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G . C. E. (Advanced Level)
PHYSICS

Grade 12
Resource Book

Unit 1: Measurement
Unit 2: Mechanics

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Faculty of Science and Technology
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Physics
Resource Book
Grade 12

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Message from the Director General

The National Institute of Education takes opportune steps from time to time for the development of quality in education. Preparation of supplementary resource books for respective subjects is one such initiative.

Supplementary resource books have been composed by a team of curriculum developers of the National Institute of Education, subject experts from the national universities and experienced teachers from the school system. Because these resource books have been written so that they are in line with the G. C. E. (A/L) new syllabus implemented in 2017, students can broaden their understanding of the subject matter by referring these books while teachers can refer them in order to plan more effective learning teaching activities.

I wish to express my sincere gratitude to the staff members of the National Institute of Education and external subject experts who made their academic contribution to make this material available to you.

Dr. (Mrs.) T. A. R. J. Gunasekara
Director General
National Institute of Education
Maharagama.

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Message from the Director

Since 2017, a rationalized curriculum, which is an updated version of the previous curriculum is in effect for the G.C.E (A/L) in the general education system of Sri Lanka. In this new curriculum cycle, revisions were made in the subject content, mode of delivery and curricular materials of the G.C.E. (A/L) Physics, Chemistry and Biology. Several alterations in the learning teaching sequence were also made. A new Teachers' Guide was introduced in place of the previous Teacher's Instruction Manual. In concurrence to that, certain changes in the learning teaching methodology, evaluation and assessment are expected. The newly introduced Teachers' Guide provides learning outcomes, a guideline for teachers to mould the learning events, assessment and evaluation.

When implementing the previous curricula, the use of internationally recognized standard textbooks published in English was imperative for the Advanced Level science subjects. Due to the contradictions of facts related to the subject matter between different textbooks and inclusion of the content beyond the limits of the local curriculum, the usage of those books was not convenient for both teachers and students. This book comes to you as an attempt to overcome that issue.

As this book is available in Sinhala, Tamil, and English, the book offers students an opportunity to refer the relevant subject content in their mother tongue as well as in English within the limits of the local curriculum. It also provides both students and teachers a source of reliable information expected by the curriculum instead of various information gathered from the other sources.

This book authored by subject experts from the universities and experienced subject teachers is presented to you followed by the approval of the Academic Affairs Board and the Council of the National Institute of Education. Thus, it can be recommended as a material of high standard.

Dr. A. D. A. De Silva
Director
Department of Science

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Unit 1
Measurement

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Introduction to physics

If we go back to human history we can see that most discoveries which are useful to human development are based on science, information networks, electric communication media which spread around the world transformed the whole community as staying in a small village. Science is the basis of all miracles in medicine and the development of modern technology. It is not an exaggeration to say that discoveries in physics are conspicuous.

Figure .1.1- Electron microscope

Figure 1.2- Hubble telescope

Physics is involved in constructing instruments from electron microscopes which can be used to see viruses clearly, (for which light microscope with even a higher magnification cannot be used) to the Hubble telescope, which can be used to see originating and diminishing stars at far distances in the universe. These instruments help widen the vision range of humans.

Figure 1.3- Concord air plane

Because of the discoveries in physics, man who travelled by a cart at the very beginning developed the transport facilities to travel by a concord air plane, which exceeds the speed of sound.

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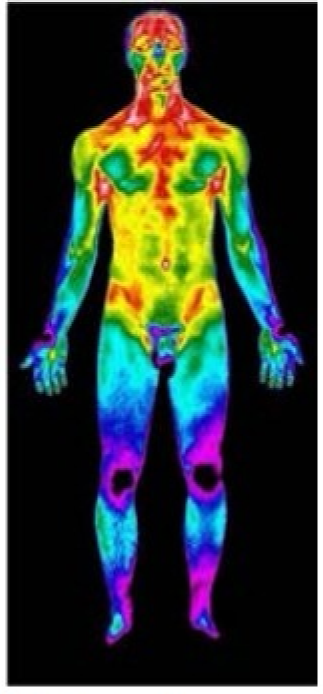

Figure 1.4- A photograph taken in the dark by infrared rays

Figure 1.5- High pressure water jet

Physics is directly involved with constructing instruments such as infrared binoculars which are used for far night vision and also high pressure water jets used to cut concrete.Physics is the basis for discoveries that has led to the development of the stethoscope to modern medical devices such as ultra sound scan, CT scan and MRI scan which are used for diagnosis in medicine.

The contribution of the discoveries of scientists in producing such excellent constructions should be appreciated. In this connection more service has been renderd by scientists such as Galileo, Newton, Robert Boyle, Albert Einstein and Stephen Hawking.

Among the pioneer scientists who were responsible for important discoveries in the field of physics by performing experiments in physics, Galileo Galilei (1564-1642) can be the first to be mentioned. He is the one who discovered the characteristic properties of the simple pendulum which led to revolutionary changes in measuring time. The service done by him for the future advancement of physics was more important because of the discoveries such as principles of motion and Galilean telescope.

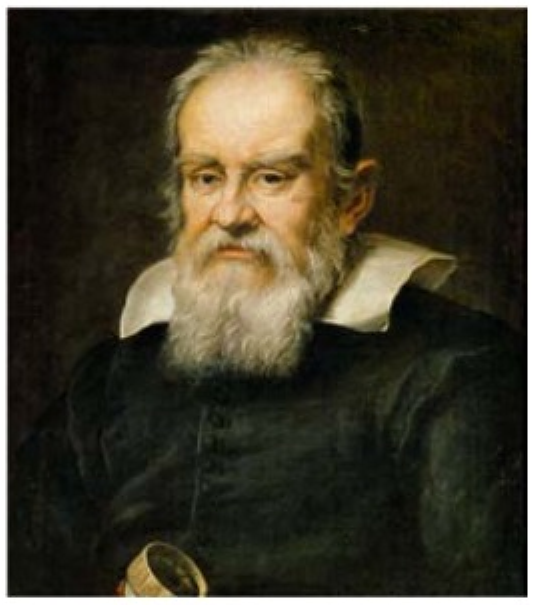

Figure 1.6- Galileo Galilei

Figure 1.7-Sir Isaac Newton

Sir Isaac Newton (1642-1727) can be introduced as a great scientist who contributed to a large number of discoveries in physics. Among his discoveries, gravitation, basic concepts of calculus which transformed mathematics in a new way and the discovery of the colour components of sunlight can be considered to be important.

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Robert Boyle (1627-1691) who also lived in this period of time did research relating to gases and the basic discoveries that followed is more used in chemistry though outlined in the work of physics.

Albert Einstein (1879-1955) produced the modern constructive concept of the special theory of relativity which even changed the former accepted theories to explain day-to-day activities and experiments in physics. In the twentieth century, he was awarded the Nobel prize for modern mathematical physics and photoelectric effect.

Figure 1.8-Albert Einstein

Figure 1.9-Stephen Hawking

Stephen Hawking clearly understood and managed to explain black holes that even most recent astronomical scientists were unable to explain as a very high gravitational field, using physical and mathematical principles and concepts.

Light and other electromagnetic waves were considered until recent times to be continuous flowing of energy. A phenomenon like photoelectric effect cannot be explained using wave nature of light only. As a solution for these problems, Max Planck (1854-1947) introduced the Quantum Theory. According to his idea, when an electron falls from an outer orbit of an atom to an inner orbit, a packet of energy is released as a quantum which is equal to the difference in energy between the two levels.

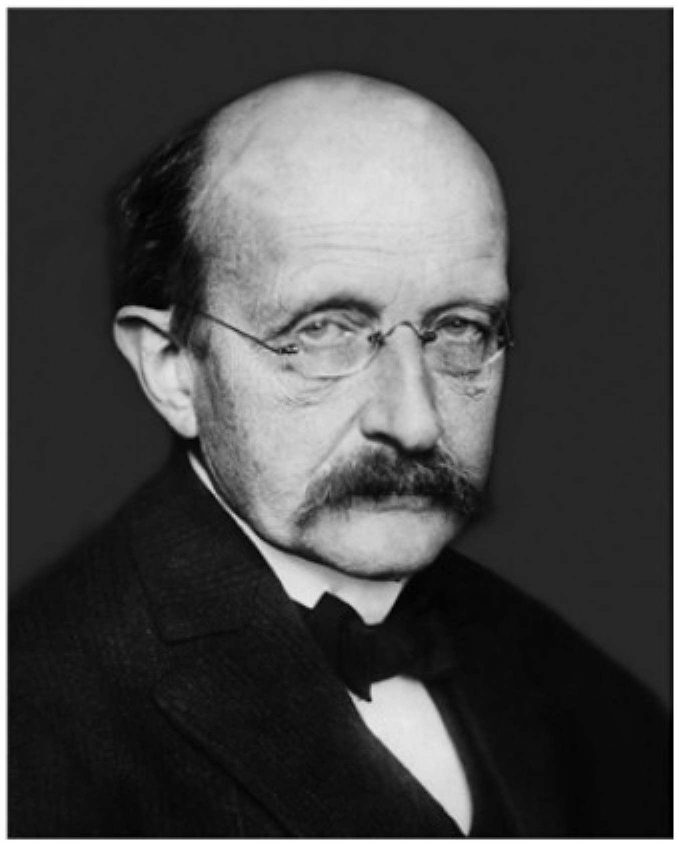

Figure 1.10 - Max Planck

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Physics rapidly developed from various discoveries and theories presented by different scientists. The basic foundation for these developments was the scientific method adopted for this.

It is important for a person who studies science to have an understanding about the scientific method. Collecting information and knowledge through experiences in a formal way by observing the nature can be considered a basic step. To explain those phenomena, hypotheses, principles and laws have to be developed. Models are being prepared which can explain these hypotheses and principles. To test the correctness of the models, formal experimental methods can be applied. Predictions can be made by models using results obtained by experiments and according to the progress or failure of them. Models are developed using new experiences. This continuous development process is called the scientific method.

Measurements done using a correct approach is important in building most theories in physics. As such, we can consider measurement to be the foundation of physics.

Scientific methodology

The steps of scientific methodology

- Observation
- Hypothesis
- Experiment
- Theory
- Prediction

Observation

The first step in scientific method is to make careful observations to collect data. The data may be drawn from a simple observation, or they may be obtained from experiments.

Hypothesis

From an analysis of these observations and experimental data, a model of nature is hypothesized. The hypothesis is an assumption that is made in order to draw out and test its logical or empirical consequences. We should be able to confirm it by testing. Testing of the hypothesis is called the experiment.

Experiment

An experiment is a controlled procedure carried out to discover, test, or demonstrate something. An experiment is performed to confirm that the hypothesis is valid. If the results of the experiment do not support the hypothesis, the experimental procedure must be checked. If the procedure is alternate and results still contradict the hypothesis, then the original hypothesis must be modified. Another experiment is then designed to test the modified hypothesis.

Theory

If the experimental results confirm the hypothesis, the hypothesis becomes a new theory about some specific aspect of nature, a scientifically acceptable general principle based on observed facts.

Prediction

After a careful analysis of the new theory, a prediction about some unknown aspect of nature can be made.

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Chapter 02

Physical quantities and units

A property which can be measured directly or indirectly, in a physical system can be expressed as a physical quantity.

Example 1

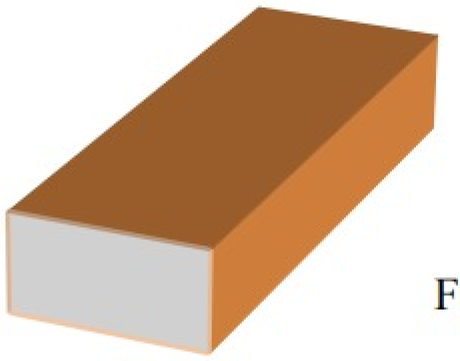

Figure 2.1

Think about the description of a log of wood as shown in Figure 2.1. For this, physical quantities such as its length, breadth, height, mass, volume and density can be considerd. Of these, length, breadth and height can be measured directly whereas volume and density can be calculated.

Example 2

When describing the motion of a vehicle, quantities such as the distance travelled by the vehicle in between two points, the time spent to travel that distance, speed or velocity of the vehicle and acceleration can be used.

Measurements have a magnitude and a unit and sometimes a direction as well. “metre” is the international unit for measuring length. Lengths have to be measured from a very small value to a very large value. A tabulation is shown below, about several objects within that range of measurement.

Table 2.1 Ranges of distance relavant to various objects

Object	**Range of distance (m)**
diameter of a proton	10^{-15}
diameter of the nucleus of a heavy atom	10^{-14}
wavelength of γ rays	10^{-12}
average distance between atoms of a crystalline solid substance	10^{-10}
distance between air molecules inside a room	10^{-8}
wavelength of visible light	10^{-7}
diameter of a red corpuscle	10^{-5}
thickness of a paper	10^{-4}
thickness of a window glass plate	10^{-3}
diameter of a pencil	10^{-2}
length of a pencil	10^{-1}
height of a child	$10^{0}=1$
height of a three storied building	10^{1}

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Object	**Range of distance (m)**
length of a football ground	10^2
maximum depth of the sea	10^4
diameter of the moon	10^6
diameter of the Earth	10^7
distance from the Earth to the moon	10^8
diameter of the sun	10^9
distance from sun to the Earth	10^{11}
distance from sun to the planet Saturn	10^{12}
distance to the nearest star	10^{17}
end of the observed universe	10^{27}

In measuring time there are measurements from a very small value to a very large value. A tabulation about them is given below.

Table 2.2 Measurments related with time

Incident	**Time spent (s)**
light to travel through an atomic nucleus	10^{-24}
time for one revolution of a proton inside an atomic nucleus	10^{-22}
time for one revolution of an electron around the nucleus of inner orbit of a heavy atom	10^{-20}
time for one revolution of an electron around the nucleus of a hydrogen atom	10^{-15}
light to travel through a window glass plate	10^{-11}
light to travel across a classroom	10^{-8}
for one vibration of a high frequency sound note	10^{-4}
for one rotation of an electric fan	10^{-2}
for rifle bullet to travel across a football ground	10^{-1}
periodic time of the bob of a pendulum clock	$10^0 = 1$
shorter distance runner to run 100 meters	10^1
for light to travel from sun to the earth	10^3
for earth to rotate one revolution about its axis (one day)	10^5
for one revolution of the earth around sun (one year)	10^7
life time of a human	10^9
half life of radium	10^{10}
time from Christian era to present	10^{11}
time from ancient human era to present	10^{13}
time for one revolution of the sun around the galaxy	10^{15}
age of an ancient fossil	10^{17}
expected life time of sun as a normal star	10^{18}

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In general the measurement of mass involves relatively large values though some are very small

Table 2.3 Mesurments related with mass.

Object	**Order of the mass (kg)**
galaxy	10^{41}
star	10^{32} - 10^{28}
sun	10^{30}
the Earth	10^{25}
moon	10^{22}
large air plane	10^{6}
an elephant	10^{4}
a human	10^{2}
a dog	10^{1}
one litre of water	$10^{0} = 1$
one apple fruit	10^{-1}
a simple living cell	10^{-10}
a red blood corpuscle	10^{-22}
a heavy atom	10^{-25}
a proton	10^{-27}
an electron	10^{-31}

International system of units – SI

According to international system of units seven basic quantities and seven corresponding units have been approved.

Table. 2.4 - Seven basic quantities and corresponding SI Units

Basic (fundamental) quantity	**Unit**	**Symbol of the unit**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Thermodynamic temperature	kelvin	K
Electric current	ampere	A
Amount of substance	mole	mol
Luminous intensity	candela	cd

Table 2.5 - Supplementary SI units

Quantity	**Unit**	**Symbol of unit**
Plane angle	radian	rad
Solid angle	steradian	sr

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Definition of basic SI units

metre (m) - metre is the length of 1,650,763.73 wavelengths of radiation emitted in a vacuum due to the transfer of an electron between energy levels $2p^{10}$ and $5d^5$ of the Krypton – 86 atom.

second (s) - second is the time required for 9,192,631,770 periods of radiation due to the transfer between the two micro levels in the ground state of Cesium – 133 atom.

ampere (A) - ampere is that current that, exsists in each of two infinitely long parallel straight wires with negligible circular cross section, placed 1m apart in a vacuum which results in a force of exactly of 1m length of each conductor.

kelvin (K) - Thermodynamic unit of temperature kelvin is the fraction 1/273.16 of the thermodynamic temperature at triple point of water.

candela (cd) - candela is the luminous intensity that is produced perpendicular to a surface of $\frac{1}{600,000}$ square metre of a black body at a temperature of freezing platinum under a pressure of $101,325$ newtons per metre.

Units which can be obtained using basic units are called derived SI units. SI unit of a derived physical quantity can be obtained by using the definition of that physical quantity. In writing the units, there should be a gap between the symbols of basic units. There should not be a dot or a coma in between basic units. There are special names for some derived units, but those units can also be expressed by SI base units.

Some derived units without special names are given in the following table.

Quantity	**Definition/Survey**	**SI unit**
Area	length × breadth	m^2
Volume	length × breadth × height	m^3
Velocity	$\frac{\text{displacement}}{\text{time}}$	$m\ s^{-1}$
Acceleration	$\frac{\text{difference in velocity}}{\text{time}}$	$m\ s^{-2}$
Density	$\frac{\text{mass}}{\text{volume}}$	$kg\ m^{-3}$
Momentum	mass × velocity	$kg\ m\ s^{-1}$

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Several SI units with special names are given below:

Table 2.7 Several SI Units with special names

Quantity	**Special name of units**	**Symbol**	**Definition using expression**	**Other SI units**	**Basic units**
Force	newton	N	mass × acceleration		$kg\ m\ s^{-2}$
Pressure	pascal	Pa	$\frac{\text{force}}{\text{area}}$	$N\ m^{-2}$	$kg\ m^{-1}s^{-2}$
Work	joule	J	force × displacement	N m	$kg\ m^2s^{-2}$
Energy	joule	J	ability to work	N m	$kg\ m^2s^{-2}$
Power watt		W	$\frac{\text{work}}{\text{time}}$	$J\ s^{-1}$	$kg\ m^2\ s^{-3}$
Frequency resistance	hertz	Hz	$\frac{\text{no.of vibrations}}{\text{time}}$		s^{-1}
Electrical charge	coulomb	C	current × time		A s
Electric potential	volt	V	$\frac{\text{work}}{\text{charge}}$	$J\ C^{-1}$	$kg\ m^2\ s^{-3}\ A^{-1}$
Electrical resistance	ohm	Ω	$\frac{\text{potential}}{\text{current}}$	VA^{-1}	$kg\ m^2\ s^{-3}\ A^{-2}$
Electrical capacitance	farad	F	$\frac{\text{charge}}{\text{potential}}$	$C\ V^{-1}$	$A^2s^4kg^{-1}m^{-2}$
Magnetic flux density	tesla	T	$\frac{\text{magnetic flux}}{\text{area}}$	$Wb\ m^{-2}$	$kgs^{-2}A^{-1}$
Luminous flux	lumen	lm	luminous intensity × solid angle	cd sr	

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SI derived units expressed using special names, are given below.

Table 2.8 Derived SI units with special names

Quantity	**Name of the SI unit**	**Symbol of SI unit**	**Expression using SI base units**
Entropy/thermal capacity	joules per kelvin	$J\ K^{-1}$	$m^2\ kg\ s^{-2}\ K^{-1}$
Thermal conductivity	watt per metre per kelvin	$W\ m^{-1}\ K^{-1}$	$m\ kg\ s^{-3}\ K^{-1}$
Permeability	henry per metre	$H\ m^{-1}$	$m\ kg\ s^{-2}\ A^{-2}$
Permittivity	farad per metre	$F\ m^{-1}$	$m^{-3}\ kg^{-1}\ s^4\ A^2$
Surface tension	newton per metre	$N\ m^{-1}$	$kg\ s^{-2}$
Moment of a force	newton metre	$N\ m$	$m^2\ kg\ s^{-2}$
Electric field intensity	volt per metre	$V\ m^{-1}$	$m\ kg\ s^{-3}\ A^{-1}$
Electric flux density	coulomb per square metre	$C\ m^{-2}$	$m^{-2}\ s\ A$
Specific entropy	joules per kilogram per kelvin	$J\ kg^{-1}\ K^{-1}$	$m^2\ s^{-2}\ K^{-1}$
Specific heat capacity	joules per kilogram per kelvin	$J\ kg^{-1}\ K^{-1}$	$m^2\ s^{-2}\ K^{-1}$

If the value of a physical quantity is very small or very large it is not possible to write or read it as it is. In such instances prefixes are used to indicate the multiples or submultiples of SI units. Name and symbols of multiple factors of several prefixes are given in the following table.

Table 2.9-Names and symbols of multiple factors of several prefixes

Multiple factor	**Name of prefix**	**Symbol**
10^{18}	*exa*	E
10^{15}	*peta*	P
10^{12}	*tera*	T
10^{9}	*giga*	G
10^{6}	*mega*	M
10^{3}	*kilo*	k
10^{2}	*hecto*	h
10^{1}	*deca*	da
10^{-1}	*deci*	d
10^{-2}	*centi*	c
10^{-3}	*milli*	m
10^{-6}	*micro*	μ
10^{-9}	*nano*	n
10^{-12}	*pico*	p
10^{-15}	*femto*	f
10^{-18}	*atto*	a

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Rules to obey in writing SI units.

- In writing base units, the unit should be written on the right side of the value and close to it.

  e.g. ten metres - 10 m

- No plurals are used in writing SI units even if the value is greater than one.

  e.g. five kilograms - 5 kg

- In writing units as a product of base units there should be a single gap between base units.

  e.g. ten metres per second - 10 m s^{-1}

- In writing the value of a physical quantity with a prefix the symbol corresponding to the prefix should be written in front of the SI unit symbol and there should not be a gap between the two symbols.

  e.g. millisecond - ms

- In writing temperature in kelvin scale no need to put a small zero to indicate degrees.

  e.g 303 kelvin - 303 K

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Chapter 03

Dimensions

In the system of SI units, mass, length, time, electric current, thermodynamic temperature, luminous intensity and amount of substance are the basic units. Other quantities such as energy and acceleration which can be derived using basic units are called derived units. Symbolic expressions which show the relation of a physical quantity to basic units is called its dimension. In studying mechanics and properties of matter, mostly used dimensions are mass M, length L, and time T. Also there are dimensions for other quantities like temperature and electric current.

The way of finding the dimensions of several physical quantities we meet in mechanics are given below.

Table 3.1 dimensions of several physical quantities

Physical quantity	**Basic relation**	**Dimension**
Area	length × breadth	L^2
Volume	length × breadth × height	L^3
Density	mass / volume	ML^{-3}
Velocity	displacement / time	LT^{-1}
Acceleration	velocity / time	LT^{-2}
Force	mass × acceleration	MLT^{-2}

Quantities without units such as refractive index, coefficient of friction have no dimensions. There are quantities with units but without dimensions.

e.g. plane angle and solid angle

Several uses of dimensions are given below;

1. To check the correctness of a given relation among physical quantities.
2. To derive a relation among physical quantities.

The following can be given as examples for the above.

1. Checking the correctness of an equation using dimensions.

How several physical quantities are related to one another is expressed mathematically by a physical equation. If the equation is correct, the dimension of its both sides should be equal. If the equation has several terms, the dimensions of all the separate terms should be the same.

If the equation connecting physical quantities a, b, c, d and e is,

$$a = bc + \frac{d}{e}$$

dimensions of a = dimensions of bc = dimensions of $\frac{d}{e}$

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Worked examples

If an object that travels along a straight line with an initial velocity *u* and uniform acceleration *a* for a time *t* and attains a final velocity *v* and the displacement experienced by it is *s,* equations relating to its motion are given below.

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

(iv) $s = \left[\frac{v+u}{2}\right]t$

(i) In the equation

$v = u + at$

dimension of $[v] = \mathrm{LT}^{-1}$

$[u] = \mathrm{LT}^{-1}$

$[at] = [\mathrm{LT}^{-2}] \times [\mathrm{T}]$

$= \mathrm{LT}^{-1}$

$\therefore$ dimensions of $[v] = [u] = [at]$

Dimensions of both sides of the equation is the same and all terms have the same dimensions.

$\therefore$ Equation (i) is dimensionally correct.

(ii) In the equation

$s = ut + \frac{1}{2}at^2$

dimensions of $[s] = \mathrm{L}$

$[ut] = [\mathrm{LT}^{-1}] \times [\mathrm{T}]$

$= \mathrm{L}$

$[at^2] = [\mathrm{LT}^{-2}] \times [\mathrm{T}^2] = \mathrm{L}$

$\therefore$ dimensions of $[s] = [ut] = [at^2]$

Dimensions of both sides of the equation are the same and all terms have the same dimensions.

Equation (ii) is dimensionally correct.

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(iii) In the equation

$$v^2 = u^2 + 2as$$

dimensions of $[v^2] = [LT^{-1}]^2 = L^2T^{-2}$

$[u^2] = [LT^{-1}]^2 = L^2T^{-2}$

$[as] = [LT^{-2}] \times L^2T^{-2}$

$\therefore$ dimensions of $\left[v^2\right] = \left[u^2\right] = \left[as\right]$

Dimension of both sides of the equation is the same and all terms have the same dimensions.

$\therefore$ equation (iii) is dimensionally correct.

(iv) From the equation (iv)

$$s = \left[\frac{v+u}{2}\right]t$$

another important property of dimensional analysis can be introduced. If there is a term in an equation expressing sum or difference, the two quantities in the term with sum or difference should be of the same dimensions. i.e.

dimensions of $[u] = [v]$

dimensions of $[s] = L$

$[vt]$ and $[ut] = [LT^{-1}] \times [T]$

$= L$

Dimensions of both side of the equation is the same and all terms have the same dimensions.

