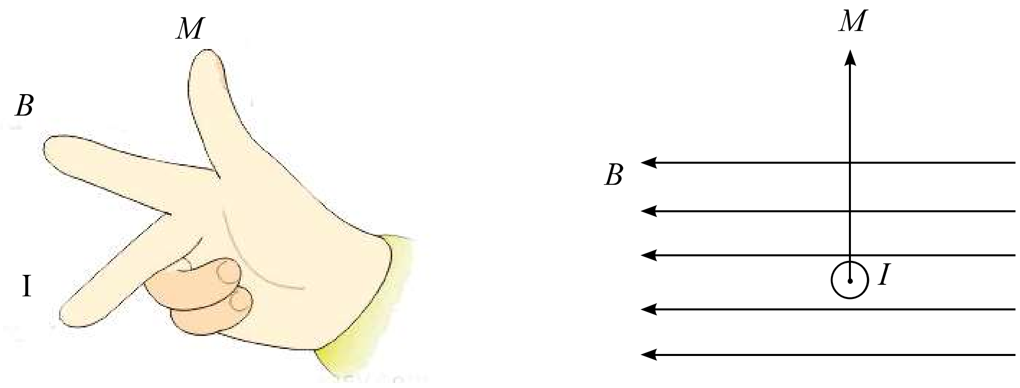


Figure 6.8

When the forefinger, the thumb and the middle finger of the right hand are streched at right angles to each other with the forefinger indicating the direction of the magnetic field and the thumb indicating the direction of motion of the conductor, then the middle finger indicates the direction of the current induced in the conductor.

6.5 Electromotive force induced in a conducting rod rotating across a magnetic field

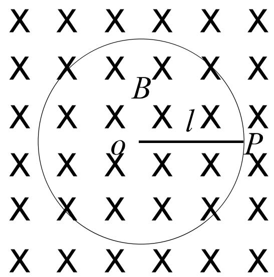


Figure 6.9

Consider a conducting rod of length 'l' rotating in a plane normal to a uniform magnetic field of flux density 'B' at a rate of 'f' rounds per second (r.p.s)

Area intersected by the rod when it rotates one round $A = \pi l^2$

$\therefore$ Magnetic flux intersected by the rod when it rotates one round $\Delta\phi = AB$

$$= \pi l^2 B$$

Periodic time of rotation of the rod $t = \frac{1}{f}$

$\therefore$ e.m.f. (voltage) induced between the ends of the rod,

$$E = \frac{\delta\phi}{\delta t} = \frac{\pi l^2 B}{\frac{1}{f}}$$

$$= \pi l^2 B f$$

If ω (rad s^{-1}) is the angular velocity of the rod ,

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

∴ Induced e.m.f $E = \pi l^2 B.\frac{\omega}{2\pi}$

$$E = \frac{1}{2} l^2 B\omega$$

6.6 Electromotive force induced in a conducting disc rotating in a uniform magnetic field

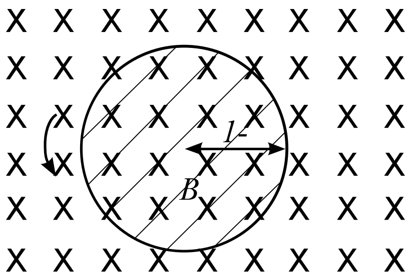


Figure 6.10

A circular conducting disc of radius 'r' rotates about its centre at a rate of f (r.p.s) at a place normal to a uniform magnetic field of flux density 'B'.

Since the disc is a conductor, there exists a conducting rod between the centre of the disc and at every point on the circumference of the disc this rod keeps on turning across the magnetic field. As a result, an e.m.f. (or a voltage) is induced between the ends of this rod between the centre of the disc and its circumference.
As shown previously, this e.m.f. $E = \pi r^2 Bf$
Hence an electric current can be obtained by setting up a circuit between the centre of the disc and at any point on the circumference.

Worked example

(1) A telegraph wire 40 metres long placed horizontally at a height of 10 metres from the ground in a direction normal to the Earth's magnetic meridian falls from rest on to the ground . What is the electromotive force induced between the two ends of the wire at the instance it strikes the ground.
(Horizontal component of the Earth's magnetic field, $B_0 = 6 \times 10^{-5}$ T, $g = 9.8$ m s^{-2})

Solution

If v is the velocity of the wire when striking the ground,

$$v^2 = u^2 + 2\,gh$$
$$= 0 + 2 \times 9.8 \times 10 = 196$$
$$v = 14 \text{ m s}^{-1}$$

E.M.F. induced between the ends of the wire when it strikes the ground,

$$E = Blv$$
$$= 6 \times 10^{-5} \times 40 \times 14$$
$$= \underline{\underline{3.36 \times 10^{-3}}}\ \text{T}$$

(2) A flat coil of 50 turns and area 40 cm^2 is placed in a plane normal to a uniform magnetic field of flux density 100×10^{-4} T. What is the e.m.f. induced in the coil in each of the following situations?