$\therefore$ equation (iv) is dimensionally correct.

2. Derivation of equation by the method of dimensional analysis

Suppose we want to develop an equation for the periodic time T of a simple pendulum. First we have to identify experimentally or logically the quantities related to the periodic time. Here we can assume that the periodic time depends on the following quantities.

(i) Mass of the pendulum bob (m)

(ii) Length of the pendulum (l)

(iii) Gravitational acceleration (g)

Since we have no idea about the correct relation among the quantities we can write,

$T \;\alpha\; m^x l^y g^z$ here x, y and z are real numbers

$T = k\, m^x l^y g^z$ where k is the proportionality constant without a dimension.

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Applying dimensions to both side of the equation,

$$T = M^x L^y (LT^{-2})^z$$

$$T = M^x L^{y+Z} T^{-2z}$$

Equating the indices of M

$x = 0$............... (1)

Equating the indices of L

$y+z = 0$........... (2)

Equating the indices of T

$-2z = 1$............ (3)

Solving the above equations

$$x = 0,\ y = \frac{1}{2},\ z = -\frac{1}{2}$$

The relation among quantities can be written as

$$T = km^0 l^{1/2} g^{-1/2}$$

$$\therefore\ T = k\sqrt{\frac{l}{g}}$$

The value of the constant k cannot be found by dimensional analysis.

When a stone is tied to a string and it moves in a circle in a horizontal plane, we can assume that the tension F gained by the string depends on the mass m of the stone, speed v of the stone and the radius r of the circle in which it moves. Since we have no idea about the correct relation among the quantities, we can write,

$F\ \alpha\ m^x v^y r^z$ where x, y and z are real numbers.

$F\ =\ k\ m^x v^y r^z$ where k is a proportionality constant without dimension.

Applying dimensions to both sides of the equation

$$MLT^{-2} = M^x (LT^{-1})^y L^Z$$

$$MLT^{-2} = M^x L^{y+z} T^{-y}$$

Equating indices of M

$x = 1$........... (1)

Equating indices of L

$y+z = 1$......... (2)

Equating indices of T

$-y = -2$......... (3)

solving the above equations,

$$x = 1,\ y = 2,\ z = -1$$

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The relation among quantities can be written as

$$F = kmv^2r^{-1}$$

$$F = k\frac{mv^2}{r}$$

Different tools are used to collect data in the field of science. Experiments should be done for the verification of models built by organizing the collected data, and in the quantitative analysis main activity is the measurement. There is a development of theories from measurements so obtained. Several such instances can be introduced like this.

The solar system was found from the measurements of the position of planets taken over a long period of time. Measurement of forces between masses is the reason for the total development of the law of universal gravitation. Measurements about the motion of objects help the development of mechanics. Measurement of force between charges, forces between currents and other phenomena about electricity and magnetism is the reason for the development of electricity, magnetism and electromagnetism.

It is clear that different measurements made according to the observed natural phenomena is the reason for new discoveries and development of theories so formulated in physics.

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Chapter 04

Measuring instruments

Working with errors (Working with uncertainties)

Systematic errors (systematic uncertainties)

These occur due to faulty apparatus such as an incorrectly labelled scale, an incorrect zero mark on a meter or a stopwatch running slowly. Repeating the measurement a number of times will have no effect on this type of error and it may not even be suspected until the final result is calculated. To eliminate this type of error, a correction can be introduced to the final reading. The instrument can be recalibrated or replaced.

Random errors (random uncertainties)

The size of these errors depends on how well the experimenter can use the apparatus. The better the experimenter, the smaller will be the random error reflected in an experiment. Making a number of readings of a given quantity and taking an average will reduce the overall error.

The maximum error that can occur during a measurement is the least count of the scale. The size of the error needs to be considered together with the size of the quantity which is being measured.
For example,
(208 ± 1) mm is a fairly accurate measurement.
(2 ± 1) mm is highly inaccurate.
In order to compare errors, use is made of absolute, fractional and percentage error.

For the reading (208 ± 1) mm;
1 mm is the absolute error.
1/208 is the fractional error (= 0.0048).
0.48% is the percentage error.

As we usually require error to only one significant figure, the last two values given above would be used as 0.005 and 0.5%, respectively.

The accuracy of a measurement is considered sufficient if the percentage error is 1 % or less than 1%. When we use a metre ruler for measuring a length of 100 mm, the percentage error is $\frac{1}{100} \times 100 = 1\%$. Therefore, the accuracy provided by a meter ruler in measuring a length which is les than 10 cm, is not sufficient. In such a case, a measuring instrument with a least count less than 1mm should be used. For this, the instruments which use the principle of vernier or screw are required. When calculating the value of a quantity y given by $y = a^n b$, the error of 'a' influences much higher than the error of 'b' towards the error of 'y'. Therefore, when measuring a term which has been raised to some power, an additional care should be taken.

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When we calculate a final result like $y = a^n b$, the error of the quantity "a" will have a greater effect in the error in y. Therefore, we have to take an extra care in measuring terms raised to powers.

Various measuring instruments are used to measure various physical quantities. But here we consider only the measuring instruments used to measure physical quantities such as length, mass and time which are mostly used in mechanics.

A measuring instrument has a scale and there is a least count which can be obtained from that scale. The instrument is not used to measure more accurate values than this least count. As an example of a metre ruler calibrated in mm scale the least count is 1 mm. Because of this, we cannot obtain an accuracy more than 1mm from a metre ruler. According to this, we can express a measurement such as 17.3 cm or 17.4 cm but not as17.35 cm.

Principle of vernier and vernier scale

Scales which are constructed by dividing the range of certain number of divisions of a millimetre scale into equal parts greater than that numbers of divisions are called Vernier scales. The millimetre scale used is called the main scale.

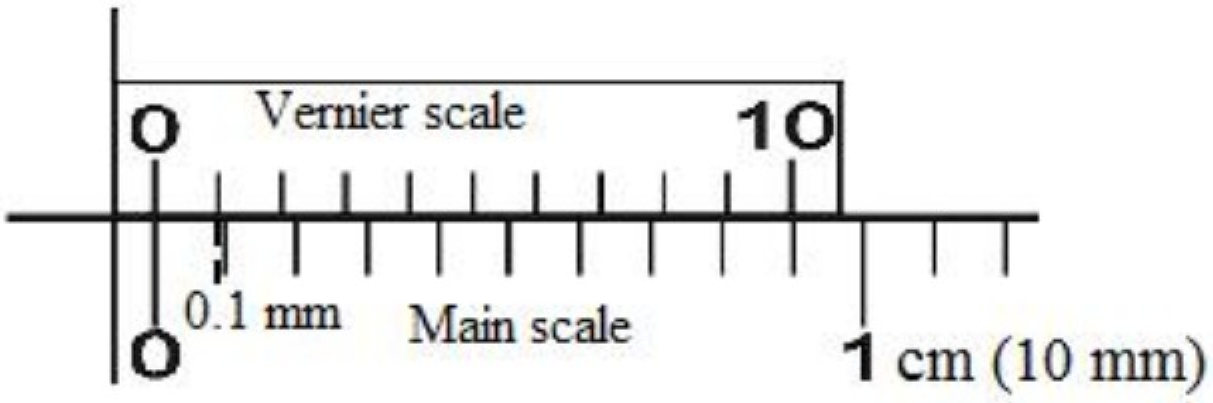

Figure 4.1

The diagram shows a vernier scale constructed by dividng the range of 9 mm in a milli metre scale into 10 equal parts. In the instrument with vernier scale, the vernier scale can be moved along the main scale. When the instrument is adjusted so that the front ends of both scales coincide, then the mark '0' of the vernier scale should coincide with mark '0' of the main scale and mark 10 of the vernier scale coincide with mark 9 mm of the main scale. No other marks of the vernier scale coincide with any other mark of the main scale.

To coincide mark 1 of the vernier scale with mark 1 of the main scale the distance which the vernier scale should be pushed forward is the difference in length between one part of the vernier scale and one part of the main scale. This length is called the least count of the vernier scale.

least count = length of one part of the main scale – length of one part of the vernier scale

According to this the least count of the above vernier scale can be found.

Length of one part of the main scale = 1 mm

Length of one part of the vernier scale = $\frac{1\times 9}{10}$ mm

$$\text{Least count} = \left(1-\frac{9}{10}\right) \text{mm}$$

$$= 0.1 \text{ mm}$$

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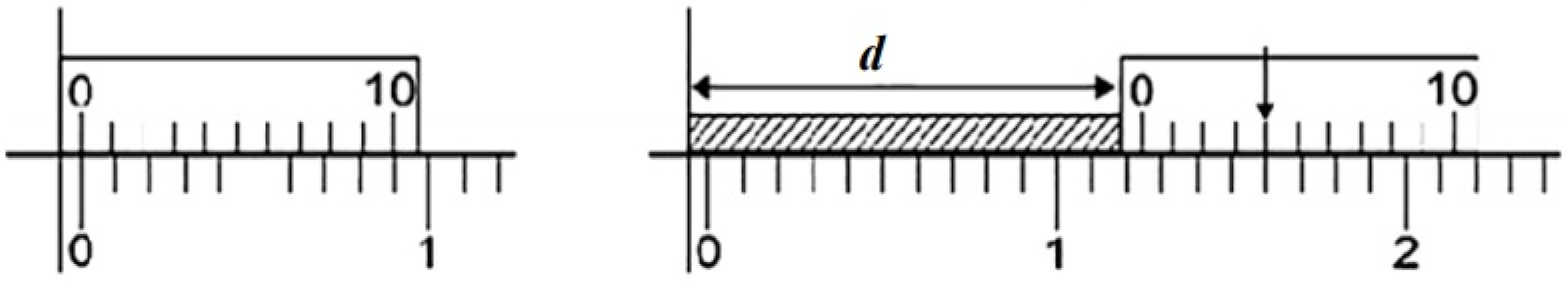
When taking a measurement such as the thickness of a normal ruler (2 mm) using an instrument with the above mentioned vernier scale, the percentage error is very large.

That percentage error $= \frac{0.1}{2} \times 100\,\% = 5\%$

To minimize the percentage error of such a measurement we need vernier scales with other fractions of millimetre. We know that Vernier scales can be constructed by dividing the range of certain divisions of a linear scale into another equal divisions. But as the numbers of division are large it is difficult to observe from the naked eye, which division of the vernier scale coincides with a division of the main scale. Sometimes it is difficult to observe using a magnifying glass also. Because of this, as a result of experiments done to obtain measurements like thickness of a paper, the screw instruments were produced. The principle of screw instruments are discussed later.



(a) **Figure** 4.2 (b)

To measure the length d of a rod using a vernier arrangement as shown in Figure 4.2 (a), one end of the rod should be kept coinciding with the first edge of the main scale and the vernier scale is in contact with the other end of the rod as shown in the Figure 4.2 (b). The length d of the rod is equal to the distance moved by the first edge of the vernier scale. This distance is also equal to the distance moved by the mark '0' of the vernier scale. Therefore, the length of the rod can be obtained from the reading corresponding to the mark '0' of the vernier scale. Consider the way of observing that reading.

The value of the main scale in front of the mark '0' of the vernier scale is 1.1 cm, which is the reading of the main scale. The distance between this reading and mark '0' of the vernier scale should be read by the vernier scale. We know that the distance through which the vernier scale should be moved to coincide first mark of the vernier scale with first mark of the main scale is 1 × least count. To coincide the 2nd mark of the vernier scale with the 2nd mark of the main scale, the distance through which the vernier scale should be moved is 2 × least count. Here the 4th mark of vernier coincides with a mark of the main scale (Figure 4.2 (b)) and hence the distance moved by the vernier scale is 4 × least count = $4 \times 0.1 = 0.4$ mm. Therefore, the reading given by the vernier scale is 0.04 cm.

$\therefore\ d = (1.1 + 0.04) = 1.14$ cm

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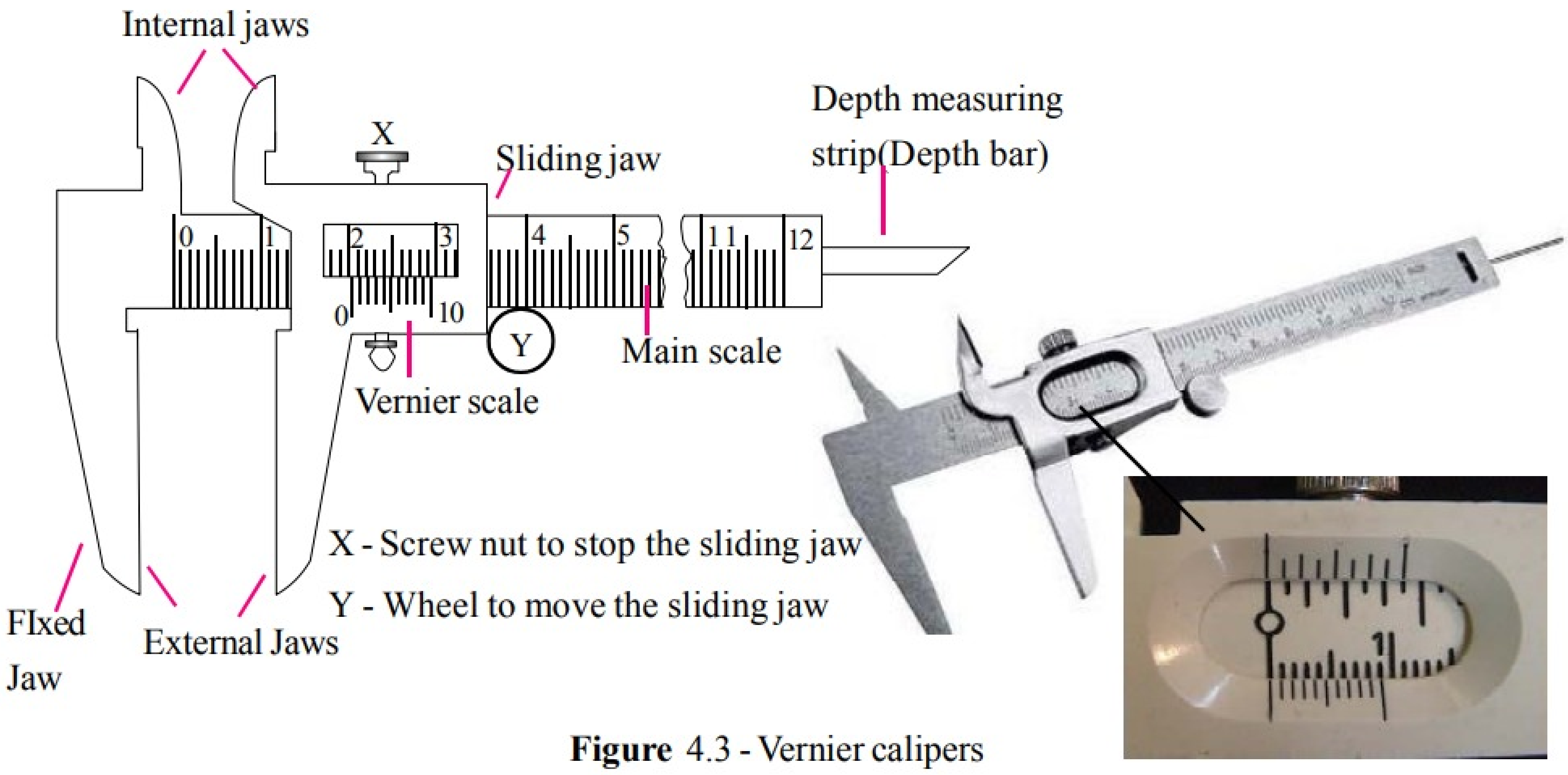
The length can be measured accurately up to $\frac{1}{100}$ cm = 0.01 cm using this vernier scale.

Vernier calipers



X - Screw nut to stop the sliding jaw

Y - Wheel to move the sliding jaw

Figure 4.3 - Vernier calipers

Figure 4.3 shows vernier calipers constructed using a similar vernier scale. It consists of a fixed jaw attached to a main scale calibrated in mm and a movable jaw attached to a vernier scale which can slide along the main scale. A rod is also attached to the vernier scale. External jaws can be used to measure external diameter of a cylinder, internal jaws can be used to measure internal diameter of a hollow cylinder and the rod can be used to measure the depth of a hole. By rotating the wheel, vernier scale with the movable jaws can be moved to and fro along the main scale and by tightening the nut, vernier scale can be kept in a fixed position before taking a reading.

Vernier calipers mostly used in laboratories for experimental work, have a main scale calibrated in mm and the vernier scale is marked by dividing the 9 mm range of the main scale into 10 equal parts. According to this the least count of the vernier calipers is 0.1 mm or 0.01 cm.

Zero error

When the vernier scale of the vernier calipers is adjusted such that the jaws are in contact with the zero mark of the vernier scale should coincide with the zero mark of the main scale. But in some vernier calipers of the above type, due to some reasons such as corrosion or wasted jaws, the zero mark of the vernier scale lies to the right side or to the left side of the main scale zero mark. Because of this there is an error in the instrument. This error is called the zero error of the vernier calipers.

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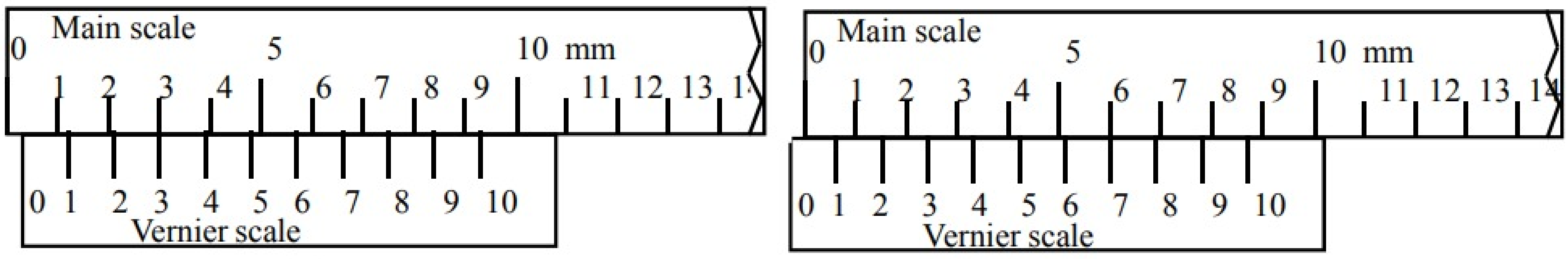

(*a*) (*b*)

Figure 4.4

Example 1

When the jaws of the vernier calipers are in contact, suppose that the 2nd mark of the vernier scale coincides with a mark of the main scale as shown in the Figure 4.4 (a). Then the reading indicated by the vernier scale is 3×0.1 mm. Because of this, the zero error is denoted by +0.3 mm. When there is a zero error, reading of the measurement obtained by the instrument is incorrect. The reading obtained from the vernier calipers indicated in the Figure 4.4(a) is more than the true value. To obtain the true value if the zero error is positive, zero correction is negative. That is, for the correction, zero error should be deducted from the relevant reading.

Example 2

The reading obtained from the vernier calipers indicated in the Figure 4.4 (b) is less than the true value. If the zero of the vernier is moved to the left side of the zero of the main scale, zero error is a negative number and for the correction, this value should be added to the relevant reading. Because of this, the zero correction is a positive number. Therefore, the magnitude of zero error of the vernier calipers indicated in Figure 4.4 (b) is;

$$7 \times 0.9 \text{ mm} - 6 \times 1 \text{ mm} = (6.3 - 6)\text{mm} = 0.3 \text{ mm}$$

Whether the zero error is positive or negative is not an important issue. Now the concept of positive and negative is hardly in use. When the vernier scale is adjusted so that the jaws of the vernier calipers are in contact and if the zero mark of the vernier scale lies to the right side of the zero mark of the main scale, the zero error should be deducted from the relevant reading. If the zero mark of the vernier scale lies to the left side of the zero mark of the main scale, the zero error should be added to the relevant reading.

Vernier scales with least counts smaller than 0.1 mm is used to measure very small lengths. In the travelling microscope the main scale is divided upto 0.5 mm parts. Vernier scale is formed by dividing the range of 49 parts of the main divisions into 50 equal parts. Consider the way of calculating the least count of this vernier.

Length of one division of the main scale = 0.5 mm

$$\text{Length of one division of the vernier scale} = \frac{0.5 \times 49}{50} \text{ mm}$$

$$\begin{aligned}\text{Least count of the vernier scale} &= \left(0.5 - \frac{0.5 \times 49}{50}\right) \text{mm} \\ &= 0.5 \times \frac{1}{50} \text{ mm} \\ &= \frac{1}{100} = 0.01 \text{ mm}\end{aligned}$$

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Travelling microscope

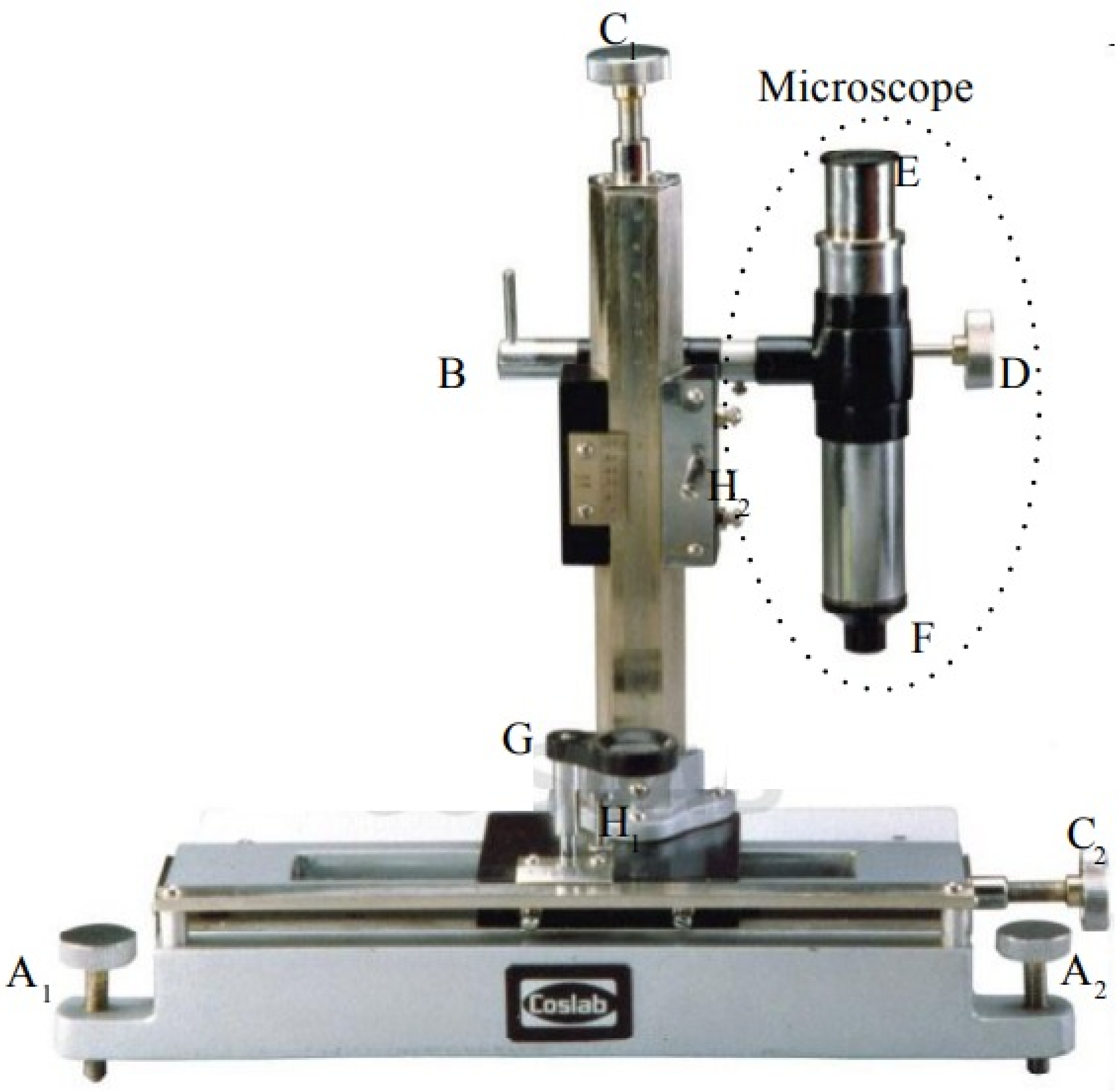

Figure 4.5-Travelling microscope

A_1, A_2 - Screws for levelling the plane of the travelling microscope horizontally (levelling screws)

B - Screw to fix microscope vertically and horizontally

C_1 - Screw for the fine adjustment of vertical sliding jaw

C_2 - Screw for the fine adjustment of the horizontal sliding jaw

D - Screw for focusing the microscope

E - Eye piece

F - Objective

G - Magnifying glass

The travelling microscope is shown in Figure 4.5 and its main parts are named. It consists of a base with a main scale calibrated in 0.5 mm divisions and a vertical scale calibrated in 0.5 mm parts attached to a platform with a vernier scale which can move up and down along the vertical scale. A small microscope is attached to this platform and the microscope can be fixed in a horizontal or vertical position according to the needs.

By moving the horizontal platform along the horizontal scale microscope can be moved horizontally and the horizontal distance moved can be obtained from the horizontal scale. By moving the vertical platform up and down along the vertical scale the microscope can be moved vertically and the vertical distance moved can be obtained from the vertical scale.

Tiny movements along the vertical scale can be done by the screw C_1 and tiny movements along horizontal scale can be done by the screw C_2. Before using the instrument the base should be levelled horizontally and that can be done by using the levelling screws A_1 and A_2 . Miscroscope can be focused on the object of which a measurement has to be taken, by adjusting the screw D. The least count of the vertical scale of travelling microscope is 0.01 mm as mentioned above. Vernier division which coincides with a scale division cannot be read from the naked eye. Therefore, a magnifying glass is used.

Travelling microscope is used to obtain measurements such as the length of a mercury thread inserted in a capillary tube, horizontal and vertical diameters of a capillary tube base, real and apparent depth of a mark when viewed through a glass block etc.

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Extended vernier scale

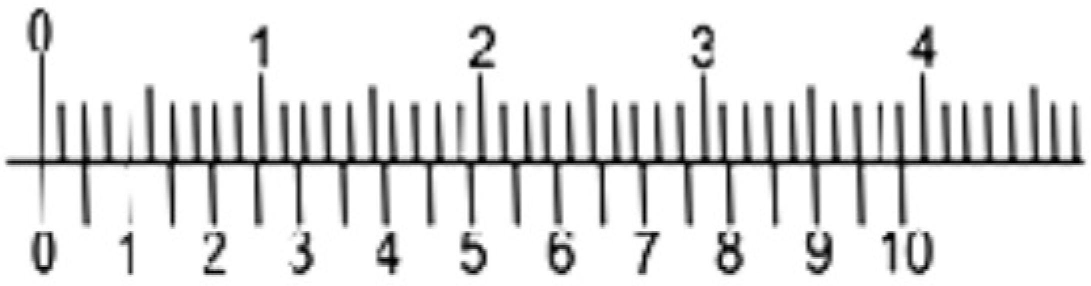

Figure 4.6

Extended vernier scale is designed to obtain very small measurements and an easily readable vernier scale with gaps of calibration not so close to each other. In such verniers the number of vernier divisions is 20. The 20 divisions include 39 mm of the main scale. The least count of the vernier is the difference between two divisions of the main scale and one division of the vernier scale. Consider the method of calculating the least count of this vernier.

Value of one vernier division $= \frac{39}{20}$ mm

$= 1.95$ mm

Range of two divisions of the main scale $= 2.00$ mm

Least count $= (2.00 - 1.95)$ mm

$= 0.05$ mm

Circular vernier

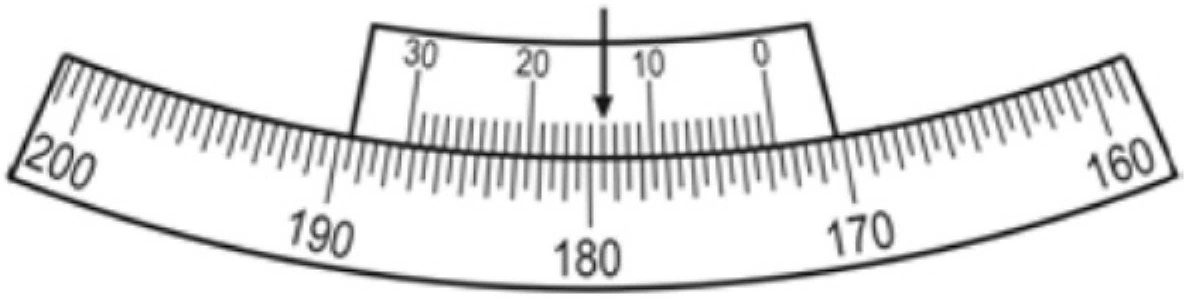

Figure 4.7

Circular verniers are used in instruments such as spectrometer and theodolite. Figure 4.7 shows such a circular scale calibrated in half a degree divisions. The vernier scale has 30 divisions. 30 vernier divisions are included in 29 divisions or $14^{\circ}30'$ of the circular scale. Because of this calibration, the vernier can be read up to $\frac{1}{60}$th of a degree, that is one minute $(1')$. Consider how to obtain the reading indicated in the diagram.

Zero of the vernier lies between $172^{\circ}30'$ and 173°. Circular scale can read up to $\frac{1}{2}^{\circ}$ or $30'$.

Since 14^{th} mark of the vernier coincides with a mark of the circular scale the correct reading is $(172^{\circ}\ 30' + 14')$, that is $172^{\circ}\ 44'$.

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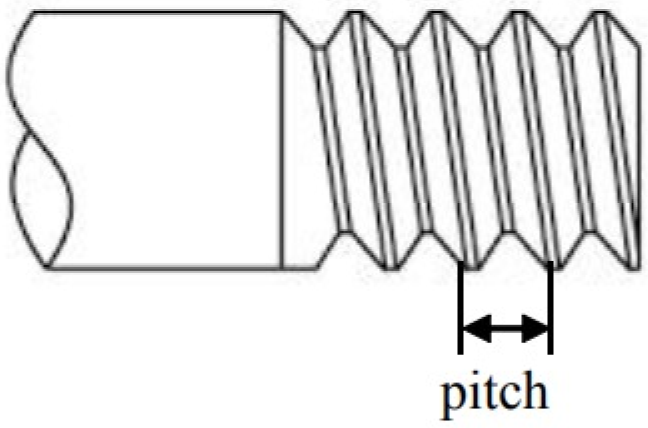
Screw principle

A measurement like the thickness of a paper cannot be measured by vernier calipers which are in common use. As a result for experiments on these measurements screw instruments are produced. Screw instruments consist of a fine thread screw which can pass through a nut. When the screw is rotated one revolution it moves to a distance equal to the gap between two consecutive threads. This is called as the pitch (screw gap) of the screw.



Circumference of the screw head is divided into equal parts and the distance moved by the screw forward or backward can be obtained from a linear scale calibrated in 0.5 mm parts attached to the nut.

Figure 4.8

Micrometer screw gauge

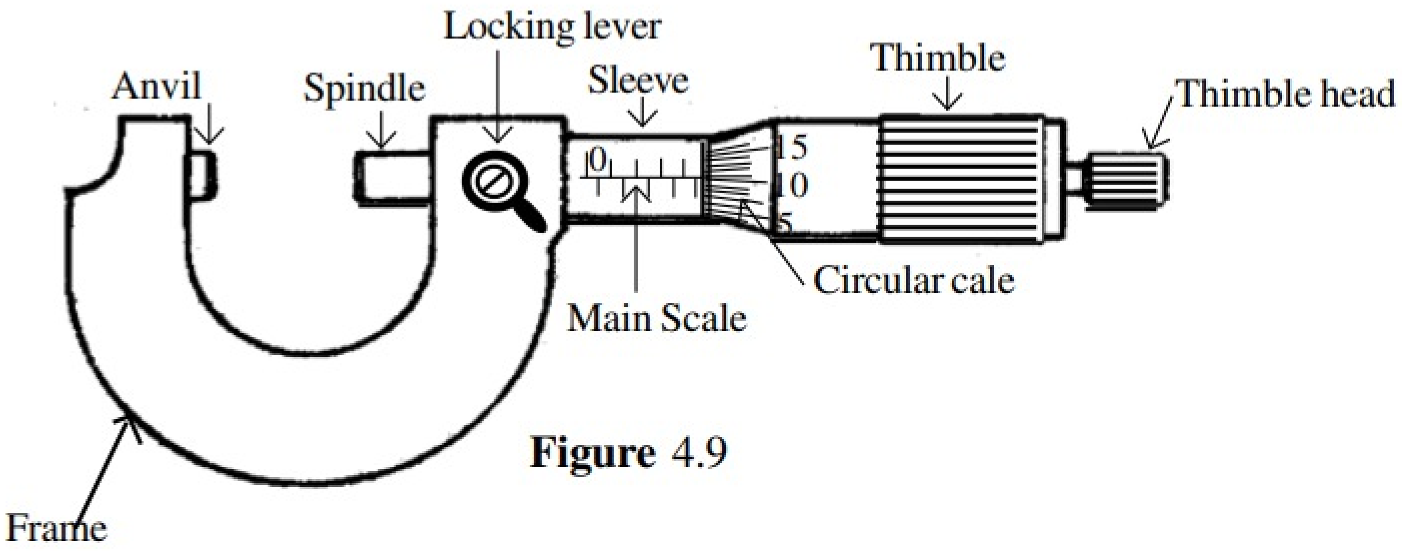


Figure 4.9

Micrometer screw gauge is shown in the Figure 4.9 and its main parts are named. Anvil is attached to the end of a frame which is connected to a nut . The spindle is connected to a screw with fine threads passing through the nut. Main (linear) scale is calibrated on the sleeve attached to the nut and the screw can move on the sleeve while rotating. The thimble head is connected to the screw. By rotating the thimble head, the thimble can be moved along the main scale. At the end of the thimble there is a circular scale.

The pitch of a micrometer screw gauge is 0.5 mm and the circular scale is divided into 50 equal parts. By rotating one division of the circular scale the distance moved by the screw is the least count. While rotating by holding from thimble head, when the spindle just touches the anvil or when the spindle and anvil just touch an object kept between them, the thimble head freely rotate emitting a sound due to a mechanism inside it. This avoids unnecessary pressing on the measuring object by spindle and anvil. Before taking the reading the spindle can be kept in a fixed position by the locking lever.

Zero error

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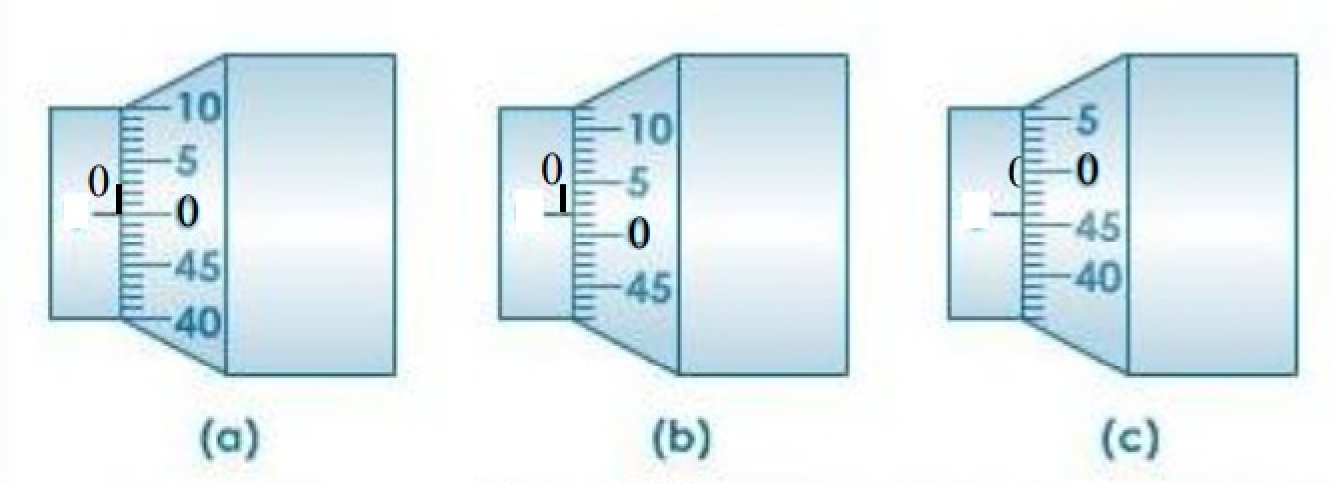

Figure 4.10

When it is rotated by holding from the thimble head so that the spindle just touches the anvil, the zero of the circular scale should coincide with the main scale line as shown in the Figure 4.10 (a). But in some screw gauges due to some reasons such as corrotion or waste of the spindle and anvil, when the thimble head is rotated so that the spindle just touches the anvil, the zero of the circular scale is in a position as indicated in the Figure 4.10 (b) and Figure 4.10 (c). Because of this, an error will occur . This error is called the zero error of the screw gauge.

According to Figure 4.10 (b) the reading of the circular scale starts from the 2nd division, that is 0.02 mm. Therefore, the zero error is 0.02 mm and for the correction this value should be deducted from the relevant reading.

According to Figure 4.10 (c) the zero of the circular scale coincides with the main scale line only when four divisions of the circular scale is rotated. That is, 0.04 mm is rotated. Therefore, the zero error is 0.04 mm. When taking the reading, since the values are counted from zero of the circular scale, the above value is not included in the reading obtained. Therefore, for the correction, the zero error should be added to the relevant reading. Micrometer screw gauge is used to obtain measurements such as thickness of a paper, diameter of a small bicycle ball bearing, diameter of a thin wire or thickness of a blade.

Spherometer

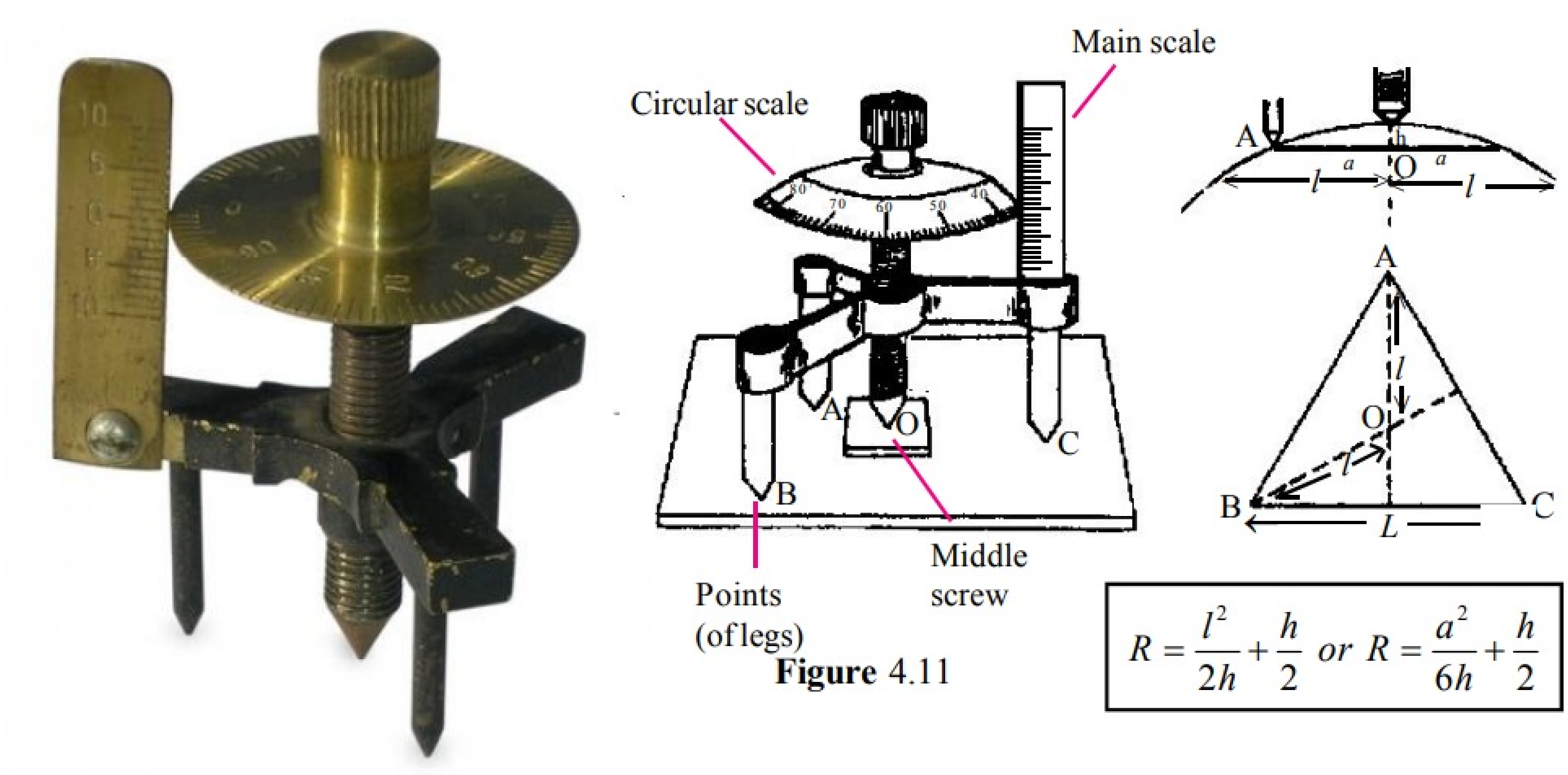

Figure 4.11

$$R = \frac{l^2}{2h} + \frac{h}{2} \quad or \quad R = \frac{a^2}{6h} + \frac{h}{2}$$

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Spherometer is another important measuring instrument which uses the screw principle. Since this is used to measure the thickness of a cover slip or a microscopic slide as well as to take measurements needed to calculate the radius of curvature of a spherical surface (convex and concave), it is called a spherometer.

Spherometer consists of three equal legs with their points situated at the vertices of an equilateral triangle and a screw with fine threads and a point passing through a nut situated at the centre of a circle passing through the three legs. Main scale is attached vertically to one leg of the instrument and the circular scale is attached to the head of the screw. Since the main scale is a centre zero scale, movement of the screw pointer above or below the plane passing through the pointers of the legs can be read.

The main scale (vertical scale) of the spherometer which is mostly in use is calibrated in 0.5 mm parts and the circular scale is divided into 50 equal parts. Its pitch is 0.5 mm.

One vernier division = 0.01 mm

Least count = 0.01 mm

When using the spherometer first we have to keep the points of the three legs to touch the supplied optically flat glass surface and rotate the screw head so that the screw point just touches the glass surface. In that instant the zero of the circular scale of instrument without defect, will lie along the main scale.

If it is not, the circular scale indicates a non-reading reading. That reading is the zero error. The correction can be positive or negative according to whether the initial reading of the spherometer is above or below the foot plane and also whether the reading is taken using a convex or concave surface. In finding the radius of curvature of a spherical surface, if the distance in which the screw pointer is raised up (or moved down) from the foot plane is h, the gap between the points of two legs of the spherometer is 'a' and the radius of curvature of the spherical surface is 'R' then,

$$R = \frac{a^2}{6h} + \frac{h}{2}$$

Measuring h and a, R can be calculated using the above expression.

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Measurement of time

Time spent for a certain incident can be measured upto one second using a normal clock or a wrist watch, which cannot be used to measure a fraction of a second. For measurements such as the period of oscillation of a simple pendulum, or the time spent for a short distance running event, stop watch is used from which a fraction of a second can be measured.

(a) **Figure** 4.12 (b)

A stop clock which is used to measure short time range is shown in the Figure 4.13 (a). It can be started by pushing the leaver A downwards, and by pushing the leaver upwards it can be stopped. By pushing the leaver B downwards the indicator can be brought to the initial zero position. An electron stop watch used to measure short time ranges is shown in Figure 4.13 (b). It can be started by pressing the button C and can be stopped by pressing the same button again. By pressing the button D the indicator numbers can be brought back to the initial zero value. Time can be measured upto 0.1 second using this stop watch.

When measuring time using instruments like this, the accuracy of the measurement depends on the reaction time of the person who operates the instrument.The reaction time of a person is the time range between the observation of some incident and responding to it.

Measurement of mass

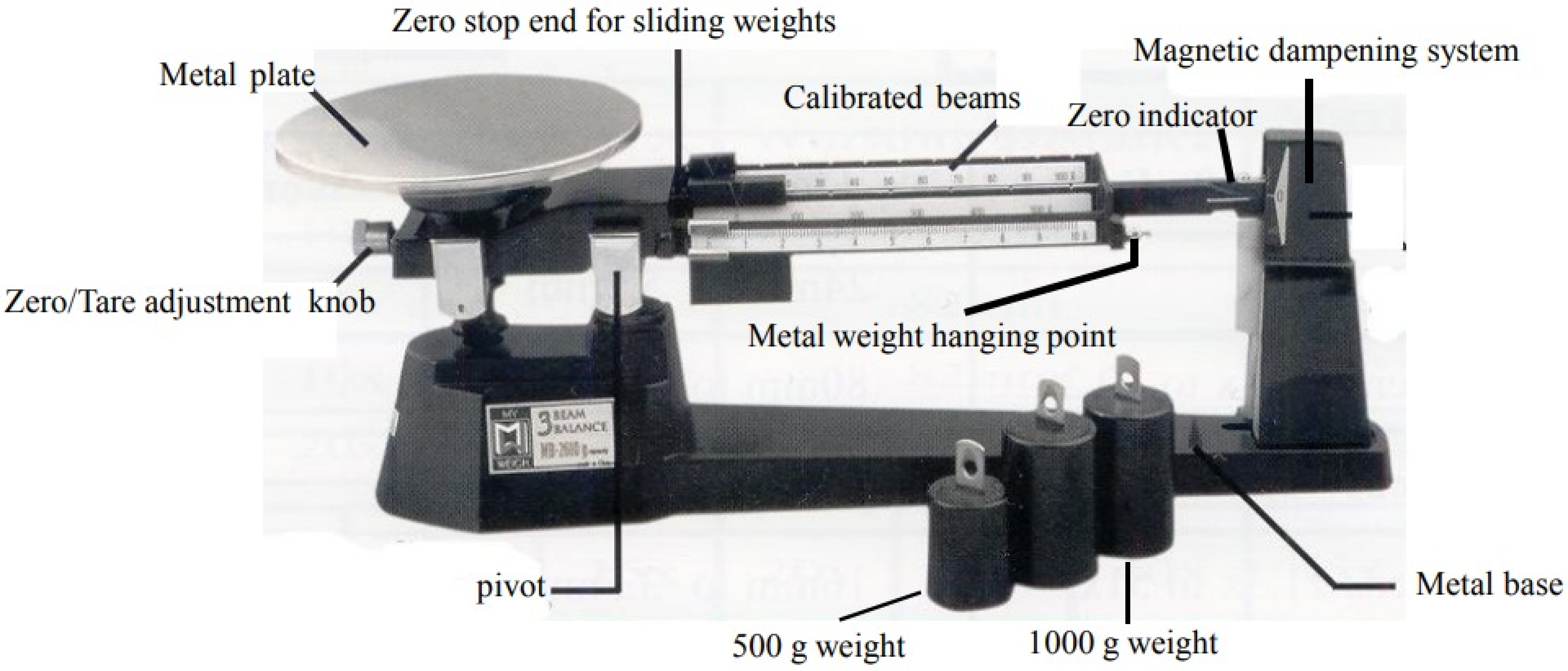

Figure 4.13(a)

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Figure 4.13 (b)

A triple beam balance is shown in Figure 4.14 (a). Recently in laboratory work the triple beam balance is mostly used to measure masses. It consists of a pivoted system having a metal plate on one side and on the other side a combination of three calibrated beams. Three beam are calibrated as (0-10) g, (0-500) g and (0-1000) g. There are small weights which can easily slide to and fro along the beams. From the pointers on the small weights we can read the corresponding position of the weights. If all the weights slide to the zero stop end, zero indicator at the end of the beams comes to a horizontal position. If there is a change, beams can be brought to a horizontal position by adjusting the zero adjustment knob.

When the beams of the balance are in horizontal positions the object to be weighed is kept on the metal plate and the weights are moved on the beams to the right side so that the beams come to horizontal positions again.

The value of mass of the object can be obtained from the values corresponding to the pointers of the weights on the beams. The minimum mass that can be measured from the triple beam balance is 0.1 g. The value of the maximum mass that can be measured, can be increased by hanging the extra weights 100 g, 500 g and 1000 g form the relevant points. Very small masses can be measured using electronic balance as shown in Figure 4.14 (b). This is like a compression balance. When we keep an object to be weighed on the plate there is a digital electronic circuit inside it, which indicates the value of the mass numerically on the indicator screen according to the thrust exerted by the object on the plate. Using this instrument we can measure masses up to one milligram.

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Chapter 05

Scalar quantities and vector quantities

Physical quantities can be divided into two categories, namely vector quantities and scalar quantities.

Scalar quantities

A scalar quantity is completely specified by its magnitude.

eg. Mass, time, distance, pressure, energy, density, speed, area, volume, work, power

Vector quantities

They have both magnitude and direction and obey the law of vector addition.

eg. Displacement, acceleration, impulse, moment, momentum, magnetic flux density, velocity, force, electric field intensity, gravitational field intensity, weight.

Scalar quantities can be added mathematically (algebraically), but for vector addition, the direction must also be considered.

A vector can be represented by a line segment geometrically. The length of the line is proportional to the magnitude of the vector and the direction of the arrow head shows the direction of the vector (Figure 5.1).

Magnitude of a vector = $\left|\overrightarrow{AB}\right| = AB$

Vector $\overrightarrow{AB} = \underline{a}$

Figure 5.1

Equal vectors

(i) If $AB = CD$ (Figure 5.2).

$AB \,//\, CD$

the direction of A to B is same as the direction of C to D.

Then $\overrightarrow{AB} = \overrightarrow{CD}$

$\Rightarrow \underline{a} = \underline{b}$

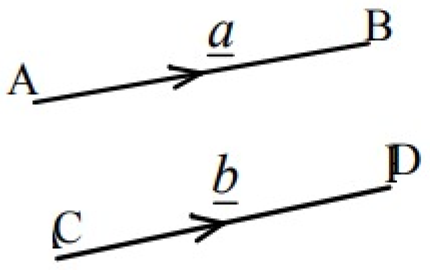

Figure 5.2

Note:- If $AB = CD$ (Figure 5.3)

$AB \,//\, CD$

And the direction of A to B is opposite to that of C to D.

Then $\overrightarrow{AB} = -\overrightarrow{CD}$

$\Rightarrow \underline{a} = -\underline{b}$

Note:- $\overrightarrow{BA} = -\overrightarrow{AB}$

$\overrightarrow{AB} = -\overrightarrow{BA}$

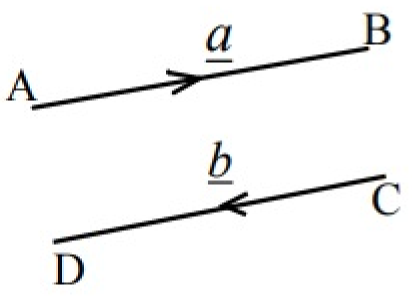

Figure 5.3

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Addition of vectors

Addition of two parallel vectors

When adding two parallel vectors magnitude of the resultant will be the sum of magnitudes of the two vectors.