(i) Magnetic field reduces to zero in $\frac{1}{10}$ seconds
(ii) Magnetic field increases to 150×10^{-4} J in $\frac{1}{20}$ seconds
(iii) Magnetic field gets completely reversed in $\frac{1}{5}$ seconds

Solution

$$E = \frac{\Delta \phi}{\Delta t} = nA\frac{\Delta B}{\Delta t}$$

(i) $E = 50 \times \frac{40}{10^{-4}} \times \frac{100 \times 10^{-4}}{1/10} = \frac{5 \times 4 \times 10}{10^{4}} = 20 \times 10^{-3}\ \text{V}$

(ii) $E = 50 \times \frac{40}{10^{-4}} \times \frac{50 \times 10^{-4}}{1/20} = \frac{5 \times 4 \times 10^{-1} \times 20}{10^{4}} = 40 \times 10^{-4}\ \text{V}$

(iii) $E = 50 \times \frac{40}{10^{-4}} \times \frac{200 \times 10^{-4}}{1/5} = \frac{40 \times 5}{10^{4}} = 20 \times 10^{-3}\ \text{V}$

6.5 Electromotive force induced in a rectangular coil rotating about an axis normal to a magnet field

Consider a rectangular coil rotating about an axis normal to a uniform magnetic field of flux density B.