Direction of the resultant will be the same direction as those of the two vectors.

e.g. If the addition of vector $\underline{5}$ and $\underline{3}$ is $\underline{R}$ (Figure 5.4)

$$\underline{R} = \underline{5} + \underline{3} = \underline{8}$$

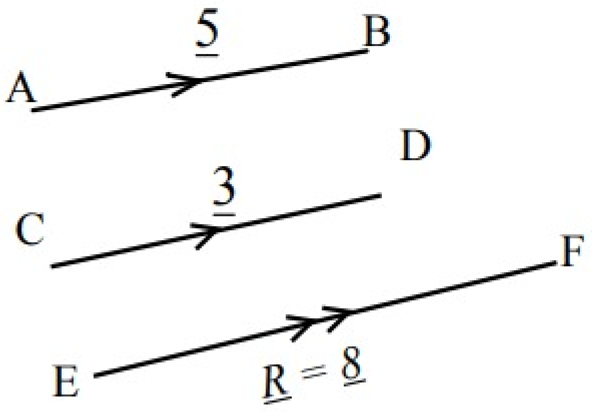

Figure 5.4

Vector triangle method

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side taken against the order will represent the sum of the two vectors in magnitude and direction (Figure 5.5).

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\underline{a} + \underline{b} = \underline{c}$$

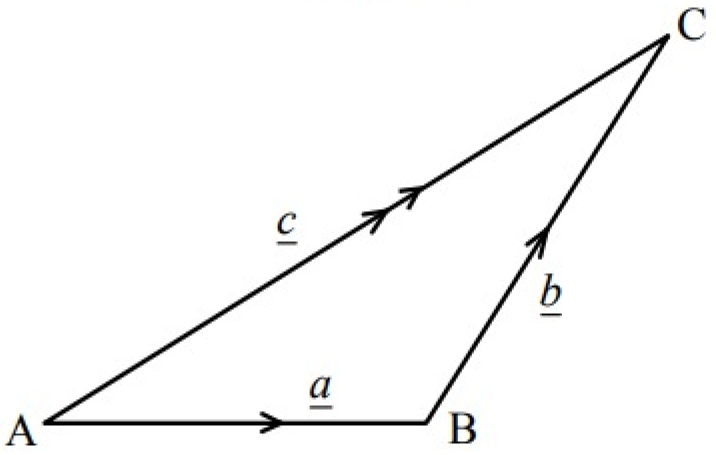

Figure 5.5

Parallelogram law of vectors

If two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point at which the two adjacent sides meet (Figure 5.6).

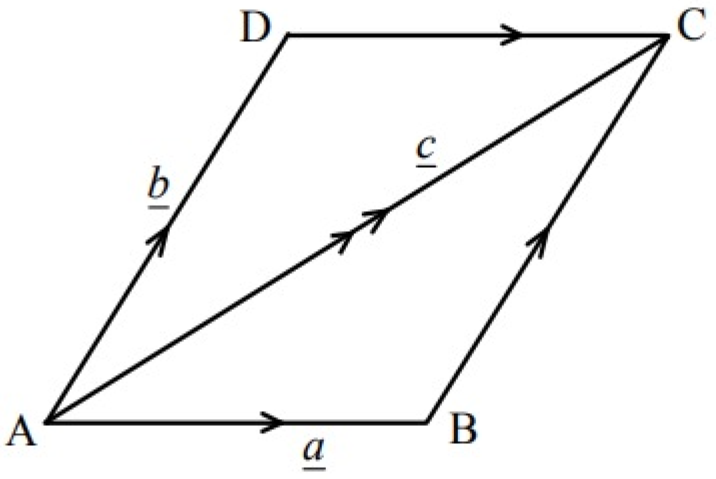

Figure 5.6

Resolution of a vector

If the side AB of a parallelogram ABCD represents the vector $\underline{a}$ and the side AD represents the vector $\underline{b}$ then the diagonal AC will represent the resultant of the two vectors denote by $\underline{c}$.

$\underline{a}$ is the resolved part of $\underline{c}$ along AB and $\underline{b}$ is the resolved part of $\underline{c}$ along AD. Unlimited parallelograms can be drawn by taking a particular side as a diagonal. Therefore , the number of resolved parts (pairs) obtainable along two prependicular directions, of a particular vector, are infinite.

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Resolution of a vector into two perpendicular components

Consider a vector inclined at an angle θ to the horizontal (Figure 5.7).

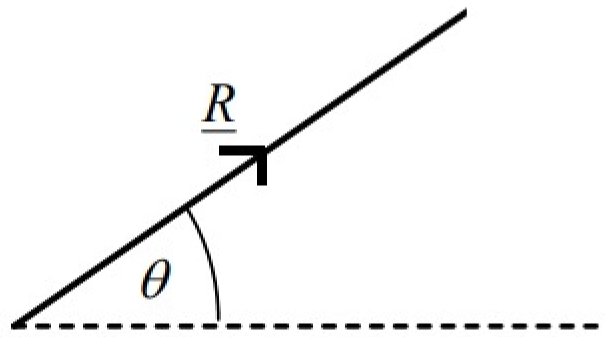

Figure 5.7

Its components, in the horizontal and vertical directions, which are perpendicular to each other are (Figure 5.8);

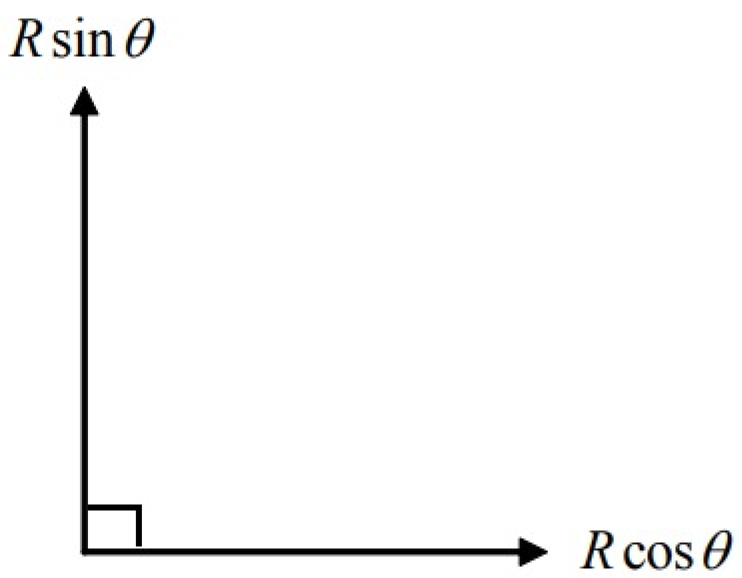

Figure 5.8

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Unit 2

Mechanics

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Chapter 01

Kinematics

"How fast" a body moves at a certain instance is described as the velocity of the body at that instance.

Definition of velocity

Velocity is the rate of change of displacement of a body in a certain direction.

Usual symbols used to express velocity is *v* and *u*.

by the definition

$$velocity = \frac{change\ of\ \ displacement}{time}$$

$$\boxed{v = \frac{\Delta s}{\Delta t}}$$

- Velocity is a vector quantity and thus has an associated direction.
- SI unit of velocity is m s^{-1}.

Relative motion

Example 1

Suppose two motor cars travel along the same direction one followed by the other, with speeds of 100 km h^{-1}. A policeman standing on the side of the road detects their speeds as 100 km h^{-1} by the speed gun. But to one driver, the other seems to be stationary.

This shows that the velocity of an object depends on the "reference frame" of whoever is observing or measuring the velocity.

In everyday life this reference frame is considered the ground.

For example the police officer detected the speeds of the motor cars as he was at rest. But he would detect the speed incorrectly if he was moving while detecting the speed.

Example 2

A passenger inside a moving vehicle observes that a tree on the ground moves with a velocity in opposite direction to his velocity with an equal magnitude. Even though the tree is actually at rest relative to the frame of reference of the ground, it has a velocity relative to the frame of reference of the moving vehicle .

The actual velocity of an object that is its velocity relative to the Earth can be indicated by, $v_{A,E}$

Likewise, the velocity of the object A relative to the frame of reference of the object B can be indicated as, $v_{A,B.}$.

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The relationship between the velocities relative to different frames of reference can be obtained as shown by the following example.

Take A and B as two bodies moving with uniform velocities v_A and v_B respectively, relative to the Earth heading in the same direction along the same path.

If they simultaneously pass the same point X with the uniform velocities v_A and v_B (if $v_A > v_B$),

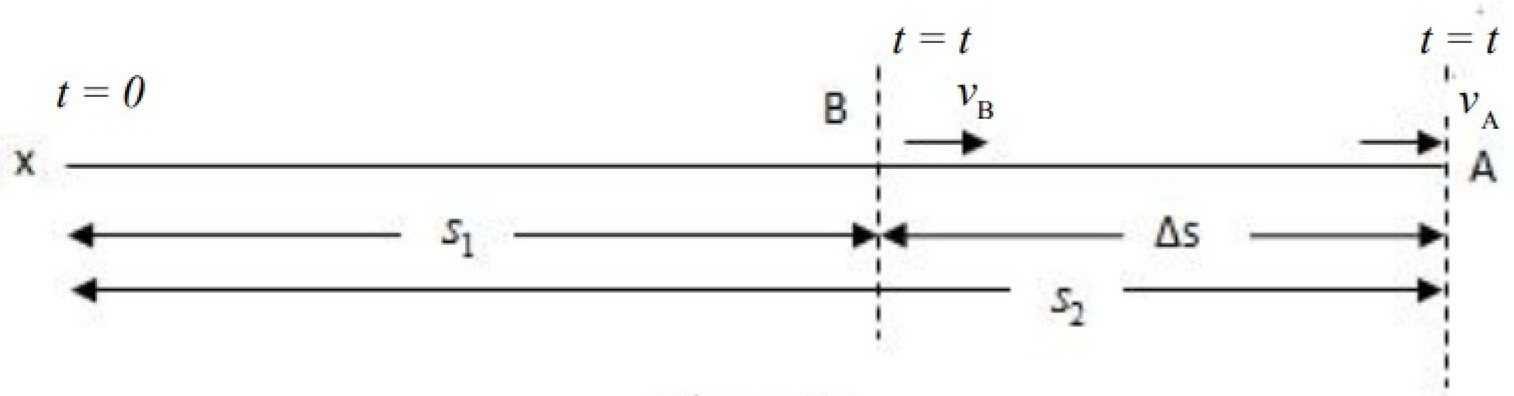

Figure 1.1

Then,

$$\text{Velocity of A, } v_A = \frac{s_2}{t}$$

$$\text{Velocity of B, } v_B = \frac{s_1}{t}$$

That is, relative to B (or as seen by B) during a time 't', A has displaced by an amount Δs,

$$\Delta s = (s_2 - s_1)$$

$$\therefore \frac{\Delta s}{t} = \frac{(s_2 - s_1)}{t}$$

$$\frac{\Delta s}{t} = \frac{s_2}{t} - \frac{s_1}{t}$$

$$\boxed{v_{A,B} = v_A - v_B}$$

$$v_{A,B} = v_{A,E} - v_{B,E}$$

(As the velocities of A and B relative to the Earth has been concerned, $v_A = v_{A,E}$ and $v_B = v_{B,E}$)

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Since by properties of vectors,

$$-\, v_{\mathrm{B,E}} = v_{\mathrm{E,B}}$$

Hence the relationship can be written as,

$$\boxed{v_{\mathrm{A,B}} = v_{\mathrm{A,E}} + v_{\mathrm{E,B}}}$$

This shows that the velocity of one body relative to another can be expressed with their respective velocities relative to a third frame of reference.

Worked examples

1. A motor boat (B) speeds at a velocity of 60 km h^{-1} towards North. A steady wind (W) blows from the North at a velocity of 40 km h^{-1} . Find the velocity of the wind felt by a passenger in the boat.

Answer:

$v_{\mathrm{B,E}} = 60\uparrow$ where then $v_{\mathrm{E,B}} = 60\downarrow$ (E represents the Earth)

$v_{\mathrm{W,E}} = 40\downarrow$

$v_{\mathrm{W,B}} = v_{\mathrm{W,E}} + v_{\mathrm{E,B}}$

$= 40\downarrow + 60\downarrow = (40 + 60)\downarrow = \underline{\underline{100 \text{ km h}^{-1}}}\downarrow$

2. A motorcycle (M) travels at a velocity of 100 km h^{-1} along a straight road. As it passes a police car (C), the car begins to chase it at a speed of 110 km h^{-1}. Find the velocity of the motor cycle relative to a policeman in the car.

Answer : $v_{\mathrm{M,E}} = \overrightarrow{100}$ km h^{-1}

$v_{\mathrm{C,E}} = \overrightarrow{110}$ km h^{-1} and $v_{\mathrm{E,C}} = \overleftarrow{110}$ km h^{-1}

$v_{\mathrm{M,C}} = v_{\mathrm{M,E}} + v_{\mathrm{E,C}}$ km h^{-1}

$v_{\mathrm{M,C}} = \overrightarrow{100} + \overleftarrow{110}$

$= \overleftarrow{-100} + \overleftarrow{110}$

$= \overleftarrow{10}$ km h^{-1}

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3. A train (T) of length 150 m travels in a straight track at a constant speed of 70 km h^{-1}. A motor car (M) travels along a straight road close and parallel to the rail track, at a constant speed of 85 km h^{-1} in the same direction as the train. Find the time taken by the motorist to pass the train.

Answer :

$$v_{T,E} = \overrightarrow{70} \quad \text{and} \quad v_{E,T} = \overleftarrow{70}$$

$$v_{M,E} = \overrightarrow{85}\ \text{km h}^{-1}$$

$$v_{M,T} = v_{M,E} = v_{E,T}$$

$$v_{M,T} = \overrightarrow{85} + \overleftarrow{70}$$

$$= \overrightarrow{85} - \overrightarrow{70}$$

$$v_{M,T} = \overrightarrow{15}\ \text{km h}^{-1}$$

$$\text{As, velocity} = \frac{\text{displacement}}{\text{time}}$$

$$15\ \text{km h}^{-1} = \frac{150\times10^{-3}\ \text{km}}{t}$$

$$t = 10^{-2}\ \text{h}$$

$$t = 10^{-2}\ \text{x}\ 3600\ \text{s}$$

$$= \underline{\underline{36\ \text{s}}}$$

Applications

1. It is observed that the sun moves around the Earth every day. But in the actual motion it is the Earth that moves (spins) about its axis.
2. When raining, the rain drops fall vertically downwards due to gravity, in still air. But a person inside a moving train will observe the rain fall at an angle.

Rectilinear motion under constant acceleration

Graphs of motion

Properties such as displacement, velocity or acceleration can be used to describe a rectilinear motion of a body. where,

$$velocity = \frac{\Delta\ displcement}{\Delta\ time}$$

$$acceleration = \frac{\Delta\ velocity}{\Delta\ time}$$

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Displacement - time graph

When the displacement of a body moving in a certain direction is plotted against time,the displacement (s) - time (t) graphs can be obtained.

If the graph of displacement (s) - time (t) is a straight line,

$$\text{gradient} = \tan\theta$$

$$\text{gradient} = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

$$\boxed{\text{gradient} = \text{velocity of the body}}$$

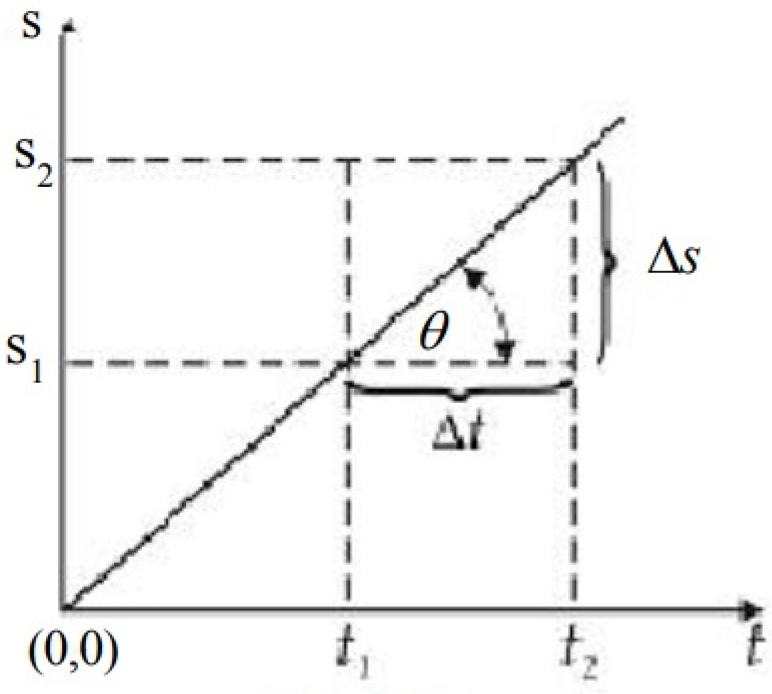

Figure 1.2

This shows that if the graph of displacement vs. time (t) is a straight line, then the described motion is a **"uniform velocity".**

Shown below are the successive positions of a vertically projected ball, when photographed during its motion under gravity, in equal periods of time.

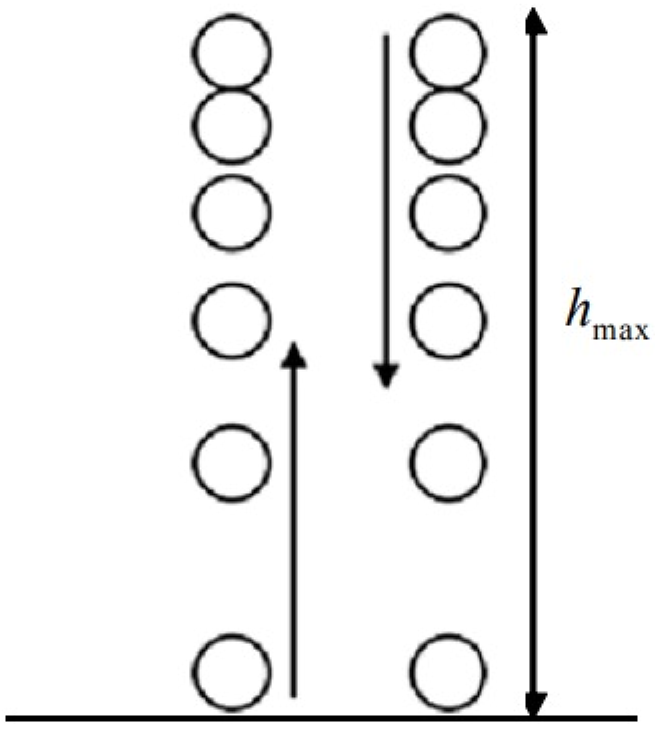

Figure 1.3

The shots are captured within equal time intervals and thus it is clear that when moving upwards its displacement decreases and when moving downwords displacement increases.

Hence if its displacement is plotted against time, the graph will be as shown in Figure 1.4.

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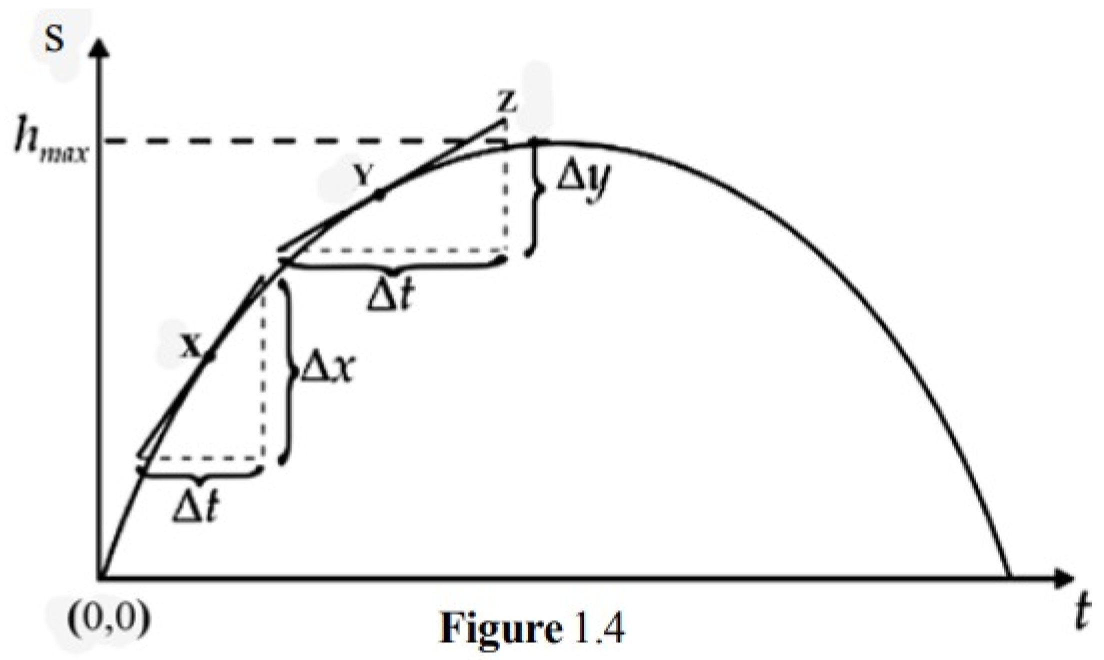

Figure 1.4

- It is clear that the ball does not travel with a uniform velocity and hence the graph is non-linear.
- When considering the two points X and Y on the graph, the velocity of the ball at these two instances will be given by the gradients of the tangents drawn to the graph at the point X and Y. where,

$$(\text{gradient})_x = \frac{\Delta x}{\Delta t} = (\text{velocity})_x$$

$$(\text{gradient})_y = \frac{\Delta y}{\Delta t} = (\text{velocity})_y$$

Since, $(\text{gradient})_x > (\text{gradient})_y$

$(\text{velocity})_x > (\text{velocity})_y$

That is, the ball has travelled with a decreasing velocity, or with a "deceleration" vertically up wards.

At the maximum height at the point Z,

$(\text{gradient}) = \theta$

Therefore, then the body has attained a zero velocity in the vertical direction.

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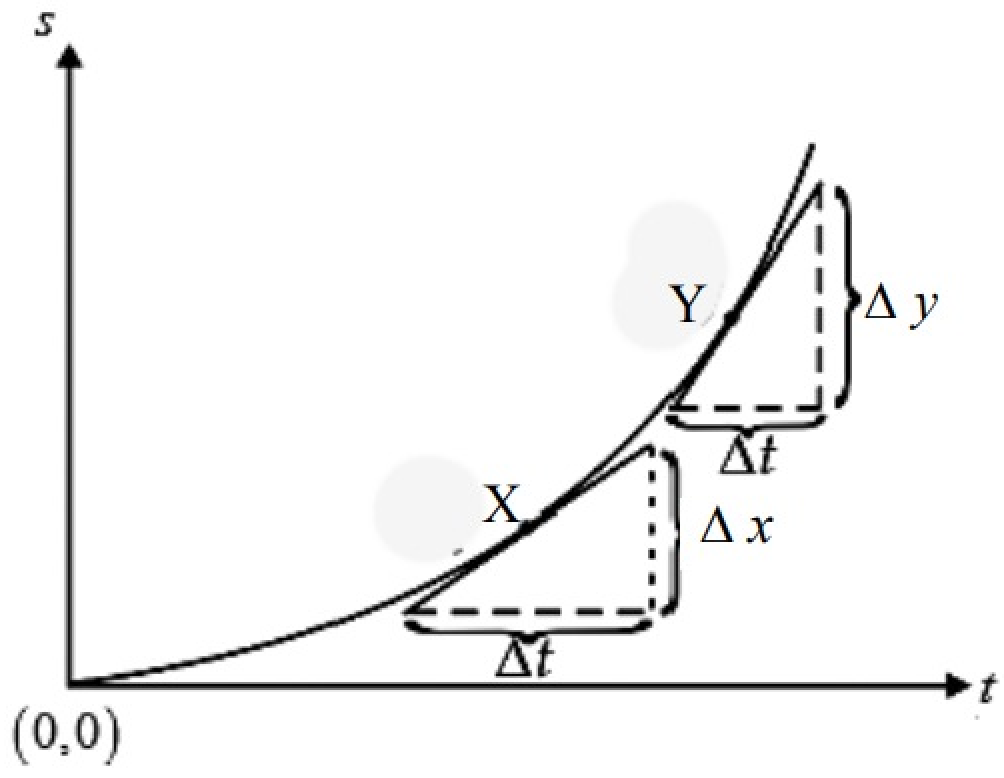
Similarly, if a motion is represented as follows,



$$v_x = \frac{\Delta x}{\Delta t}$$

$$v_y = \frac{\Delta y}{\Delta t}$$

it is clear that,

$$v_x < v_y$$

Figure 1.5

Hence, the motion represented is"**acceleration**" .

It is also clear that in the motion of the ball projected under gravity, it comes down with an acceleration.

Velocity – Time graphs

When the velocity of a body is plotted against time, the velocity (v) - time (t) graph can be obtained.

(1) Uniform velocity

This is when the velocity of a certain body remains constant with time. Hence the velocity (v) - time (t) graph will be as follows,

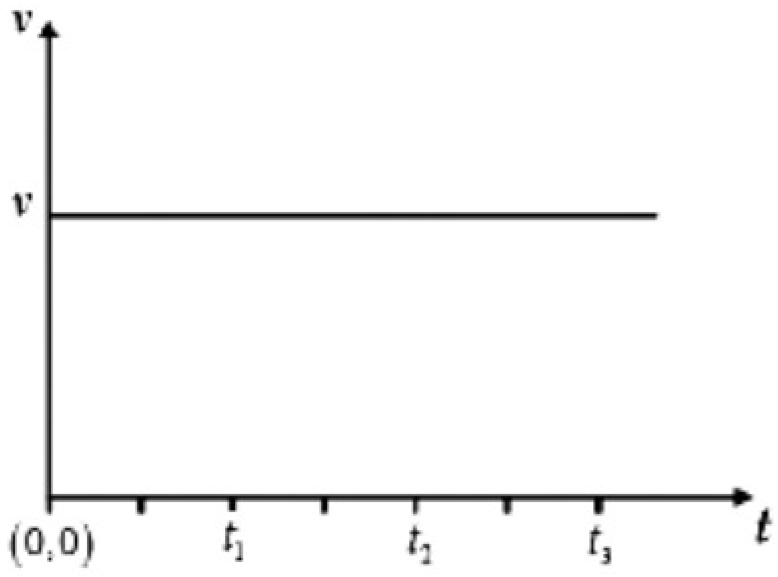


Figure 1.6

Hence a straight line parallel to time axis represents uniform velocity.

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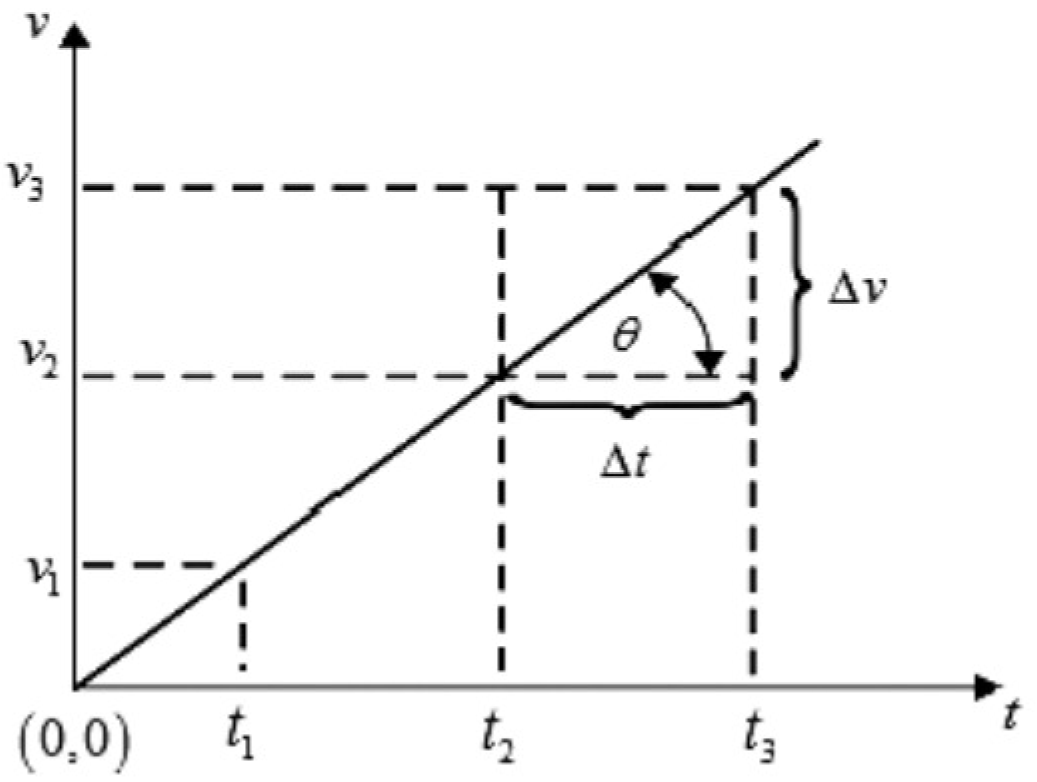
(2) If the velocity (v) - time (t) graph is a straight line inclined to the time axis,



$$\text{gradient} = \frac{v_3 - v_1}{t_3 - t_2}$$

$$\text{gradient} = \frac{\Delta v}{\Delta t}$$

$$\therefore \boxed{gradient = acceleration}$$

Figure 1.7

Figure 1.7 shows that the velocity increases with time, and hence it represents acceleration by the gradient of the graph.

Since in the above graph the gradient along the graph is constant, and also the velocity increases with time, it represents a **'uniform acceleration'**.

(3) If the inclination of the straight line is as given by the following graph, then the gradient is uniform but the magnitude of velocity decreases and therefore, it represents a **'uniform deceleration'.**

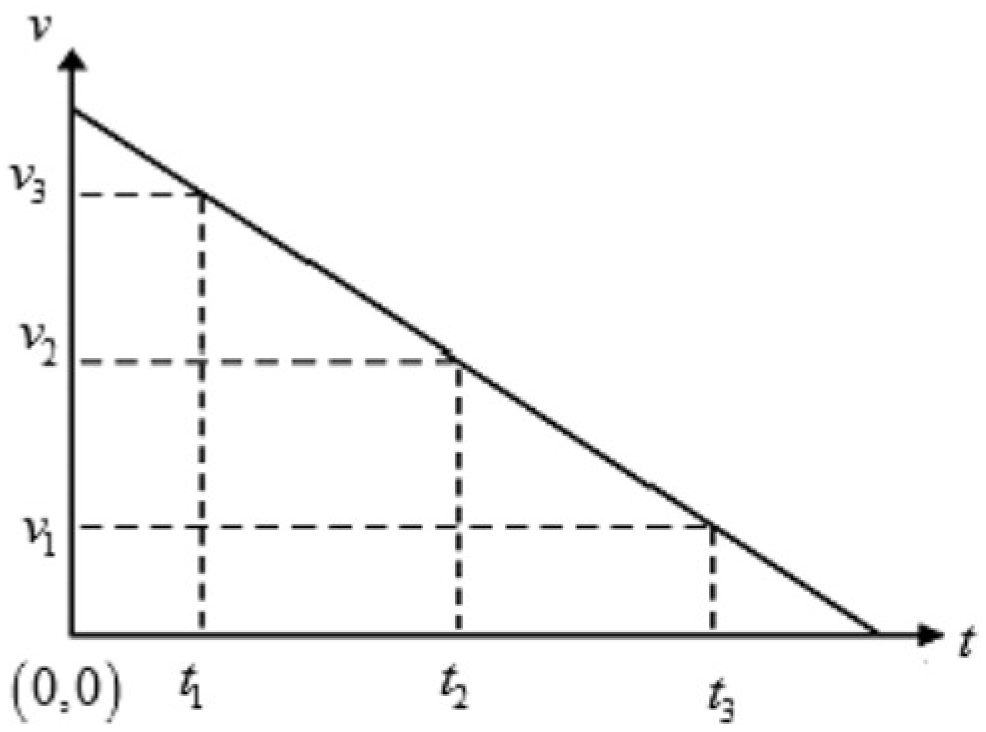


Figure 1.8

It should be noted that since displacement and velocity are vector quantities, it is important to represent the direction of motion graphically.

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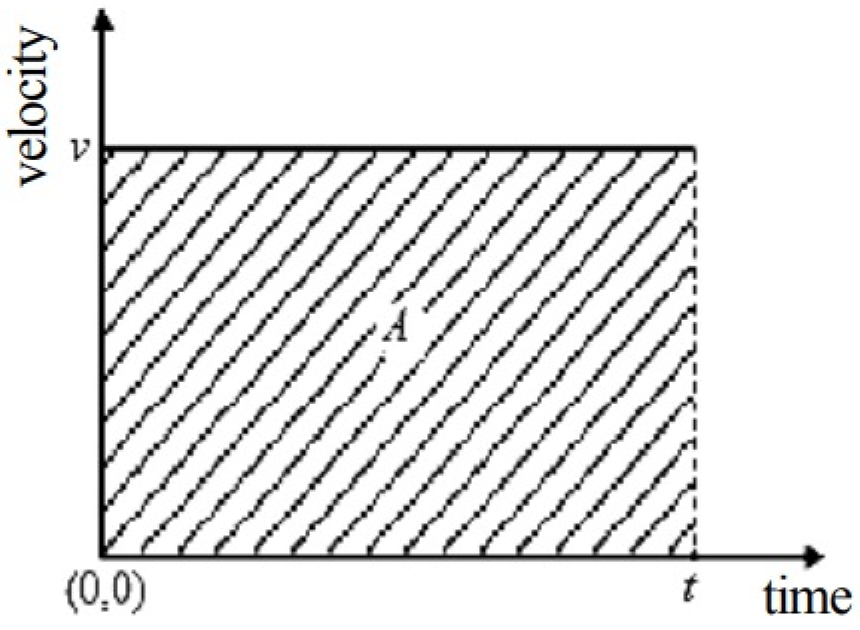
Area of a velocity - time graph

Consider motion under "uniform velocity",



From the equation

$$v = \frac{\text{displacement}}{\text{time}}$$

displacement = velocity x time

by the graph,

Area A = Area of rectangle

Area $A = v \times t$ = displacement

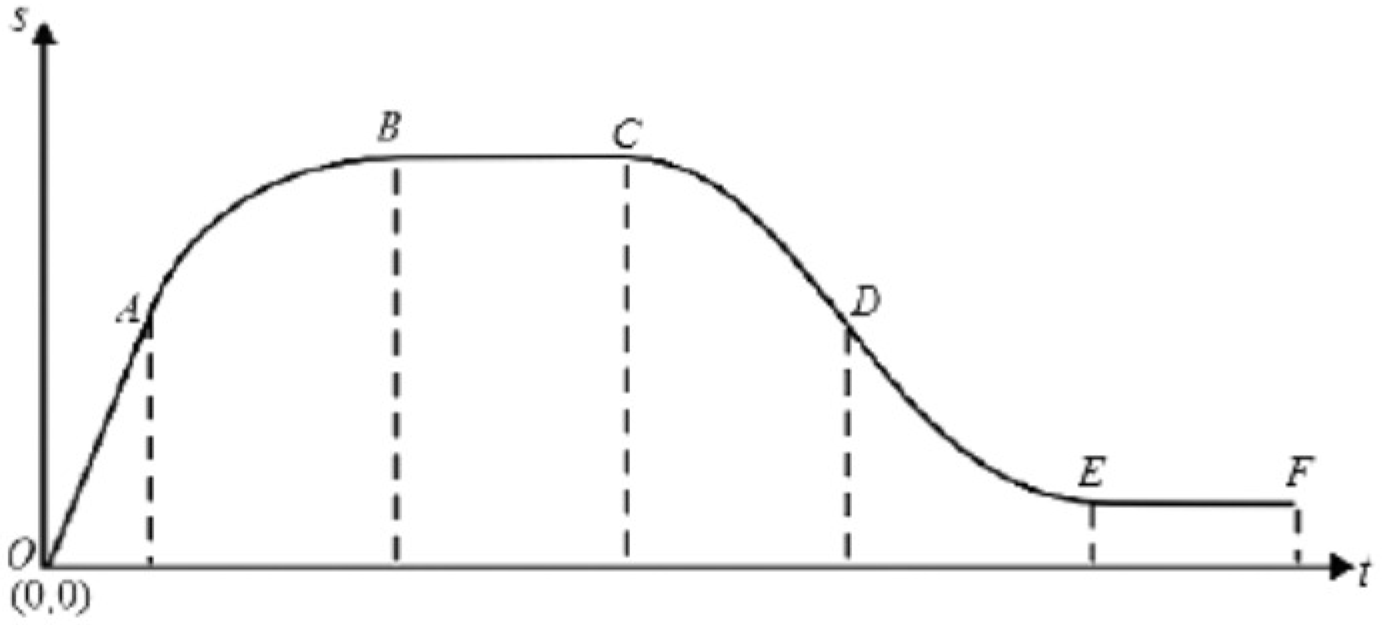
That is,

$$\boxed{\textit{The area between the curve and the time axis} = \textit{Displacement}}$$

Worked Examples :

1. Consider the following displacement (s) - time (t) graph. Explain the motion of the body in each of the following sections.

(1)	O to A	(2)	A to B	(3)	B to C
(4)	C to D	(5)	D to E	(6)	E to F



Answer :

(1) O to A :- Uniform velocity (as the gradient is constant)

(2) A to B :- deceleration or (-) acceleration (as gradient decreases)

(3) B to C :- remains at rest (as gradient = zero)

(4) C to D:- at C body turns back and moves with an acceleration in the (-) ve direction [as the gradient is (-) ve and its magnitude is increasing].

(5) D to E :- deceleration (as the magnitude of gradient decreases)

(6) E to F :- remain at rest (as gradient = zero)

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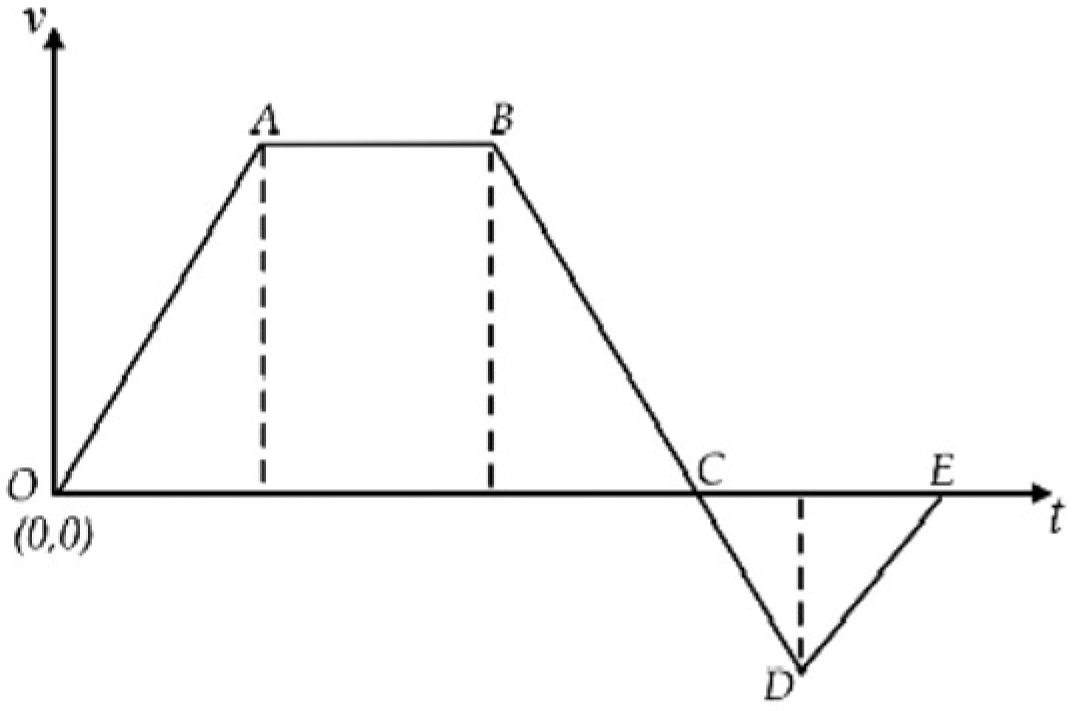
2. Consider the motion represented by the following velocity (v) - time (t) graph.



Describe the motion of the body between the following points

(1) O to A (2) A to B (3) B to C (4) C to D (5) D to E

Answer :

(1) O to A –	Starts the motion from rest and travels with a uniform acceleration (as gradient is constant and magnitude of velocity increases)
(2) A to B –	Uniform velocity (as the gradient is zero at a certain value of velocity)
(3) B to C -	Uniform deceleration and comes to rest at C (as the magnitude of velocity decreases and the gradient is constant).
(4) C to D -	Starting from rest body moves in opposite to initial direction with a uniform acceleration (as the magnitude of velocity increases and velocity is expressed as negative and gradient is constant)
(5) D to E -	Body moves with a uniform deceleration in a direction opposite to the initial direction and comes to rest at E (as the magnitude of velocity decreases and velocity values are expressed as negative and also the gradient is constant).

3. A rubber ball is thrown vertically upwards under gravity with a velocity of 40 m s^{-1}. When it returns to the level of projection, it collides with a horizontal surface and rebounds with a velocity half of the velocity of collision.

(1) Draw the displacement (s) - time (t) graph and velocity (v) - time (t) graph to express the motion described above.

(2) (a) Determine time between the starting moment and the first collision

(b) Determine the maximum height that by the ball reached.

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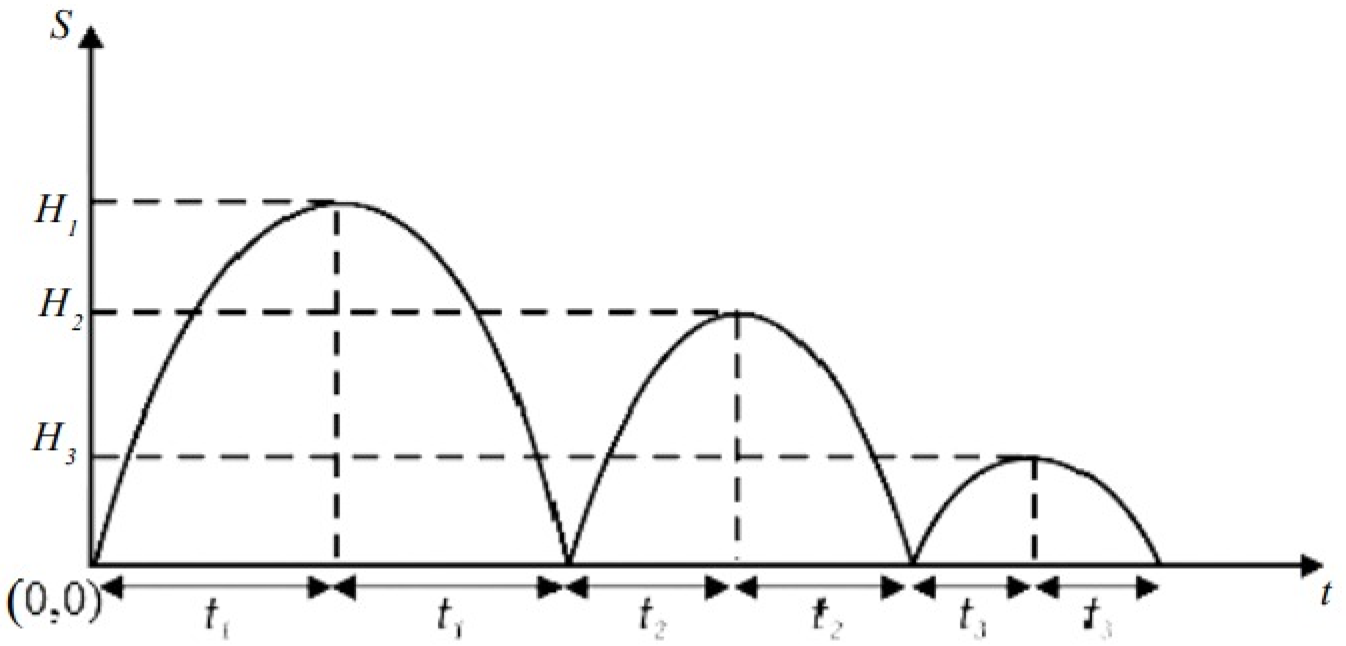
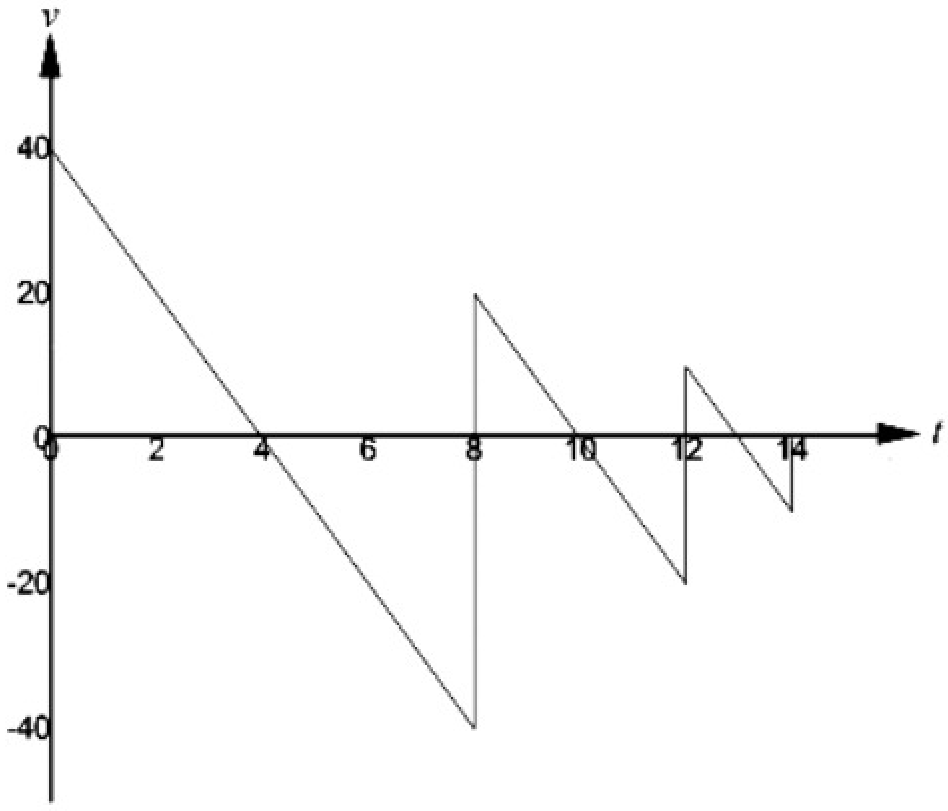
Answer :

(1) When the ball is thrown up against gravity, it moves upwards with a uniform deceleration 10 m s^{-2} (g), then reaches the maximum height and then fall along the same line in opposite direction with an equal magnitude of acceleration of 10 m s^{-2} (g) and will return the same level of projection with a velocity 40 m s^{-1} (equal to velocity of projection).

Displacement - time graph



Velocity vs. time graph



(2) (a) by the gradient of $v - t$ graph,

$$\text{gradient} = \text{acceleration} = 10 = \frac{40-0}{t_1}$$

$$t_1 = 4 \text{ s}$$

$\therefore$ time for 1^{st} collision $= 2t_1 = 8$ s

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(b) by the area (A) of the graph

area = displacement $= H_1$

$$= \frac{1}{2} \times t_1 \times 40$$

$$= \frac{1}{2} \times 4 \times 40$$

$$= \underline{\underline{80 \text{ m}}}$$

Equations of motion

The relationships between the physical properties that describe a particular motion of a body are called equations of motion.

If the initial velocity of a body is 'u', and if this body moves along a **straight line** with a **uniform acceleration** 'a' for a period 't' and obtains a final velocity 'v' making a displacement 's', its motion can be expressed by a velocity vs. time graph as in Figure 1.9.

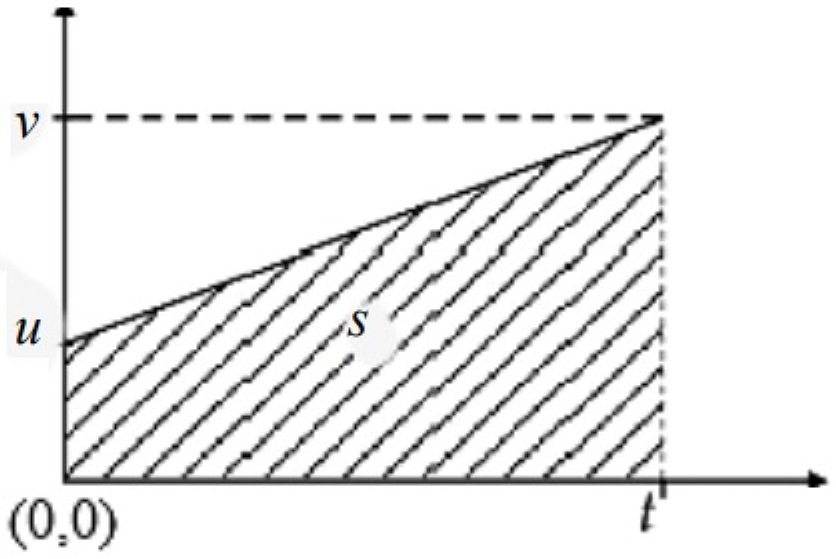

Figure 1.19

By the gradient = acceleration $= \frac{\Delta v}{\Delta \text{ t}}$

$$a = \frac{v - u}{\text{t}} \Rightarrow \boxed{v = u + at}$$

By the area = displacement (s)

$$\boxed{s = \left(\frac{u + v}{2}\right)t}$$

Hence by the above two equations.

$$s = \left(\frac{u + (u + at)}{2}\right)t \Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

$$s = \left(\frac{u + v}{2}\right)\left(\frac{v \text{ - } u}{\text{a}}\right) \Rightarrow \boxed{v^2 = u^2 + 2as}$$

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Motion under gravity

The motion under the action of gravitational force, with negligible resistances, such as air resistance, is called motion under gravity.