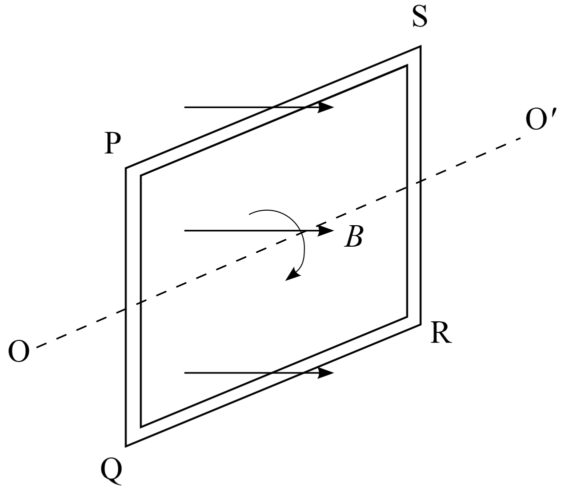


Figure 6.11

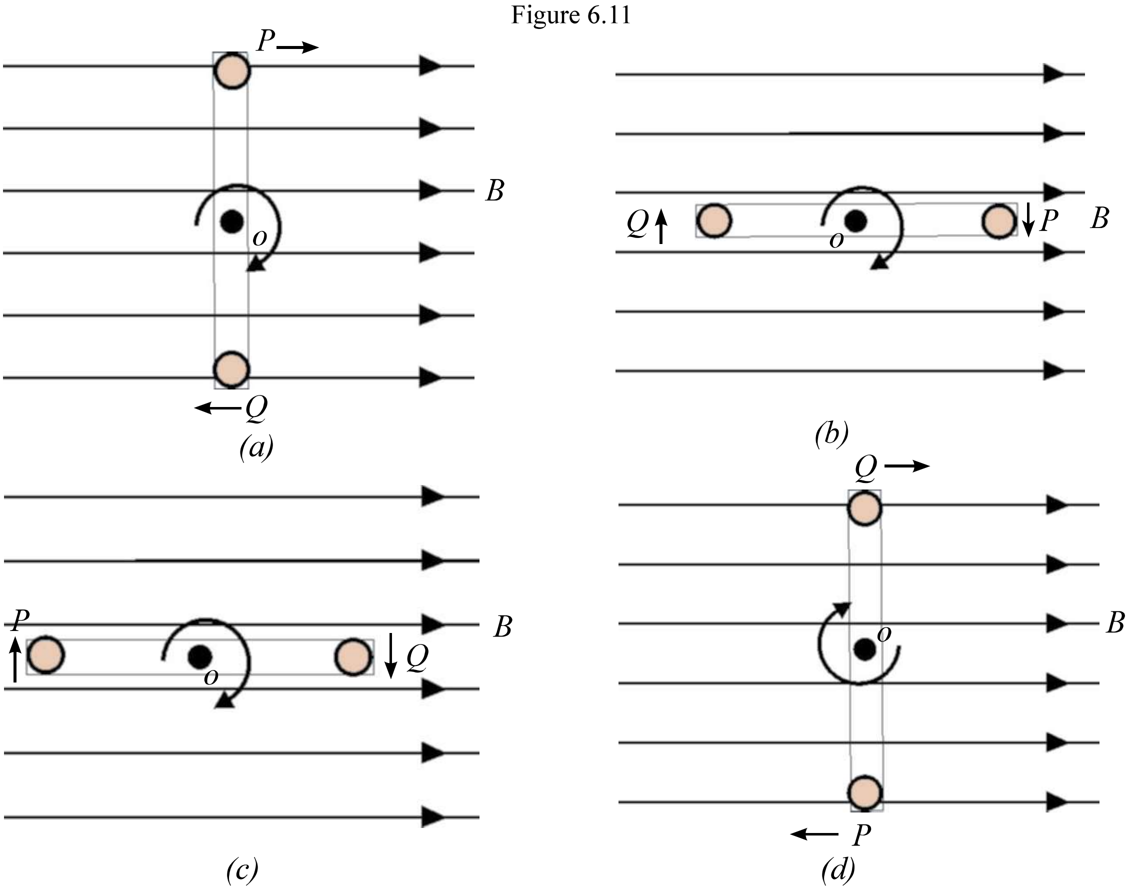


Figure 6.12

(1) Consider a rectangular coil PQRS placed in a plane normal to a uniform magnetic field of flux density B, beginning to rotate about an axis OO' with a uniform angular velocity. Figure shows only the arm PQ. At this instant both arms PS and QR move parallel to the flux in lines of the field and hence do not intersect with the flux lines. No e.m.f. or current is induced in the coil. ($E = 0$, $I = 0$)

(2). The coil has rotated through an angle of 90^0 and both arms PS and QR intersect the magnetic field normally. Hence the rate of intersection of flux lines has reached a maximum. Thus a maximum e.m.f. as well as a maximum current is induced in the coil. (E_m, I_m)

(3). After the coil has rotated through a further 90^0, the arms PS and QR are again moving parallel to the magnetic flux lines not intersecting the field. The induced e.m.f. and the current are again reduced to zero. ($E = 0, I = 0$)

(4). The coil has rotated through another 90^0 and the arms PS and QR again intersect the field lines normally. The induced e.m.f. and the current have reached maximum again. However the directions of motion of arms PS and QR relative to the magnetic flux is opposite to that in situation (2). Hence according to Lenz's law, the direction of the induced e.m.f and the current through the coil is opposite to that in situation (2). But their magnitudes are equal ($-E_m$, $-I_m$). [When the coil rotates further this cyclic process is repeated].

If T is the period of rotation of the coil, the variation of the induced current with time can be represented graphically as follows. (The variation of the induced e.m.f. with time is similarly represented).

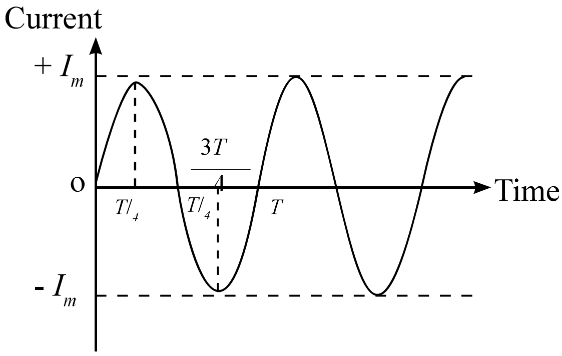


Figure 6.13

I_m above is the peak value of the induced current. This current which is induced in the rotating coil is called an alternating current (A.C.) and its main features are as follows.

1. The induced current varies periodically between a maximum and zero.

2. The current gets reversed after every half turn of the coil.

The e.m.f. which is responsible for an alternating current is known as an alternating e.m.f. In order to find the peak value of the alternating e.m.f., consider the instant when arms PQ and RS are moving normally to the magnetic field.

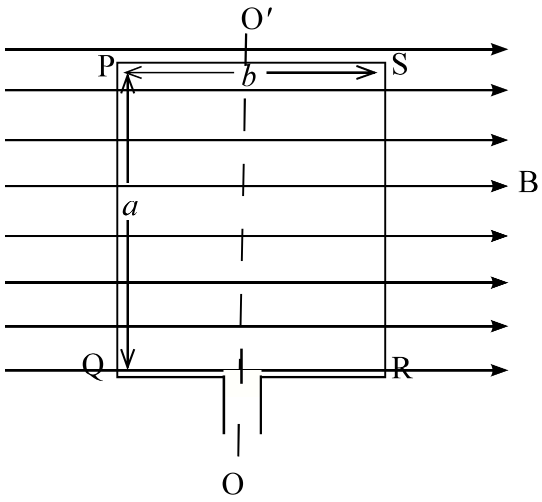


Figure 6.14

Suppose the arm PQ moves up with a velocity *v* and the arm SR moves down with the velocity v.