Example – 1

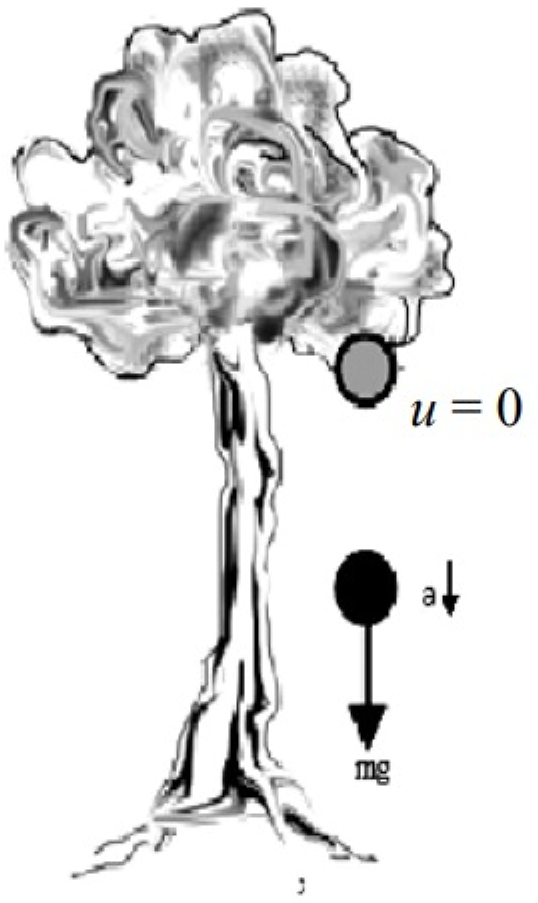
A fruit falling from a tree

$$\downarrow F = ma$$

$$mg = m \times a$$

$$\downarrow a = g \text{ m s}^{-2}\text{; } g \text{ - gravitational acceleration}$$

$$g = 10 \text{ m s}^{-2} \text{ near the surface of the Earth}$$



Example – 2

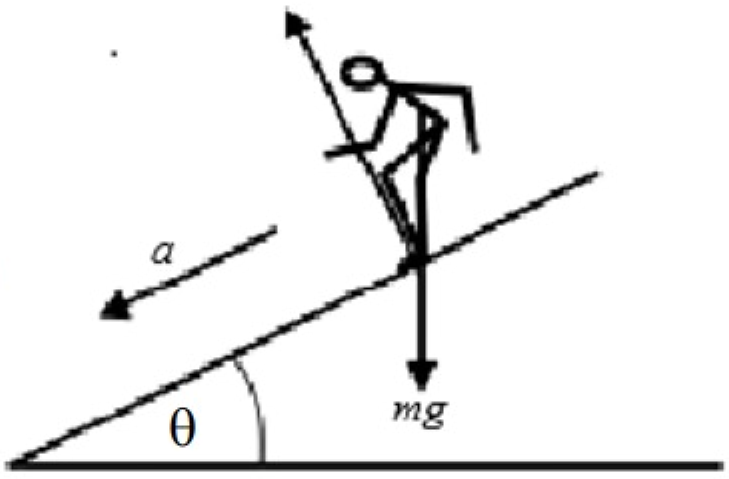
If a man skiing on ice (of negligible friction) slides down along an inclined bank,

$$F = m\ a$$

$$\cancel{m}\ g\sin\theta = \cancel{m} \times a$$

$$a = g\ \sin\theta$$

Hence when the inclination (θ) increases, his acceleration increases.



Projectile motion

You may have seen cricketers batting to earn 6 marks on the score. The ball flies along a curved path. Such a motion of an object is called a projectile motion. Have you thought about the Science behind the motion of the ball? How far the ball goes depends on several factors. They are,

1. The magnitude of initial velocity given to the object
2. Angle of projection (inclination of the direction of initial velocity to the horizontal direction)

Coaches consider these factors when guiding a batsman to hit the ball to get a sixer.

Projectile motion is a combination of horizontal and vertical motion. So we can study this by considering the effect on horizontal and vertical components.

Consider a case when a batsman strikes a ball projecting it at an angle (θ) to the horizontal.

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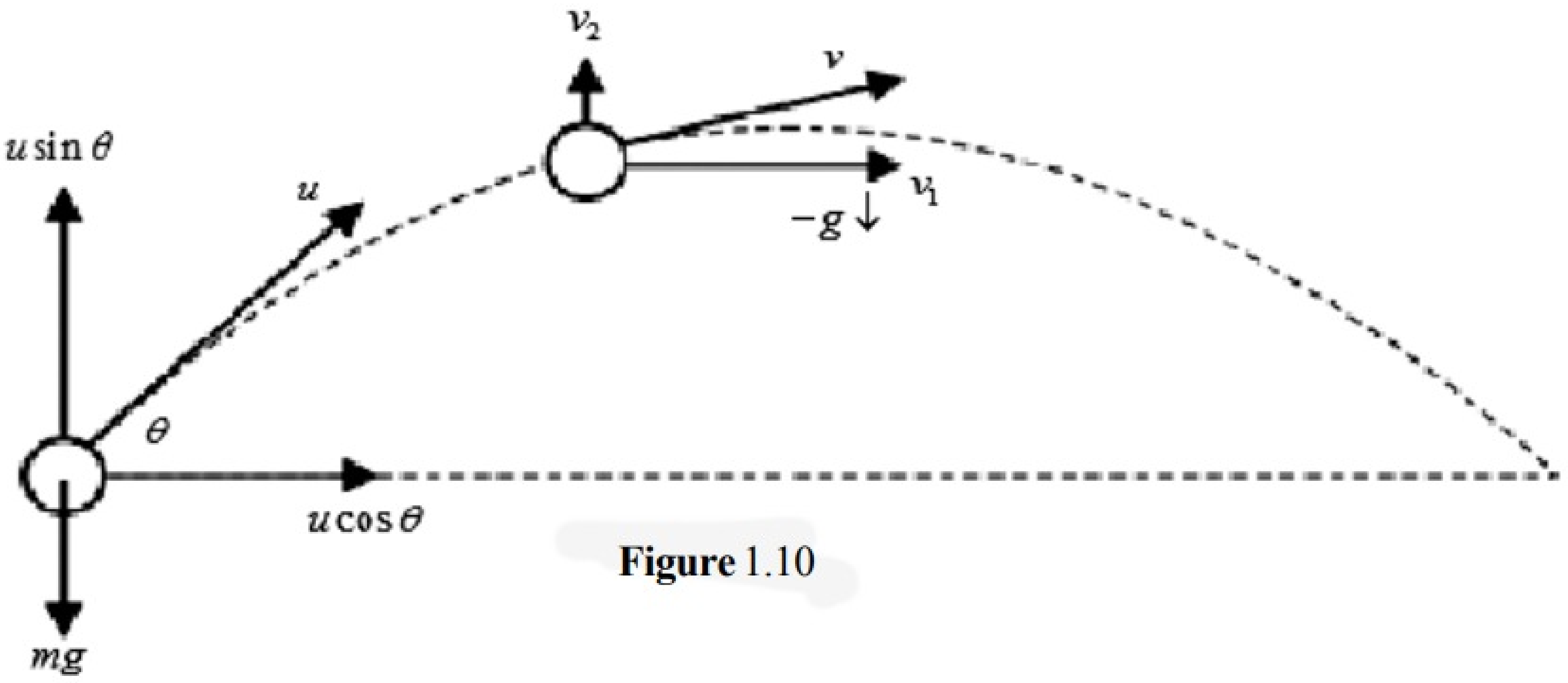

Figure 1.10

Considering the effect on the vertical component,

Vertical component of initial velocity $\uparrow u\sin\theta$

$acceleration = +g\downarrow$

$\uparrow v = u + at$

$v_2 \;= u\,\sin\theta - g \times t < u\,\sin\theta$

$\therefore$ Vertical component of velocity decreases.

This decreases until it reaches zero. After that, due to the gravitational force acting on the body, vertical component of the velocity gradually increases in downward direction until it hits the ground.

So the vertical motion of the object is simillar to a motion under gravity.

Considering the effect on the horizontal component,

$\rightarrow u\cos\theta$, Since horizontally,

Acceleration $= 0$

$v = u + at$

$v_1 = u\cos\theta$

Horizontal component remains constant. In determining the velocity of the ball at a given time using vector addition, it is clear that its trajectory is a parabola.

Worked Examples

1. A bus stopped at a bus halt takes off obtaining a velocity of 72 km h^{-1} during 10 s. Then it travels with this obtained velocity for 10 s and comes to rest at another halt during another 5 s. If all accelerations and decelerations are uniform, determine the acceleration, deceleration, total distance between the two bus halts and the average velocity of the bus.

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Answer :

$$72 \text{ km h}^{-1} = 72 \times \frac{1000}{3600} = 72 \times \frac{5}{18} = 20 \text{ m s}^{-1}$$

If acceleration is '*a*'

$$v = u + at$$

$$\therefore 20 = 0 + a_1 \times 10$$

$$a = 2 \text{ m s}^{-2}$$

Displacement during acceleration

$$s = \left(\frac{u+v}{2}\right)t$$

$$s_1 = \frac{0+20}{2} \times 10$$

$$s_1 = 100 \text{ m}$$

Displacement during uniform velocity

$$s = \left(\frac{u+v}{2}\right)t$$

$$s_2 = \frac{(20+20)}{2} \times 10 = 200 \text{ m}$$

Decceleration

$$v = u + at$$

$$0 = 20 - a_2 \times 5$$

$$a_2 = 4 \text{ m s}^{-2}$$

Displacement during decceleration

$$s = \left(\frac{u+v}{2}\right)t$$

$$s_3 = \frac{(20+0)}{2} \times 5 = 50 \text{ m}$$

Total displacement = 100 + 200 + 50 = 350 m

$\therefore$ distance between the two bus halts = 350 m

$$\text{Average velocity} = \frac{\text{Total Displacment}}{\text{Total time}}$$

$$= \frac{350}{(10+10+5)} = \frac{350}{25} = 14 \text{ m s}^{-1}$$

2. A child drops a tennis ball from the window of a high- rise building. It reaches the ground with a speed of 25 m s^{-1} .The ball rebounds upwards with a speed of 16 m s^{-1}. If air resistance is negligible and ball moves under gravity determine,

(1) The height to the child from the ground

(2) The height the ball reaches after the rebounce

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(3) The total time of flight of the ball between the first and the second collisions with the ground.

Answers :

(1) From the release, until it hits ground,

$$\downarrow v^2 = u^2 + 2as$$

$$(25)^2 = 0 + 2 \times g \times h$$

$$h = \frac{625}{20} = 31.25 \text{ m}$$

(2) After rebounce until it reaches the maximum height (h_{max})

$$\uparrow v^2 = u^2 + 2as$$

$$0 = 16^2 + 2 \times g \times h$$

$$h_{\text{max}} = \frac{256}{20} = 12.8 \text{ m}$$

(3) Between the 1st and 2nd collisions,

$$\uparrow s = ut + \frac{1}{2}at^2$$

$$0 = 16 \times t + \frac{1}{2}(-g)\, t^2$$

$$\therefore \; 0 = (32 - gt)\, t$$

$$\therefore t = 0 \text{ or } (32 - gt) = 0$$

but, $t \neq 0 \therefore t = 3.2 \text{ s}$

3. A man skiing on ice slides down along a banked cliff of angle of inclination 30° to the horizontal. If the surface has negligible friction determine,

(1) The time taken and distance he travels when his speed increases from 5 m s^{-1} to 10 m s^{-1}.

(2) The distance he will travel thereafter, during an equal period of time which you have got as an answer in (1) above.

Answers :

(1) $\swarrow a = g \sin 30^\circ = 5 \text{ m s}^{-2}$

$$v = u + at$$

$$10 = 5 + 5t$$

$$t = 1 \; s$$

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$$v^2 = u^2 + 2as$$

$$10^2 = 5^2 + 2 \times 5 \times s$$

$$s = \frac{75}{2 \times 5} = 7.5 \text{ m}$$

(2) $s = ut + \frac{1}{2}at^2$

$$s = 10 \times 1 + \frac{1}{2} \times 5 \times 1^2$$

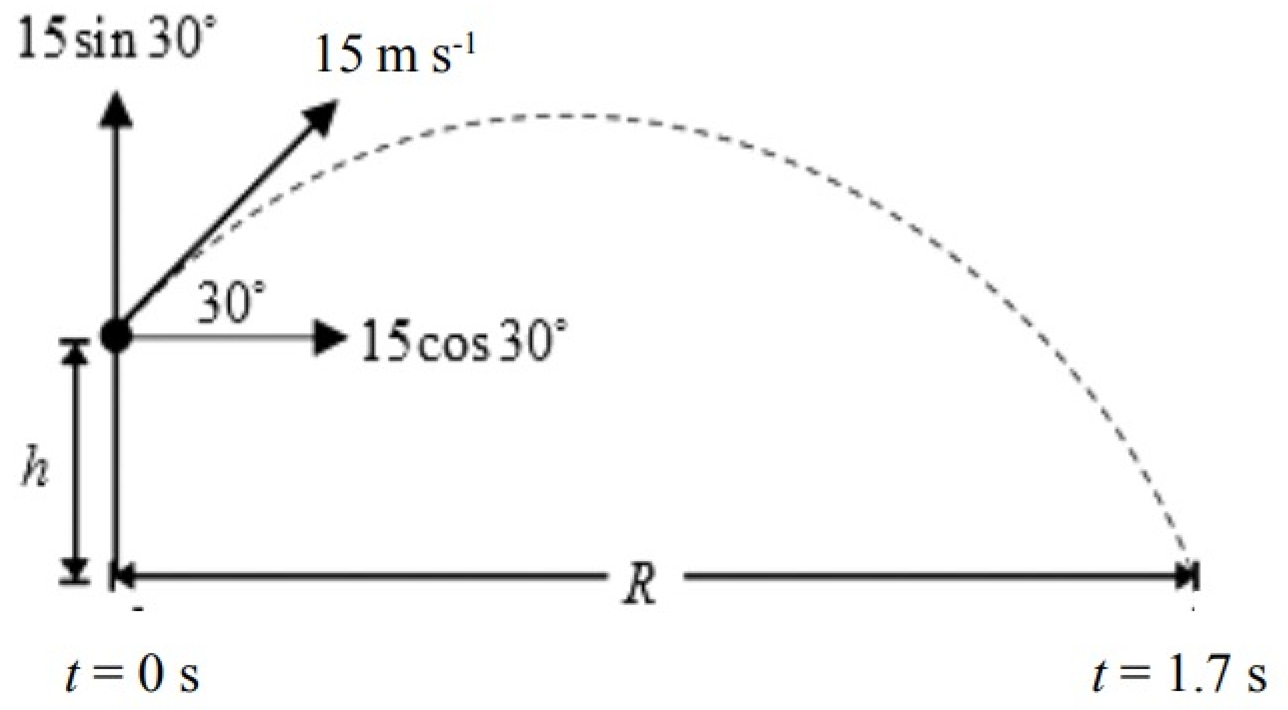
$$s = 12.5 \text{ m}$$

4. An athlete throws a discus with a speed of 15 m s^{-1} with an inclinations of 30° to the horizontal at a height '*h*' above ground. The discus lands on the ground after 1.7 s from the time of projection. Determine (take $\sqrt{3}$ =1.7)

(1) The height '*h*' above the ground the discus was initially projected

(2) The horizontal range obtained by the discus.

Answers :



(1) $\uparrow s = ut + \frac{1}{2}at^2$

$$-h = 15 \sin 30^\circ \text{ x } 1.7 + \frac{1}{2}(-g)\,(1.7)^2$$

$$h = 1.7 \text{ m}$$

(2) $\rightarrow s = ut$

$$\therefore R = 15 \cos 30^\circ \text{ x } 1.7 = 15 \frac{\sqrt{3}}{2} \times 1.7 = \frac{45}{2} = 22.5 \text{ m}$$

Applications

1. Firing a cannon ball by a cannon at a target.
2. Throwing a putt or a discus.
3. A batsman striking a ball in a game of cricket.

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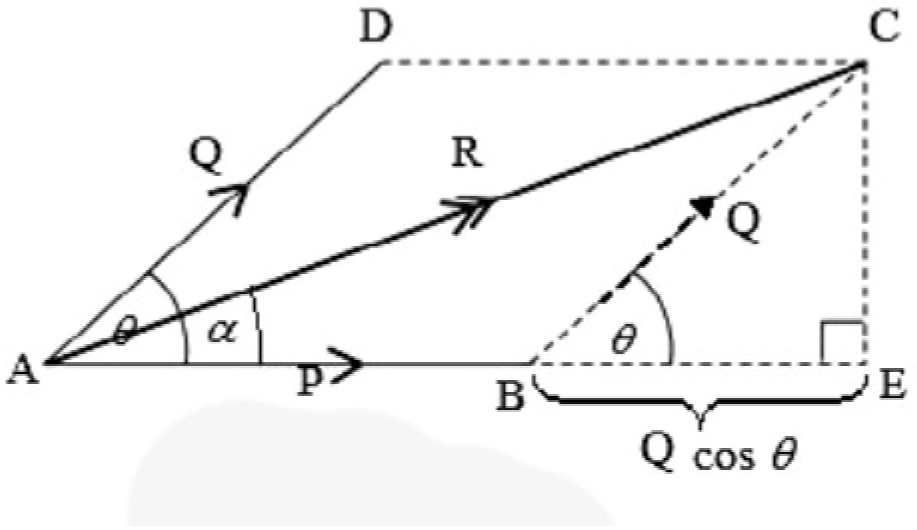
Chapter 02

Resultant of a system of coplannar forces

Principle of parallelogram of forces

If two forces are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant will be represented given by the corresponding diagonal in magnitude and direction.



'P' and '$\underline{Q}$' are two forces acting at a point with an inclination of θ to each other.

Figure 2.1

By Pythagoras theorem,

$$AC^2 = AE^2 + CE^2$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$R^2 = P^2 + 2.P.Q\cos\theta + (Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + 2PQ + Q^2\cos\theta^2 + Q^2\sin\theta^2$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right);$$ inclination of resultant to the force P.

Worked examples

Two forces 5 N and 12 N inclined to each other by 60° act at a point . Determine the magnitude and direction of the resultant.

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$$R^2 = 5^2 + 12^2 + 2 \times 5 \times 12 \cos 60^\circ$$

$$R = \sqrt{25+144+60}$$

$$R = \sqrt{229}\ \text{N}$$

$$\tan\alpha = \frac{12\sin 60^\circ}{5+12\cos 60^\circ}$$

$$\alpha = \tan^{-1}\left(\frac{6\sqrt{3}}{11}\right)$$

$$= 43^\circ 22'\ \text{inclined to the force 5 N}$$

Method -1

Force resolution method

Resolve the system of forces to any two perpendicular directions and add those resolved components using method of vector additions.

Worked example ;

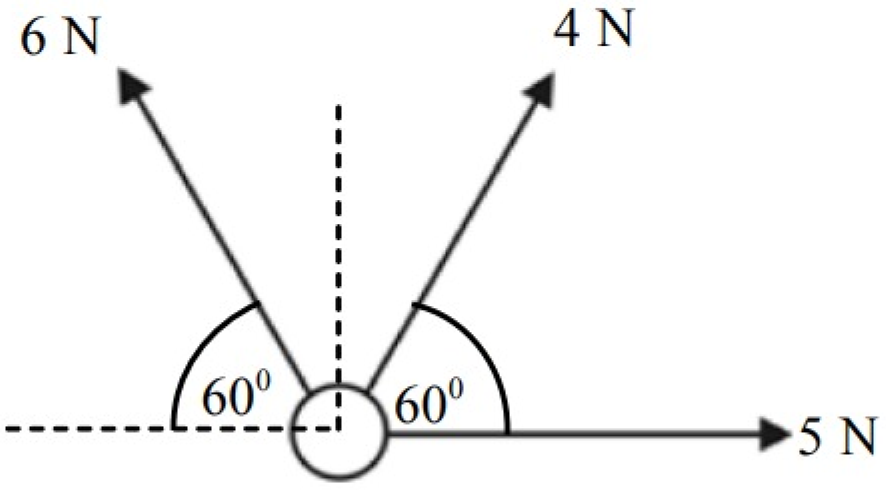

Figure 2.2

By resolving,

$$\vec{x} = 5 + 4\cos\ 60^o\ - 6\cos\ 60^o$$

$$= 5 + 4\ \times\ \frac{1}{2} - 6 \times \frac{1}{2}$$

$$=\ 4\ \text{N}$$

$$\uparrow y\ = 4\sin 60^\circ + 6\sin 60^\circ + 5\cos 90^\circ$$

$$= 2\sqrt{3} + 3\sqrt{3}$$

$$= 5\sqrt{3}$$

$$= 8.66\ \text{N}$$

$$R = \sqrt{X^2 + Y^2} = \sqrt{42 + \left(5\sqrt{3}\right)^2} = \sqrt{91}\text{N} = \underline{\underline{9.54\text{N}}}$$

Method -2

Force Polygon method

If a systems of forces is represented in magnitude and direction by adjacent sides of a polygon (taken in order) , the magnitude and direction of the resultant will be given by the final side of the polygon, in the opposite sense.

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Moment of a force

The moment of a force is the turning effect produced by the force.

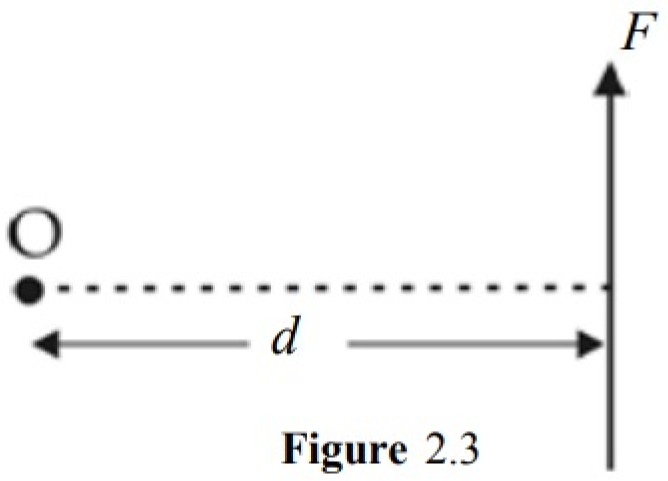


Figure 2.3

Moment of force F about the point $'\mathrm{O}' = F \times d$

where *'d'* is the perpendicular distance from 'O' to the force.

Moment of a force is a vector property. Its direction is given by the right hand screw rule."If a body turns due to a moment, and its direction of rotation is indicated by the direction of turn of gripped fingers of right hand, the direction of the moment is given by the direction of pointing of the thumb".

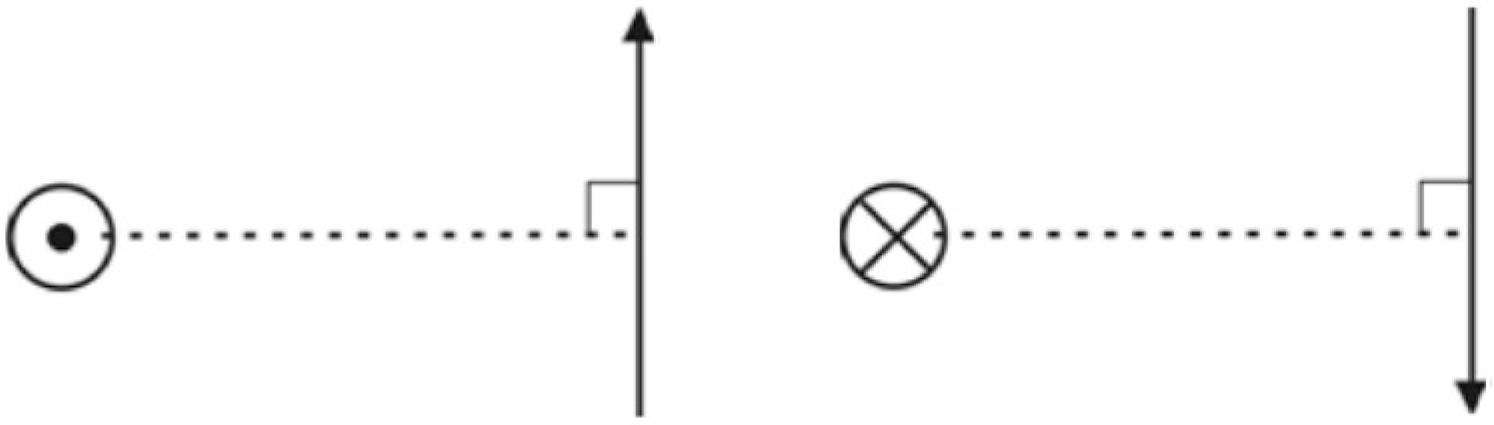
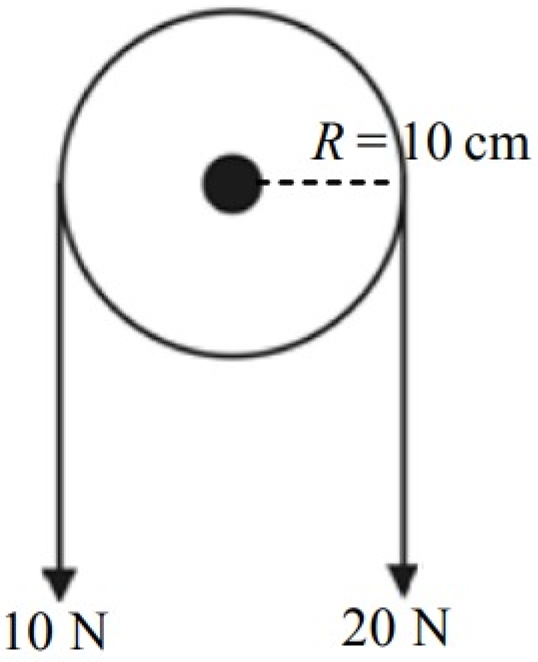


Figure 2.4

Worked example :



Two forces 20 N and 10 N are exerted at the two ends of a string passing over a pulley. Determine the net moment about the centre of the pulley.

Net moment $(\tau) = 20 \times 0.1 - 10 \times 0.1$

$$\tau = 1 \text{ N m}$$

Moment of a couple (of forces) or torque (τ)

Two equal and opposite forces whose lines of action do not coincide are said to form a couple. Consider two equal and opposite parallel forces acting with a separation *'d'* (a couple).

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Moment about point X, (τ_x)

$$\tau_x = F \times x + F(d - x)$$

$$\boxed{\tau_x = F \times d}$$

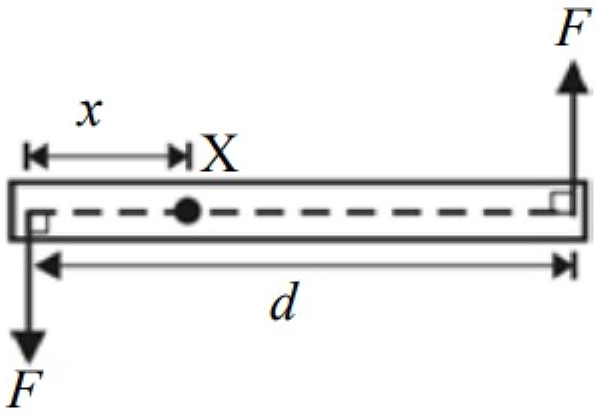

Figure 2.5

Therefore, the moment or torque of couple about <u>any point</u>

τ = Force perpendicular distance between the two forces.

Centre of gravity of a body

A body is made by several particles, each attracted towards the centre of the Earth by a force of gravity (given by Newton's universal law of gravitation) and thus the body experiences a resultant force of gravity (weight). That force is called the weight of the body.

The centre of gravity of a body is the point at which the resultant force of gravity or weight of the body acts.

Centre of gravity of regular shaped bodies

Object	Centre of gravity (G)
1. Uniform rod	G - Mid point

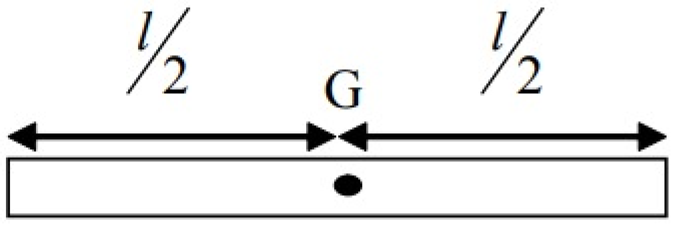

Figure 2.6

2. Uniform ring	G - Centre of ring

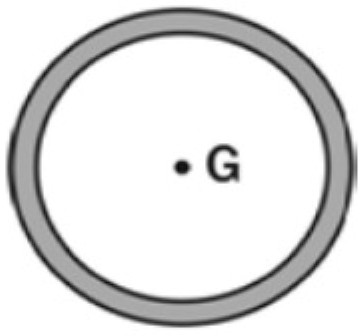

Figure 2.7

3. Uniform circular disc	G - Centre of disc

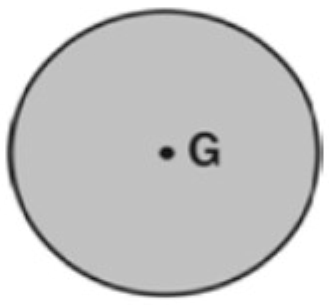

Figure 2.8

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4. Uniform rectangular shaped laminar G - centre (or point of intersection of diagonals)

Figure 2.9

5. Uniform solid /hollow cylinder G - Mid point along the axis

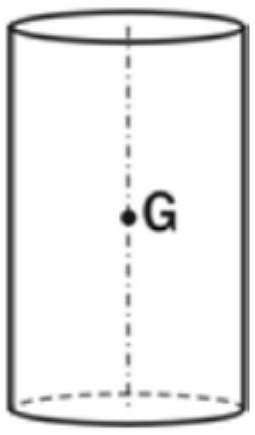
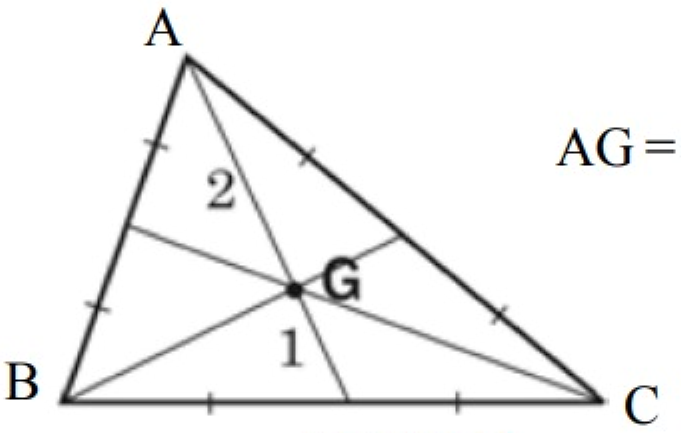


Figure 2.10

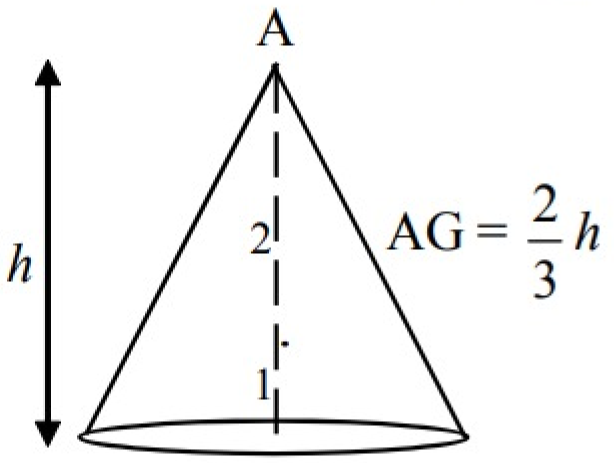
6. Uniform triangular shaped laminar G - Centroid (point of intersection of medians)



$$AG = \frac{2}{3} AD$$

Figure 2.11

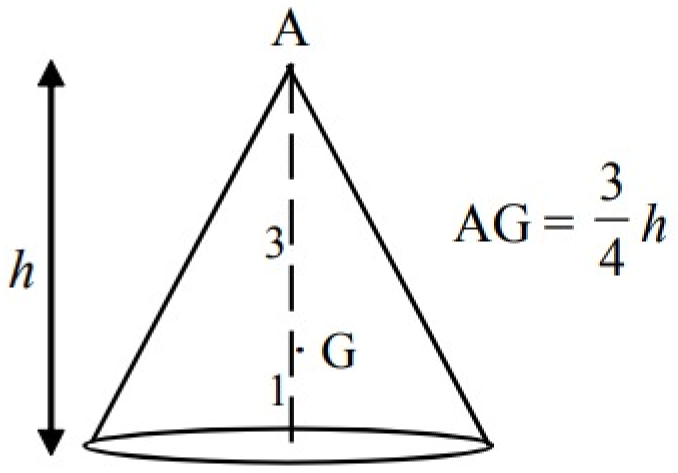
7. Uniform hollow cone G - Centroide of cone (along axis)



$$AG = \frac{2}{3} h$$

Figure 2.12

8. Uniform solid cone G - Centre of cone (along axis)



$$AG = \frac{3}{4} h$$

Figure 2.13

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9. Uniform hollow/ solid sphere G - Centre of sphere

Figure 2.14

Centre of gravity of regular shaped compound bodies

Worked examples :-

1. Uniform ring and rods fixed to form a cart wheel. Determine centre of gravity.

   Answer – Centre of gravity at X

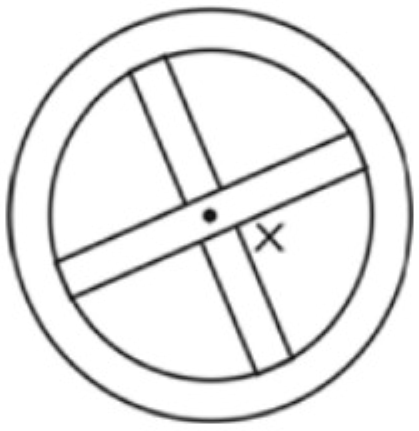

Figure 2.15

2. ABCD is a uniform rectangular shaped laminar and BCE is a uniform triangular shaped laminar made of same metal and of uniform thickness. Determine the centre of gravity of the composite laminar formed by connecting them as shown in the diagram.

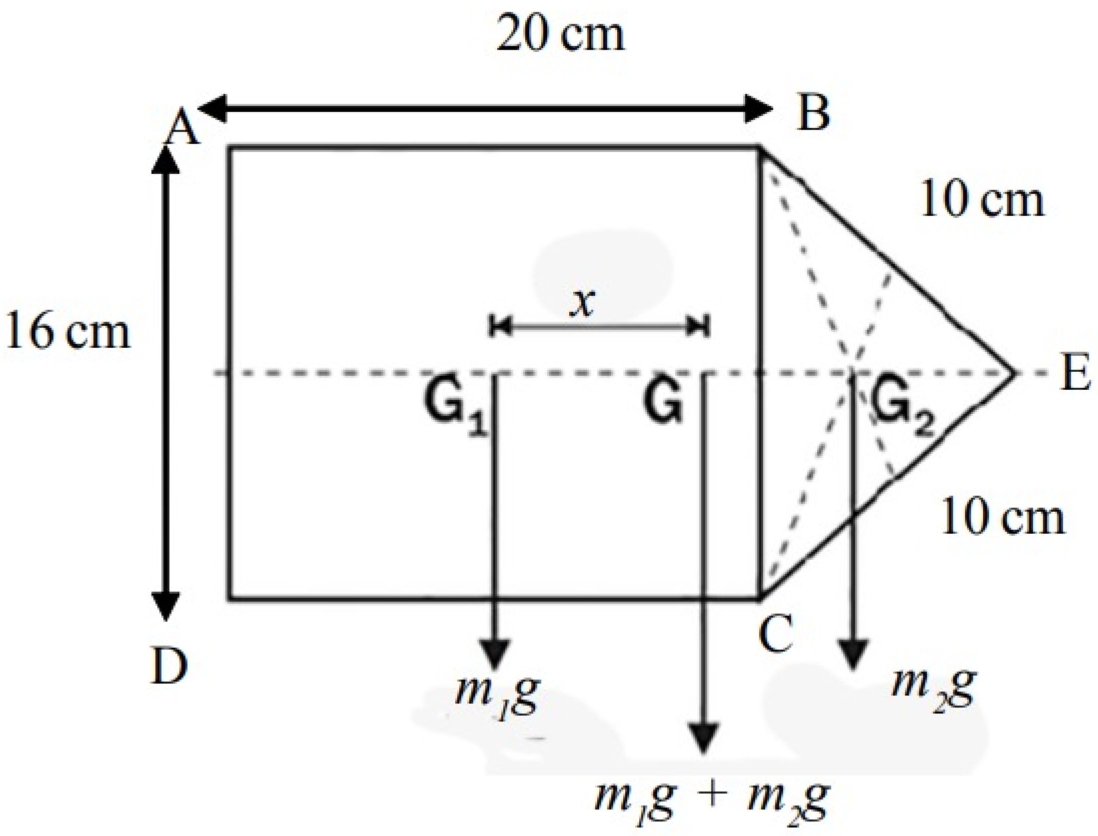

Figure 2.16

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Answer :

G_1 – Centre of gravity of rectangle (of weight m_1g)

G_2 – Centre f gravity of triangle (of weight m_2g)

G – Centre of gravity of compounded object (of weight $m_1g + m_2g$)

By taking clockwise moments,

Net moment of all forces = moment of resultant force

$$G\circlearrowright \quad m_2g \times (GG_2) - m_1g\,(G_1G) = (m_1g + m_2g) \text{ x } 0$$

$$\left(\frac{1}{2}\times 16\times 6\times \rho g\right)\times(12-x)-(20\times 16)+\rho g\times(x)=0$$

$$x=\frac{36}{23}\text{ cm } =\underline{\underline{1.57\text{ cm}}}$$

Centre of mass

Centre of mass of a body is defined as the point at which an applied force produces linear acceleration but no angular acceleration.

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Chapter 03

Force and motion

The exact link between force and motion is not easy to find in everyday life. It is not that force is needed to maintain motion. Push a heavy box across a rough floor with a uniform velocity and you may mistakenly conclude that motion needs force. In fact the "hidden" force of friction confuses the link.

The general link between force and motion is that unbalanced (resultant) force is required to change motion. In other words, to change the velocity of an object, the object must be acted on by an unbalanced force. If there is no unbalanced resultant force on an object, its velocity must stay the same. If there is an unbalanced force on an object, its velocity must change.

Inertia

Try the following activity.

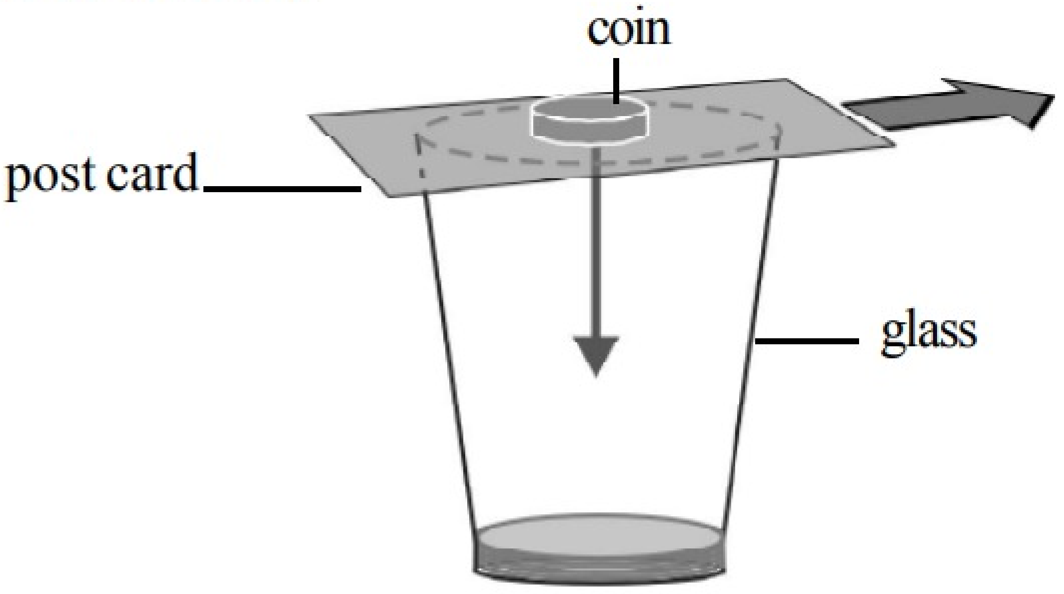

Figure 3.1

A coin is placed on a post card. Pull the card away quickly. Coin will not move away with the card. Coin will drop into the glass. What prevented it from moving with the card? It is the mass of the coin. Since the coin has more mass than the postcard, it resists moving with the card. This resistance to change the state of motion is called inertia. The more mass something has the more inertia it has.

Inertial mass

Mass is a measure of inertia. Inertial mass indicates the opposition of a body to change of motion when responding to all types of forces.

Gravitational mass

Gravitational mass is determined by the strength of the gravitational force experienced by a body when it is in a gravitational field. Experiments show that to a high degree of accuracy these two masses are equal for a given body.

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Frame of reference

Motion of a body is always described with reference to some well defined co-ordinate system. This co-ordinate system is referred to as 'frame of reference'. In three dimensional space a frame of reference consists of three mutually perpendicular lines called 'axes of frame of reference' meeting at a single point called origin.

Inertial frames

A frame of reference that remains at rest or moves with constant velocity with respect to other frames of reference is called 'Inertial Frame of Reference'. An inertial frame of reference is actually an unaccelerated frame of reference. Newton's laws of motion are valid in all inertial frames of reference. According to Newton's first law when a body or system of bodies is not accelerating, a resultant force is not acted upon it. Since the inertial frames are not accelerating, no external force is acted upon from outside the frame. So in this frame of reference, since the system is not accelerating the body is not acted upon by external forces.

Eg:-

- Our earth (Although the Earth is not exactly inertial, we assume it to be inertial)
- A space shuttle moving with constant velocity relative to the earth.
- A rocket moving with constant velocity relative to the earth.

Non-Inertial frames

A frame of reference is said to be a non-inertial frame of reference when it is moving non-uniformly (accelerating) relative to an inertial frame. In non-inertial frame of reference, Newton's laws of motion are not valid.

Newton's first law of motion

This law explains the exact link between force and motion.

Newton's first law of motion
Every object continues to be at rest or move with uniform velocity unless acted upon by a resultant force.

Momentum

When a mass is moving its momentum is given by,

$$\boxed{\text{momentum} = \text{mass} \times \text{velocity}}$$

$$\boxed{p = mv}$$

The unit of momentum is kg m s^{-1}. Momentum is a vector quantity. Its direction is the same as the direction of velocity.

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Eg :

If a model car has a masss of 2 kg and a velocity of 2 $\mathrm{m\,s^{-1}}$, its

$$\begin{aligned} \text{momentum} &= \text{mass} \times \text{velocity} \\ &= 2\ \text{kg} \times 2\ \text{m s}^{-1} \\ &= 4\ \text{kg m s}^{-1} \end{aligned}$$

Newton's second law of motion

Newton's first law states that a resultant force can change the state of motion of an object. Newton's second law states about this resultant force.

> **Newtown's second law**
>
> The rate of change of moments of an object is proportional to the unbalanced (resultant) force which acts on the object.

The direction of the unbalanced (resultant) force is the same as the direction of the momentum change.

Consider an object of fixed mass *m* acted upon by a constant resultant forc *F*. Suppose the object is accelerated from velocity *u* to velocity *v* in time *t*.

$$\text{Change of momentum} = \text{final momentum-initial momentum} = mv - mu$$

$$\text{Change of momentum per second} = \frac{mv - mu}{t}$$

From Newton's second law, the resultant force *F* is proportional to the change of momentum per second.

$$F \propto \frac{mv - mu}{t}$$

$$F \propto m\frac{(v-u)}{t}$$

but the object's acceleration

$$a = \frac{\text{change of velocity}}{\text{time taken}} = \frac{(v-u)}{t}$$

$\therefore$ Above relationship can be denoted as,

$$F = k\ ma$$

The value of *k* is set at 1 by defining the unit of force.

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Defining the unit of force - newton (N)

A force of 1 newton (N), is the force which would give a 1 kg mass an acceleration of 1 m s^{-2}.

accordingly, $1 = k \times 1 \times 1$

$\therefore \quad k = 1$

$\therefore \quad \boxed{F = ma}$

F - Resultant force in newtons

m - mass in kg

a - acceleration in m s^{-2}

Impulse and impulsive force

A large force accting in a very small period of time is an impulsive force.

Ex:- i. Shot on a nail by a hammer

ii. Stroke on a ball by a bat.

$$\boxed{\text{Impulse} = \text{Impulsive force x Time of action of the force}}$$

The unit of impulse is N s.

$$I = F \times t \text{ (N s)}$$

$$= ma \times t$$

$$= m\left(\frac{v-u}{t}\right) \times t$$

$$= mv - mu$$

The quantity on the left-hand side is called the impulse, the product of force and time, and is measured in N s. The quantity on the right-hand side is the change of momentum produced by this impulse.

$$\boxed{\text{Impulse} = \text{Change in momentum}}$$

Large force acting for a short time may cause the same change of momentum as small force acting for a long time. You use this every time you jump off something and bend your knees on landing. If you were to land rigidly, your momentum would fall to zero in a very short time. The large rate of change of momentum would exert large forces on your body and probably cause injuries. Bending your knees allows the same change of momentum to occur over a longer time and so reduces the force on your body. A similar principle applies in the use of air bags to make cars safer.

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Principle of conservation of linear momentum

According to Newton's second law resultant force is proportional to the rate of change of momentum. So if the resultant force acted upon is zero, it implies that momentum does not change.

$$F \; \alpha \; \frac{mv - mu}{t}$$

$$If \;\; F = 0 \;\; then$$

$$mv - mu = 0$$

$$mv = mu$$

Now consider a system of objects interacting with each other. If no external resultant force acts on this system of objects, the total momentum does not change. However, within the system, the interactions between the objects cause transfer of momentum between them. But the total momentum stays constant.

Principle of conservation of linear momentum

For a system of interacting objects, the momentum remains constant, provided no external force acts on the system.

Consider collision of two balls of masses m_1 and m_2. The velocities of balls before collision are u_1 and u_2, while the velocities of balls after collision are v_1 and v_2.

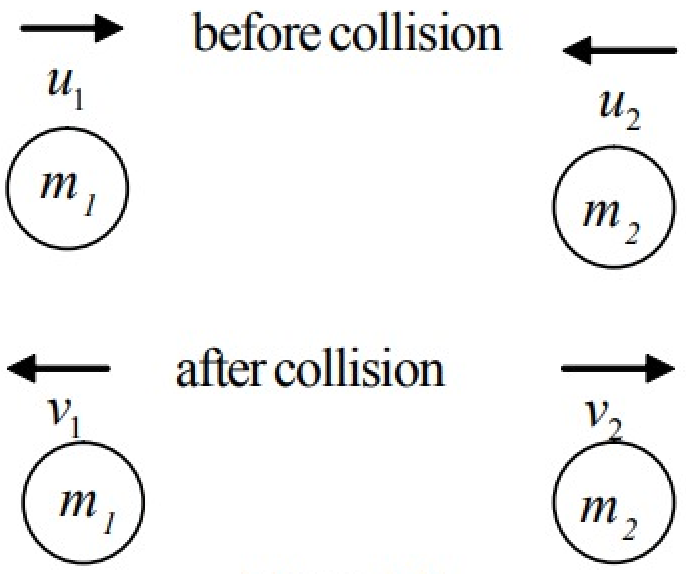

Figure 3.2

According to principle of conservation of linear momentum,
momentum of the system before collision = momentum of the system after collision

$$m_1u_1 - m_2u_2 = -m_1v_1 + m_2v_2$$

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Newton's third law of motion

When you push on a wall, did you fall through the wall? You may say no. What prevented you from going through the wall? The wall pushes back on you with an equal and opposite force. If the wall suddenly gives way, the push back on you is removed allowing you to fal towards the wall.

Newton's third law of motion

When the objects interact (collide), they exert equal and oppsite forces on one another.

Applications of Newton's laws

Worked examples:

1. An object of mass 20 kg is to be accelerated at 3 m s-2. What force is required?
   By Newton's second law,

   $$F = ma = 20 \times 3 = 60 \text{ N}$$

2. A car of mass 1500 kg, travelling at 80 km h^{-1}, is to be stopped in 11 s. What force is required?

   $$80 \text{ km h}^{-1} = \frac{80 \times 1000}{3600 \times} \text{ m s}^{-1} = 22 \text{ m s}^{-1}$$

   The deceleration can be obtained from equation $v = u + at$

   $$a = \frac{v-u}{t} = \frac{0-22}{11} = -2$$

   By Newton's second law,

   $$F = ma = 1500 \times (-2) = -3000 \text{ N}$$

   $\therefore$ A force of 3000 N oppsite to the direction of velocity should be applied to stop the car.

3. A box of mass 5 kg is pulled along a horizontal floor by a force *P* of 50 N, applied at an angle of 60° to the horizontal plane. A frictional force F' of 20 N acts parallel to the floor. Calculate the acceleration of the box.