The maximum e.m.f. induced in each arm $= Bav$

∴ The total e.m.f. induced in the arms $= 2\,Bav$

the angular velocity of the coil about axis oo', $\omega = \frac{v}{b/2} = \frac{2v}{b}$

$$\therefore \; V = \frac{b\omega}{2}$$

∴ The maximum (peak) e.m.f. induced in the coil, $E_m = 2Ba\,\frac{b\omega}{2}$

$$= 2BA\,\frac{\omega}{2}, \; A = ab$$

$$= BA\omega$$

where '*A*' is the area of the coil.

If *N* is the number of terms in the coil, $E_m = NBA\omega$.

6.8 Alternating current generation (A.C. dynamo)

The discovery of the phenomenon called electromagnetic induction by scientist Michael Faraday in 1831 is considered as the beginning of the electrical engineering technology. The sole methodology for the production of high tension electricity required for transmission through long distance cables as well as for operation of powerful machinery has been electromagnetic induction. Up - to - date what is being utilized for this purpose is the alternating current generator, a creation based on electromagnetic induction.

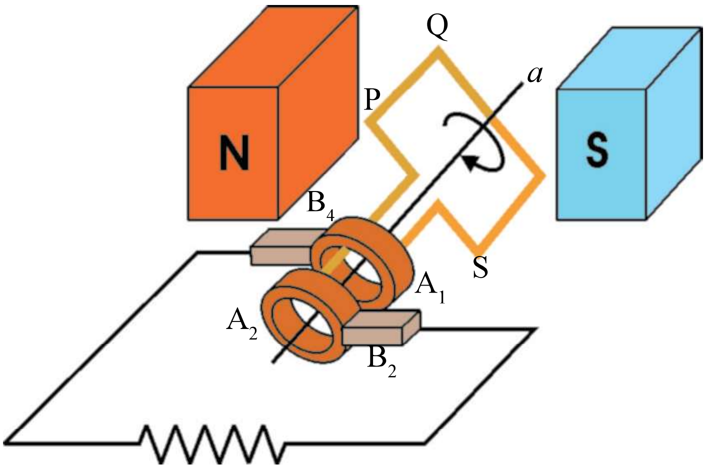


Figure 6.15

As shown in the Figure 6.11, the alternating current generator consists of a rectangular coil PQRS made to rotate between the poles of a strong horse-shoe magnet. The two terminals of this coil are connected to a commulator consisting of two slip rings. (A_1, A_2)

When the coil rotates in the magnetic field an alternating current is induced in it and this induced current is supplied to the external circuit through the commutator. The passage of the current takes place across two brushes (B_1, B_2) which are continuously in contact with the slip rings of the commutator.
In the same way as an alternating current varies with time the alternating current liberated by an A.C generator too varies with time as follows.

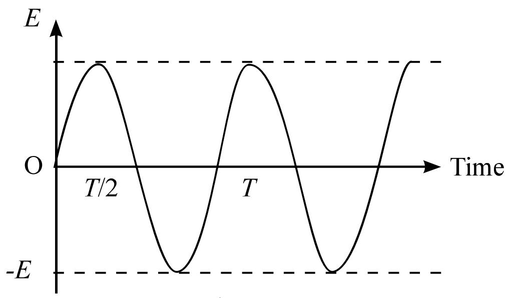


Figure 6.16

Note

1. By modifying the commutator of the a. c. generator as shown below, it can be converted to a direct current (d.c) generator.

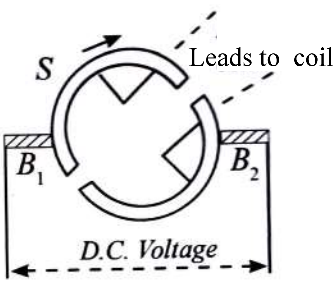


Figure 6.17

The commutator of this generator consists of one split-ring instead of two rings. The two terminals of the coil are connected separately to the two halves of the split ring. In the instant when the induced current in the coil gets reversed, the two halves of the ring get exchanged in the two carbon brushes which act as terminals of the external circuit. Thus the current received by the external circuit is not an alternating current.

However it is not a uniform current, but vary periodically between zero and a peak value as shown below.

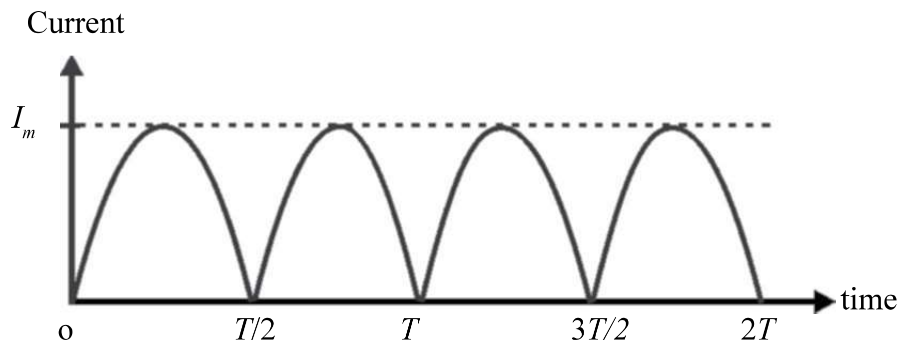


Figure 6.18

The following characteristics can be observed in instruments used as electricity generators.

1. The horse-shoe magnet which supplies the magnetic field to the rotating coil is not a permanent magnet, but an electro-magnet which turns out to be a magnet only when the generator is operating. It is converted to an electro magnet by passing a current through a coil wound round it. This current is obtained from a direct current source.

2. The coil of the generator is wrapped round a soft iron core which too rotates with the coil. By this arrangement the coil is made to rotate in a more intense magnetic field.

3. Initially although the coil rotates freely, with the development of the induced current, a torque is produced on it which according to Lenz's law acts opposite to that of the coil. This phenomenon is known as the motor effect of a dynamo. Due to this the rotating, torque of the coil has to be increased to prevent the decrease of the rotating speed of the coil.

4. When the coil rotates the magnetic flux passing through the soft iron core is subjected to a variation. When the magnetic flux passing through a conducting medium varies a type of an electric current is induced in a direction normal to this magnetic flux. This current which is known as an "eddy current" causes intense 'joule heating' (I^2Rt) of the medium. In order to prevent this the soft iron core is "laminated". In this a number of plates of the medium is taken as a pile with an insulating oxide applied between every two adjoining plates. No current is allowed to get induced across this insulation.

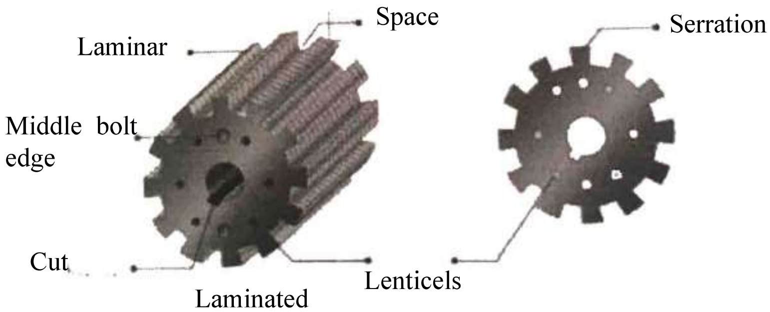


Figure 6.19

6.8.1 Eddy current

When a block of a metal moves across a magnetic field or when a varying magnetic field passes across it, an electromotive force is induced in the block causing a circular flow of current. This current is called an eddy current as mentioned earlier.

Since this eddy current passes through paths of low resistance in the metal, high currents are produced even if the induced e.m.f is low. Hence considerable heating effects and magnetic effects would occur.

6.8.2 Heating effect

This effect is made use of in inductive hearths. In these an alternating current of high frequency is passed through a coil which is wound round the metallic part to the heated. The magnetic field which varies at a high rate in the coil produces a high eddy current to heat up the metallic part.

6.8.3 Magnetic effect

According to Lenz' law eddy currents always flow in a direction opposite to the direction of motion which causes the induction of eddy currents. Thus formation of eddy currents can be made use of to control the motion of a metal. This result can be demonstrated by the following activity. Shown by the following Figure 6.14 of two pendulums.

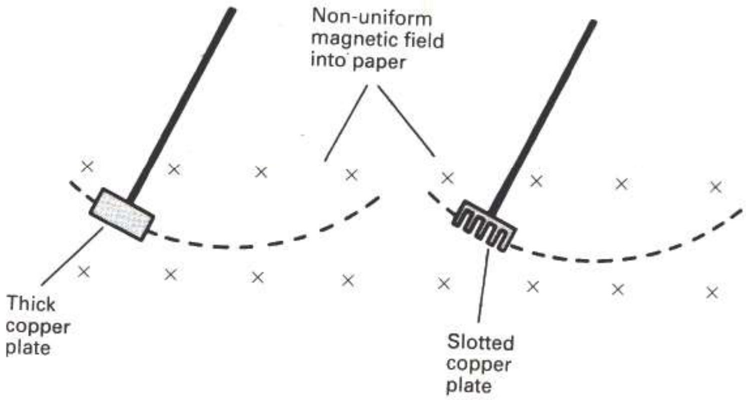


Figure 6.20

High eddy currents are formed within the thick copper sheet. Hence its motion is soon damped. In the slotted sheet the possibility of the existence of eddy currents is confined to a narrow space . Due to smaller eddy currents thus formed the oscillation takes place with lesser damping.

6.9 The root mean square value of an alternating current

When a coil of N terms and area A rotates with an angular velocity ω about an axis normal to a uniform magnetic field of flux density B, the e.m.f. induced in the coil at the instant ' t ' seconds after it begins to turn can be given by the general expression,

$$E = NAB\omega \sin \omega t$$

Since the peak value of the induced e.m.f. is given as, $E_m = NAB\omega$, the above expression can be modified as, $E = E_m \sin \omega t$

Also if I_m is the peak value of the induced current, the induced current in the above instant can be expressed as,

$$I = I_m \sin \omega t$$

Accordingly both the induced e.m.f. and the current vary sinusoidally with time. In practical usage an alternating current is measured and also expressed by its root mean square (r.m.s.) value.

The root mean square value of an alternating current is the value of the direct current when flowing through a certain resistance liberating heat at the same rate as the alternating current.

If I_m is the peak value of an alternating current its root mean square value is given by,

$$I_{\text{r.m.s.}} = \frac{I_m}{\sqrt{2}}$$

Similarly the root mean square value of an alternating e.m.f. or a voltage is given by,

$$V_{\text{r.m.s.}} = \frac{V_m}{\sqrt{2}}$$

where V_m is the peak value of the voltage.

For example, what is supplied by the national electric grid of Sri Lanka to the consuming community of the country is an alternating voltage of which the r.m.s. value is about 240 V.

Hence $240 = \frac{V_m}{\sqrt{2}}$

$\therefore$ Peak value of the supply voltage $V_m = 240 \text{ V} \times \sqrt{2}$

$$= 339.4 \text{ V}$$

Thus the peak value of the alternating voltage supplied to the Sri Lankan consumers is about 340V and the value of the voltage specified in the electric appliances used in the household is the r.m.s. value which could cope up with the above mentioned peak value.

If $I_{r.m.s.}$ is the root mean square value of a certain alternating current and I the value of the direct current which liberates heat in resistance R at the same rate, then according to the definition of $I_{m.r.s.}$,

Rate of liberation of energy $P = I^2_{\text{r.m.s.}} R = I^2 R$ (W)

Hence this rate of liberation of energy by an alternating current can be obtained by using its r.m.s. value.

Also $P = V_{\text{r.m.s.}} I_{\text{r.m.s.}} = I^2_{\text{r.m.s}} \times R = \frac{V^2_{\text{r.m.s.}}}{R}$ (W)

6.10 The Transformer

Distribution of electricity in a land is done by high tension supplies of thousands of volts through a cable system running all over the land. When the current flows through the cables joule heating causes loss of energy resulting in a drop of voltage along the cables. Hence action is taken at specific positions along the cable lines to raise the voltage to the standard value.

On the other hand when electricity is to be distributed to households etc. for human consumption, the transmitting high voltage has to be lowered to a safety value such as 240 V. For this raising and lowering of the A.C. voltage while being transmitted through cables, a special device called the "transformer" is being used.

Of the transformers mentioned above, those which are used to raise voltages are called 'step up' transformers while those used to lower voltages are called "step down" transformers.

Accordingly, the transformer is an electrical device which is used to transform an alternating voltage to another either at a higher value or at a lower value without any change of its frequency.

Since the operation of transformers is essentially by alternating currents, the electricity produced on a large scale for distribution is inevitably produced as alternate currents.

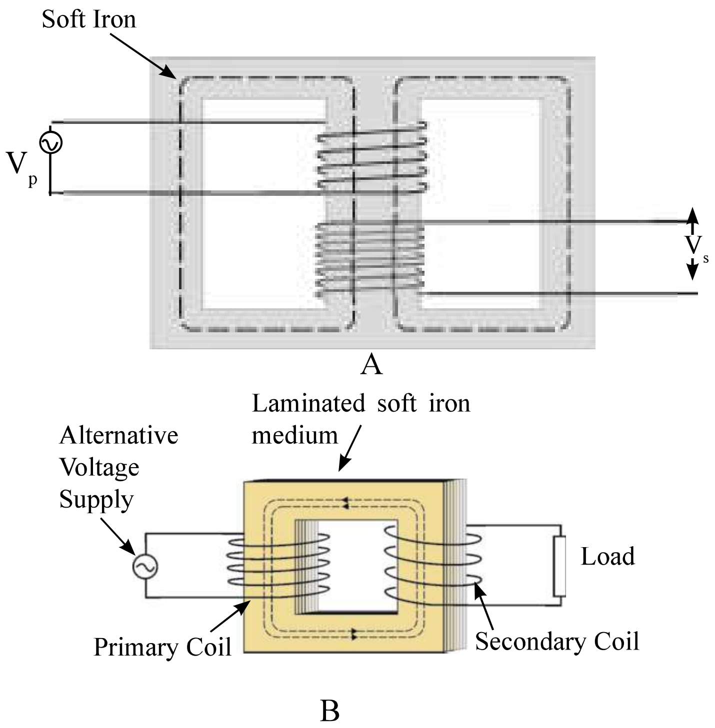


Figure 6.15

The figure 'B' in Figure 6.12 illustrates the general plan of a transformer, while the figure 'A' represents a modern transformer.

The step-up transformer consists of a soft iron frame around one arm of which is wound a lesser number of turns of insulated thick copper wire while around the other arm of the frame is wound a larger number of turns of insulated thin copper wire.

The first mentioned coil (P), known as the primary coil, is connected to an alternating current supply. Due to this current flowing in the coil, a magnetic field which alternates continuously is formed around it. Its varying magnetic flux transmitting through the soft iron wire of the frame intersects the secondary coil (S). As a result an alternating voltage is induced in the secondary coil.

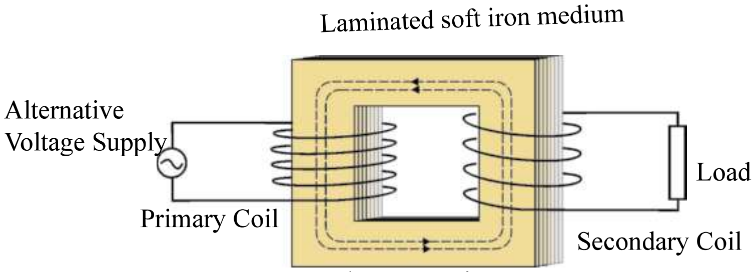
If N_p is the number of turns in the primary, N_s is the number of turns in the secondary, V_P is the input voltage and V_s is the output voltage, $\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$

Also if the transformer is an ideal one, the input power supplied from the primary will be output from the secondary without any loss,

Hence, $V_P I_P = V_S I_S$

$$\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

Thus in a step up transformer the current flowing in the secondary is less than that in the primary.



step down transformer
Figure 6.16

Note

1. In the step down transformer the primary coil has a higher number of turns and the secondary has a fewer number of turns.
2. The soft iron core of the frame guides the magnetic flux emitting from the primary to the secondary minimizing leakages of flux to the outside.
3. The transformer when operating can get heated due to following reasons.
    1. Production of eddy currents : This is minimized by laminating the soft iron core.
    2. Joule heating

1. Eddy current loss

When the magnetic field around the soft iron core on which the coil of a transformer is wound increases, a small current develops around every line of force in such a way that a magnetic field is produced in the opposite direction in the conductor. All these currents which are formed in the same direction get added to form a large eddy current. Since the magnetic field is reversing continuously the eddy current too gets reversed in the same way. As the metal has a resistance, heat is produced causing a loss of energy.

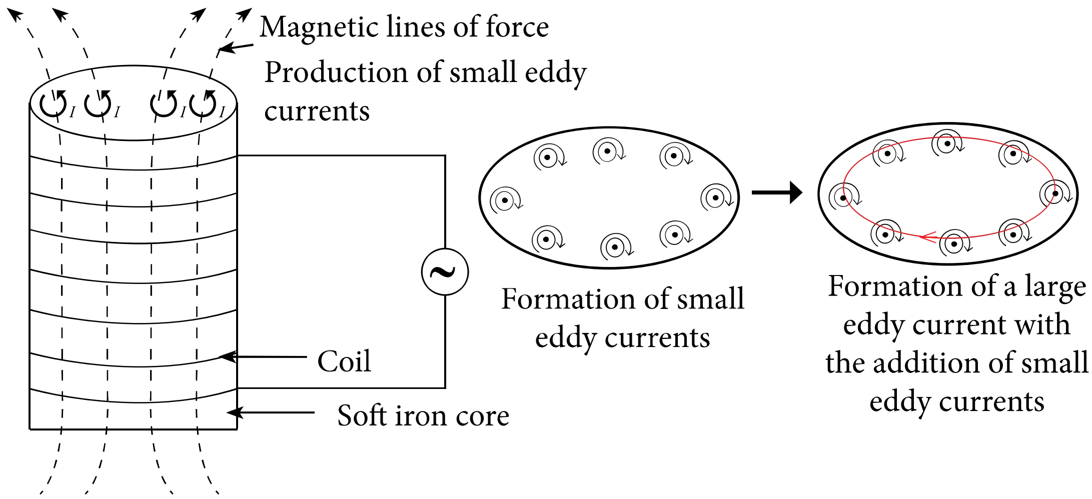


Figure 6.17

The soft iron core of a transformer is laminated as shown below. This prevents the small eddy currents getting added and hence minimizing eddy current losses. The thin conducting sheets here are insulated.

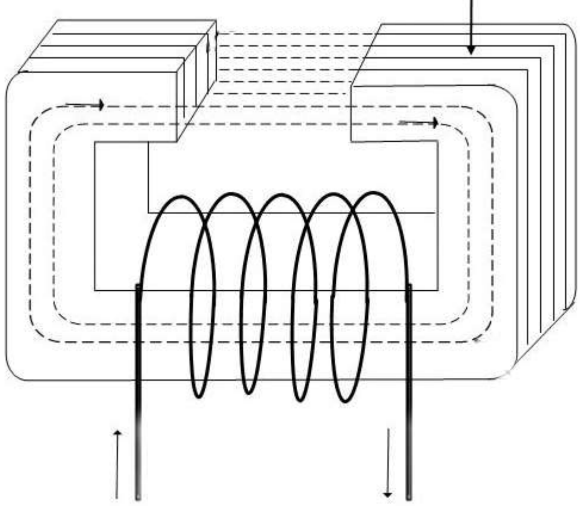


Figure 6.18

2. Joule heating

This heat (I^2Rt) is produced due to the resistance of the coils wound round the soft iron core.

6.11 Transmission of electrical energy

Electrical energy produced in electric power stations has to the transmitted to places as required for consumption. Since electrical energy is an outcome of electric current as well as electric potential difference its transmission is possible either by using a low current from a high voltage or by using a high current from a low voltage. However due to the resistance of the electric cables used for transmission of electrical energy, the loss of a part of this energy as heat cannot the avoided. If 'R' is the resistance of the electric cable, the heat produced in a time by a current 'I ' flowing in the cable can be given as I^2RT. According to this expression the energy loss is proportional to the square of the current. Hence more efficiency can be expected by using a low current under a high voltage. This way of transmission is known as high tension transmission.

An advantage of high tension transmission is that very thin electric cables can be used for the purpose and hence expenses too will be low. However, in the usage of high tension transmission thick insulation is needed for the cables which in turn would increase the expenses.

6.12 Electric Motor

The motor can be regarded as the instrument used for the conversion of electric energy to mechanical (kinetic) energy. The function of the electric motor is opposite to the simple current generator (dynamo).The armature placed between the magnetic polls of the direct current generator and the split ring commutator connected to it contains in the electric motor too. When the current from the direct current source passes through the commutator to the armature; as the current flowing through, the armature force acts on the two arms of the armature, the directions of which are decided by Fleming's left hand rule. Accordingly a couple of forces is setup on the armature which makes it to rotate.

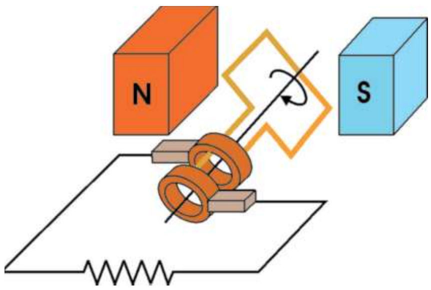


Figure 6.19

At the end of each half of a turn the positions of the arms of the armature interchange their positions in the rotating space and at the same instant, according to the plan of the commutator the current gets reversed in the armature. Due to this the sense of the couple and also of rotation, the armature continues without interruption.

6.13 Dynamo effect of a motor

When a motor is working the rotation of a coil in a magnetic field fulfills the conditions of a generator (dynamo). The result is that an electro motive force (emf) develops in the armature due to induction. According to the laws of induction, this emf acts in the sense opposite to that of the input emf. Hence it is known as a "Back emf". If the back emf is E and the input voltage is V, the armature current is,

$$I_a = \frac{V - E}{R_a}$$ where R_a is the armatures resistance.

This back emf is proportional to the flux density of the magnetic field and the speed of rotation of the armature. When the motor is activated its back e.m.f. is zero but the current develops fast along with the speed of the armature. This instantaneous increase of initial current would be harmful to the armature. Hence in order to control it, a device called the " starter switch" is set up along with a resistor in the motor circuit.

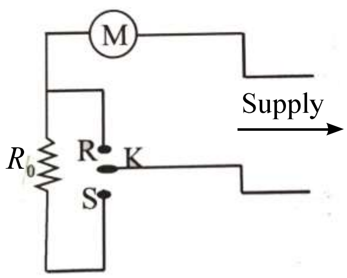


Figure 6.20

Initially the switch K is connected to the terminals (Start) and the starting current is controlled. Then, when the back e.m.f. too has developed along with the speed of the armature, the switch K is connected to the terminal R (Run).

Example

A motor with an armature of resistance 2 Ω is connected to a supply of 240 V. Without a load the armature turns at a rate of 300 r.p.m. and the armature current is 5 A with a load if the current becomes 35 A, what is the rate of rotation in r.p.m. of the armature?

Using $I_a = \frac{V - E}{R_a}$ (where E is the back e.m.f)

without the load, $5 = \frac{240 - E}{2}$

$E = 230$ V

with the load, $35 = \frac{240 - E}{2}$

$E = 170$ V

Since the speed of rotation of the armature is proportional to the back, e.m.f

$$\frac{230}{170} = \frac{300}{m}$$

$$m = \frac{300 \times 170}{230} = \underline{\underline{222 \text{ r.p.m}}}$$

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