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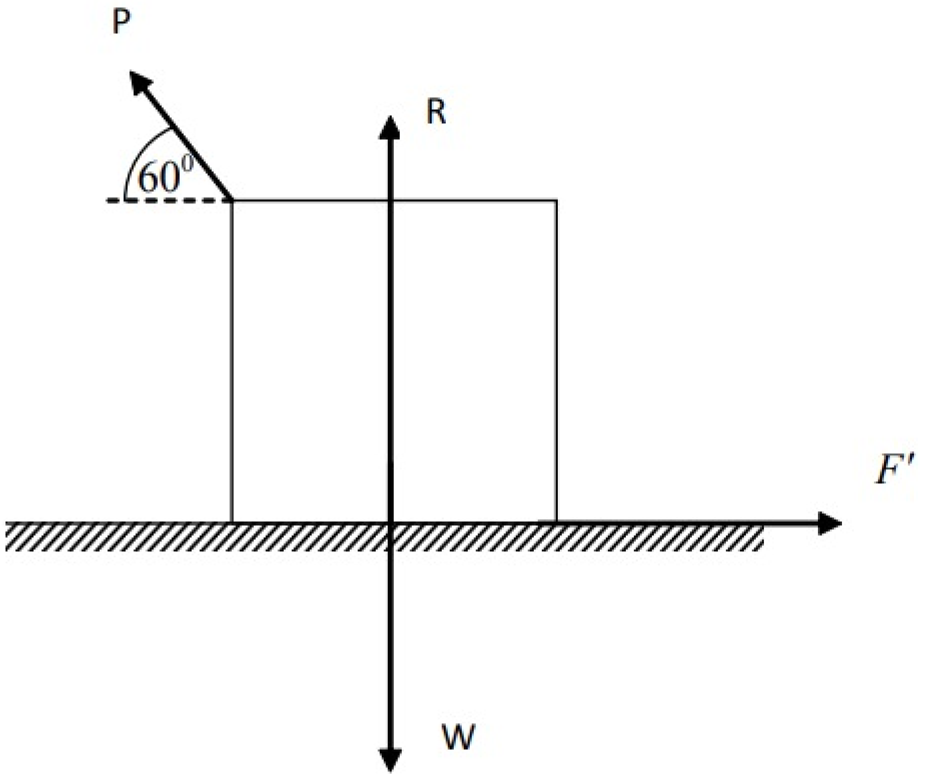
$$\leftarrow F = ma$$

$$P\cos 60^\circ - F' = ma$$

$$50 \times \frac{1}{2} - 20 = 5 \times a$$

$$a = 1 \text{ m s}^{-2}$$

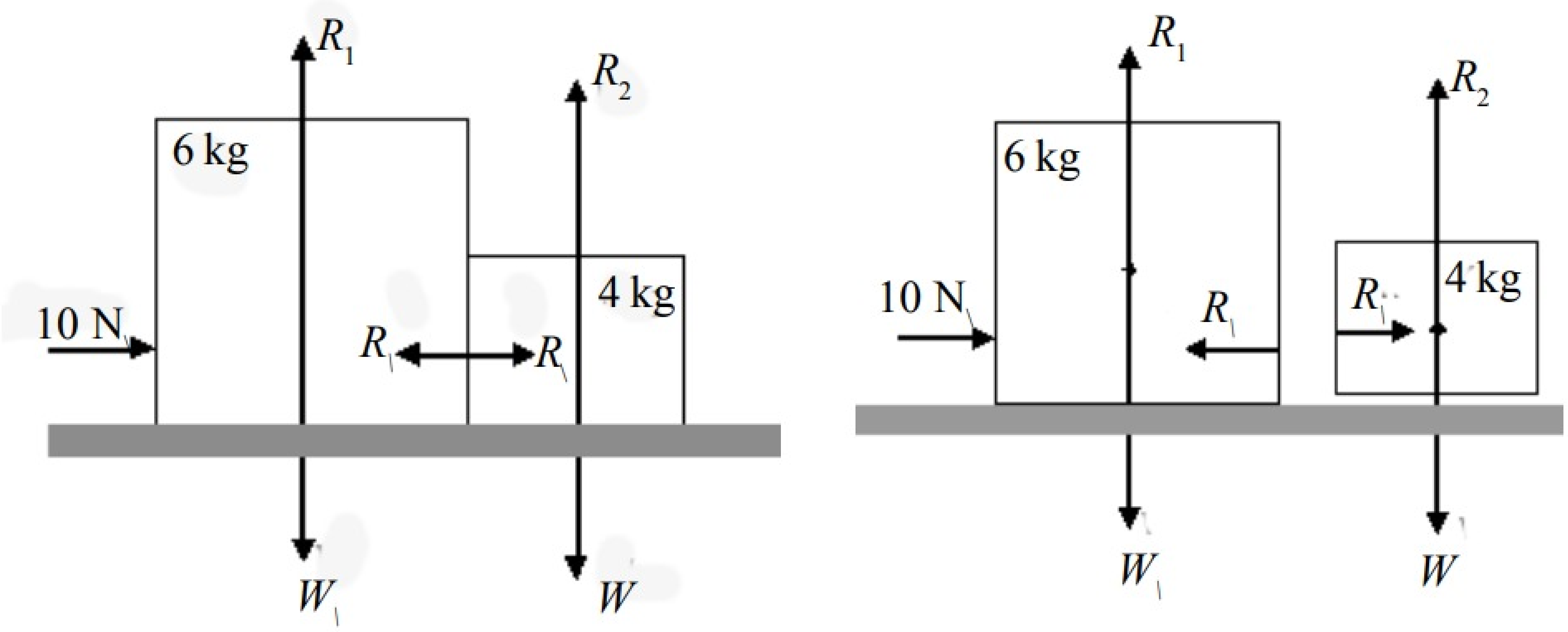


4. Two masses of mass 6 kg and 4 kg are kept in contact on a smooth horizontal floor. The 6 kg mass is pushed with a horizontal force of 10 N. Find the acceleration of the system and the reaction force between the two masses.

$\rightarrow F = ma$ to the whole system

$$10 = 10 \times a$$

$$a = 1 \text{ m s}^{-2}$$



Above diagram shows the free body force diagram for the above system

$\rightarrow F = ma$ to 6 kg mass

$$10 - R = 6 \times 1$$

$$R = 4 \text{ N}$$

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5. A string is put over a smooth pulley and two masses of 4 kg and 6 kg are attached at the ends of the string. If the masses are released from rest, find the initial acceleration of the masses and the tension of the string $\left(g = 10 \text{ m s}^{-2}\right)$.

If T is the tension of the strings and *'a'* is the initial acceleration.

$\downarrow F = ma$

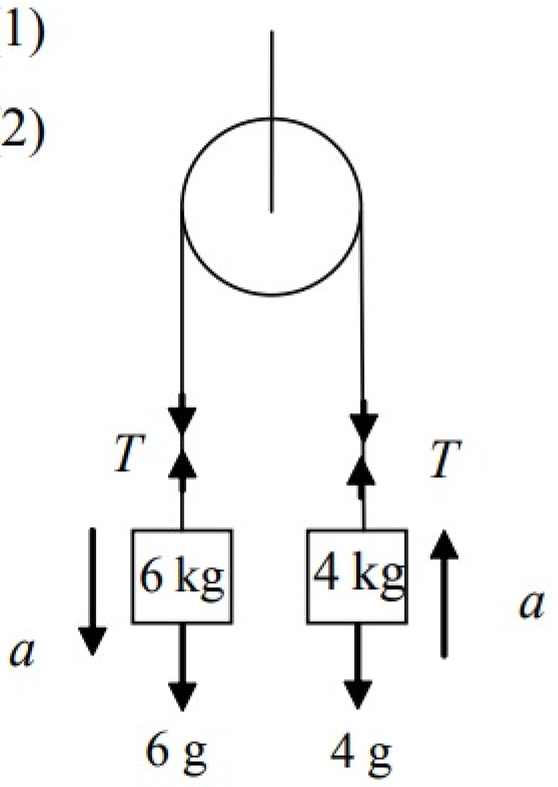
to 6 kg , $\downarrow$ $6g - T = 6a$ ....(1)

to 4 kg , $\uparrow$ $T - 4g = 4a$ ....(2)

equation (1) + equation (2)

$2g = 10\,a$

$a = 0.2\,g = 2 \text{ m s}^{-2}$



From equation (2)

$T = 4 \text{ x } 0.2 \text{ x } g + 4 \text{ x } g = 4.8g = 48 \text{ N}$

Self adjusting forces

Tension

Consider piece of rope which is stretched along. What force prevents it from breaking? It is a force called tension.

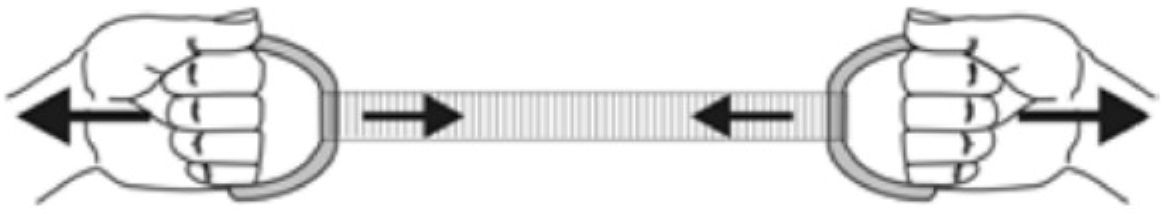


Figure 3.3

If this force is always acting in the rope, it has to be shrinked. But it does not happen. So this force appears only when it is stretched. When you pull the rope this force adjusts itself to prevent the breakage of the rope. When you increase the pulling force the tension force also will be increased. Since this tension force adjusts itself it is called as a **"self-adjusting force".**

Tension, thrust or compression and friction are self-adjusting forces.

Thrust or compression

This acts in a way of opposite to the tension. For example when you push a small piece of wood by both your hands a force of compression appears to prevent it from breaking.

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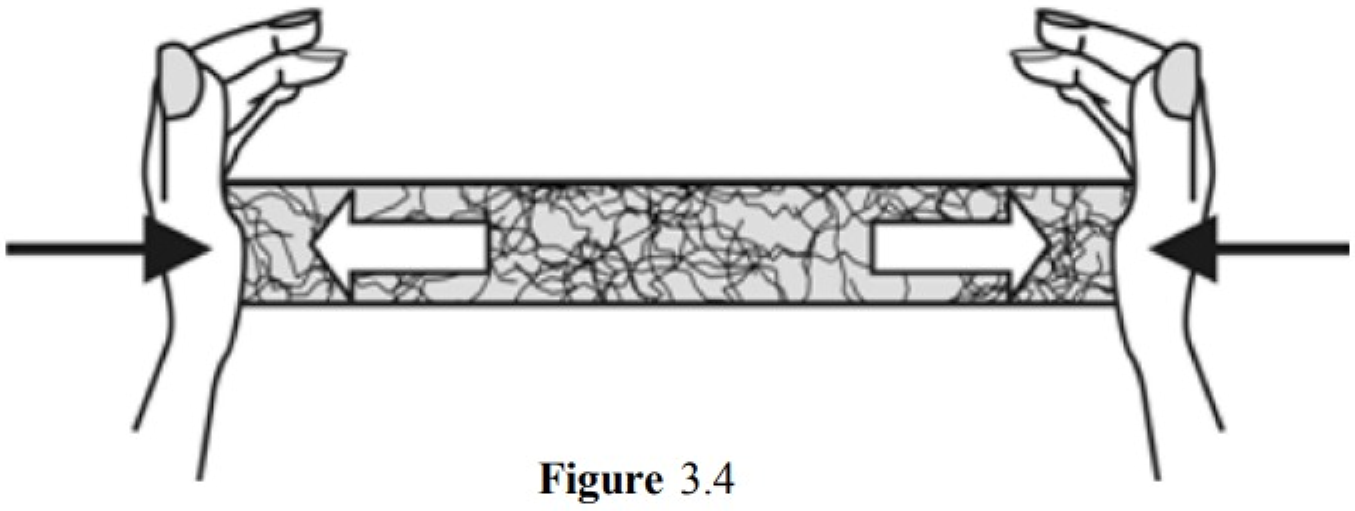
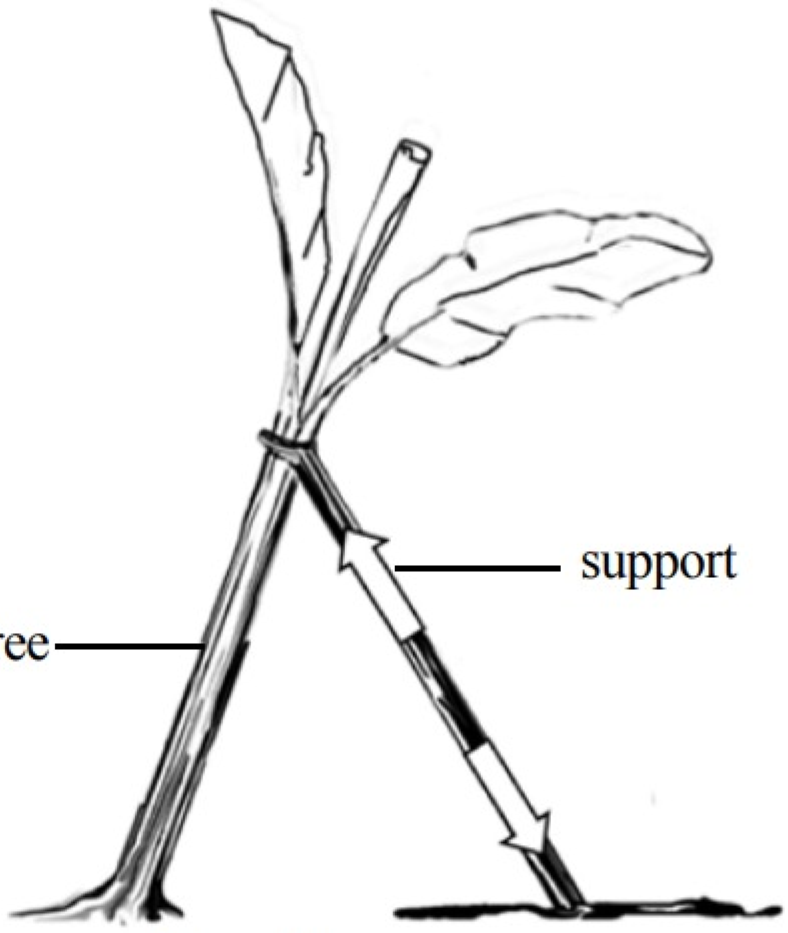


Figure 3.4

This force is changing itself to balance the external force given by hands. So it is also a self-adjusting force.

Eg :



Thrust force acts along the support to prevent the free from falling.

Figure 3.5

Frictional force

Friction is the force that tries to stop materials sliding across each other. Frictional force prevents or opposes relative motion. Microscopically all surfaces are rough. When such two surfaces placed in contact there will be regions where the irregularities interlock , and points where high pressure may result in temporary bonds. Any attempt to slide the surfaces over one another will require a certain amount of work to be done, lifting and deforming the surface. This will require a force as the surface moves.

In some circumstances friction is unwanted because it wastes energy. However at some circumstances friction is useful. For example, braking a car are applied, friction between the wheels and the road surface is essential if the car is to be slowed down.

Static friction

Sometimes the force applied is not large enough to overcome the friction. So, no movement takes place, though a force is applied. The friction acting in this situation is called static friction.

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So static friction acts between surfaces at rest when a force is applied to make them slide past one another.

Think of sliding a heavy case across a horizontal floor. If you push it gently it doesn't move. Push it a bit harder and it still doesn't move. In both cases the static frictional force has balanced the applied force. Static friction adjusts itself to equalize the applied force until the object starts to move.

Limiting frictional force

However, with enough force applied, the force of friction can be overcome and then movement takes place. The frictional force acting when the surfaces just begin to slip past one another is called the limiting frictional force.

Consider a case in which a man tries to push a heavy box on a rough horizontal surface.

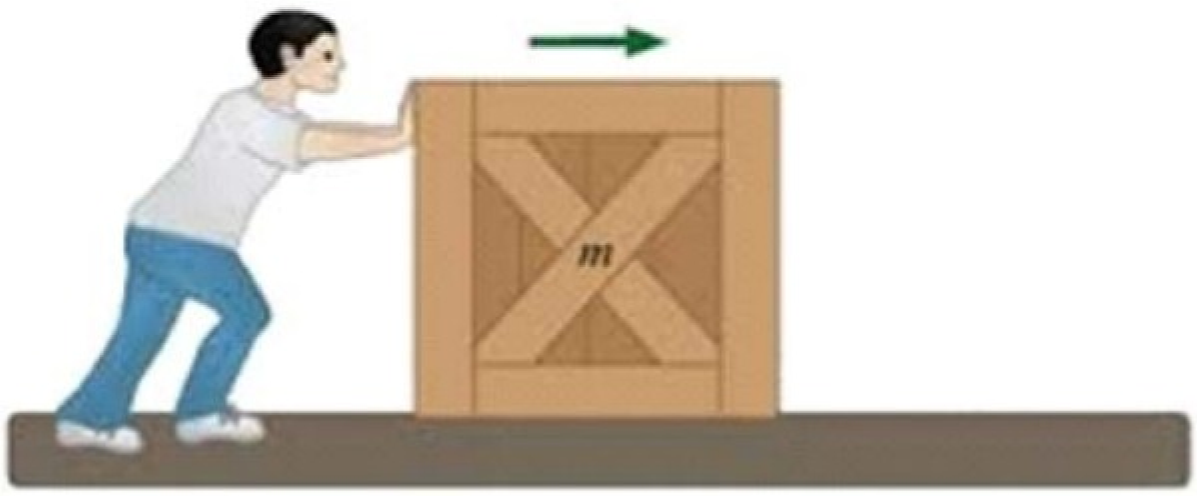

Figure 3.6

If we plot the frictional force against the applied force, the shape of the graph will be as shown in Figure 3.7.

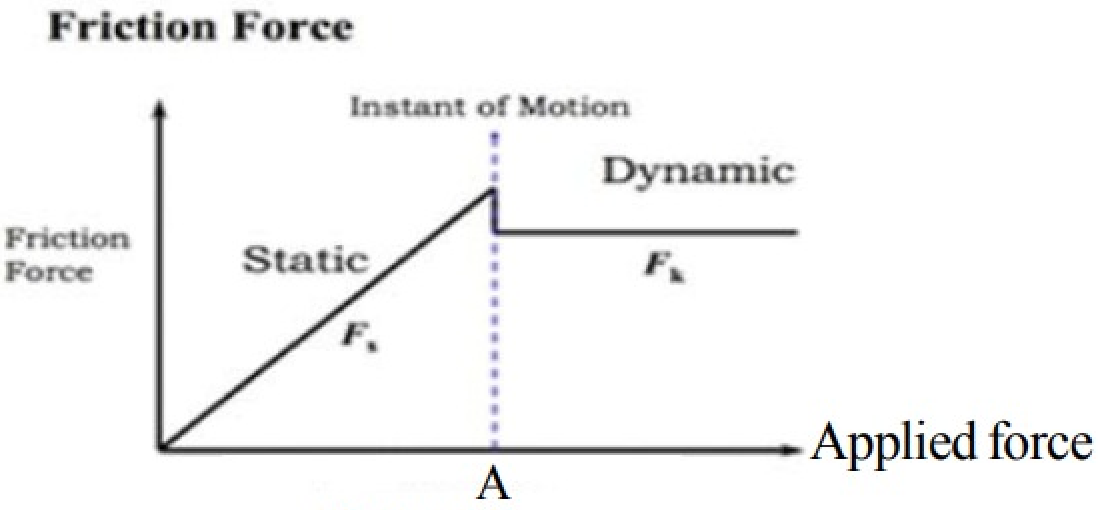

Figure 3.7

Point 'A' indicates the instant at which the box just begins to slip over the surface. So, the friction at that instant is the limiting frictional force.

After point 'A' the box moves over the surface and the frictional force remains constant. The friction acting when the box is moving is called dynamic friction.

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Although the frictional force between real surfaces will vary from one place to the next as the local surface changes, it is possible to describe sliding friction by a fairly simple rule. This rule links the normal force between the two surfaces(that is how hard they are pressed together) to the frictional force acting on them when they just begin to slip past one another(the limiting friction)

limiting friction between surfaces $\propto$ normal contact force between surfaces

$$F \propto R$$
$$F = \mu R$$

μ is called the **coefficient of static friction** and is roughly constant for a particular pair of surfaces.

Dynamic (or kinetic) friction

The friction acting between two surfaces as they slide over one another is called dynamic friction. At the instant the surfaces just begin to slide the above relationship $F = \mu R$ is valid and there the dynamic friction is equal to the limiting frictional force.

The dynamic friction acting on an object is also proportional to the normal contact force between surfaces. If the dynamic friction is F_d and normal contact force is R,

$$F_d \propto R$$
$$F_d = \mu_d R$$

μ_d is called the coefficient of dynamic friction.

According to the graph in Figure 3.7,

$$F > F_d$$

$$\mu > \mu_d$$

So, the coefficient of dynamic friction is always less than the coefficient of limiting friction.

Worked example

(i) A man is trying to push a packing case of weight 300 N across a horizontal floor. What is the minimum horizontal force required to slide it, if the coefficient of limiting friction between the case and the floor is 0.3?

To slide the case the applied force must just equal the limiting friction.

$$F = \mu R = 0.3 \times 300 = 90\text{N}$$

(ii) Finally the man was able to push the packing case over the floor. When he started pushing, he felt that the packing case is easily moving with less effort. He only needed 87 N force to maintain a constant velocity. Calculate the coefficient of dynamic friction.

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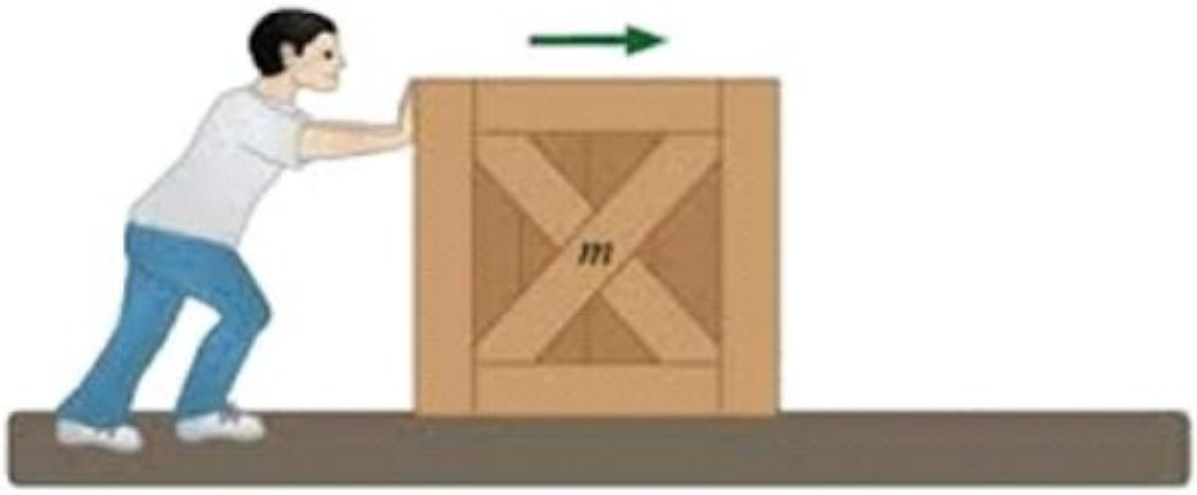
Since the man is maintaining a constant speed, the pushing force given by the man is equal to the dynamic frictional force.

Let F be the force applied by the man and F_d be the dynamic frictional force.

$$\rightarrow F = ma$$
$$F - F_d = 0$$
$$F = F_d$$



$$F_d = \mu_d R$$
$$87 = \mu_d \times 300$$
$$\mu_d = \frac{87}{300} = 0.29$$

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Chapter 04

Equilibrium of Forces

The bridge in the Figure 4.1 is built not to be collapsed under large amounts of loads. This is done by designing each part of the bridge to be under equilibrium with heavy loads. So equilibrium acts an important role when building stable structures.

Conditions for Equilibrium

Figure 4.1

As it is known, equilibrium is the state in which a body is at rest without any translatory motion or rotation when it is acted upon by a system of forces. For this to happen, it is clear that the main condition for equilibrium of a system of forces is that when subjected to composition the system should not

reduce to a single force or resultant (to prevent motion) and also to a couple of forces or torque (to prevent rotation). Considering this main condition, the necessary and sufficient conditions for equilibrium, depending on the number of forces in the system, can be stated as follows.

1. Conditions for the equilibrium of two forces.

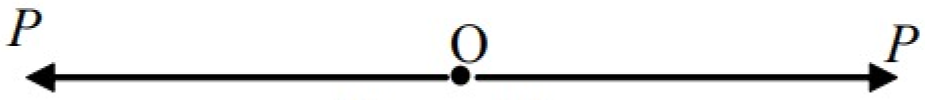

Figure 4.2

1. The two forces should be collinear.
2. The two forces should be equal in magnitude.
3. The two forces should act in opposite directions

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2. Conditions for the equilibrium of three forces

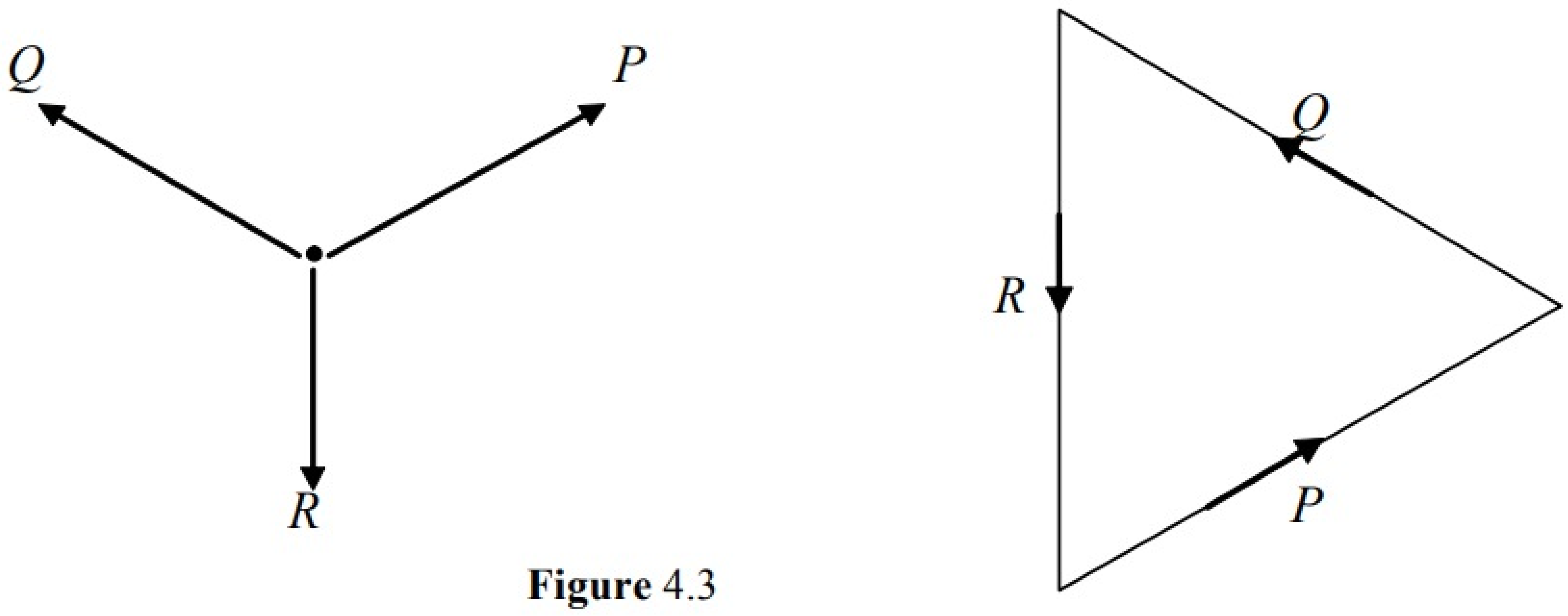
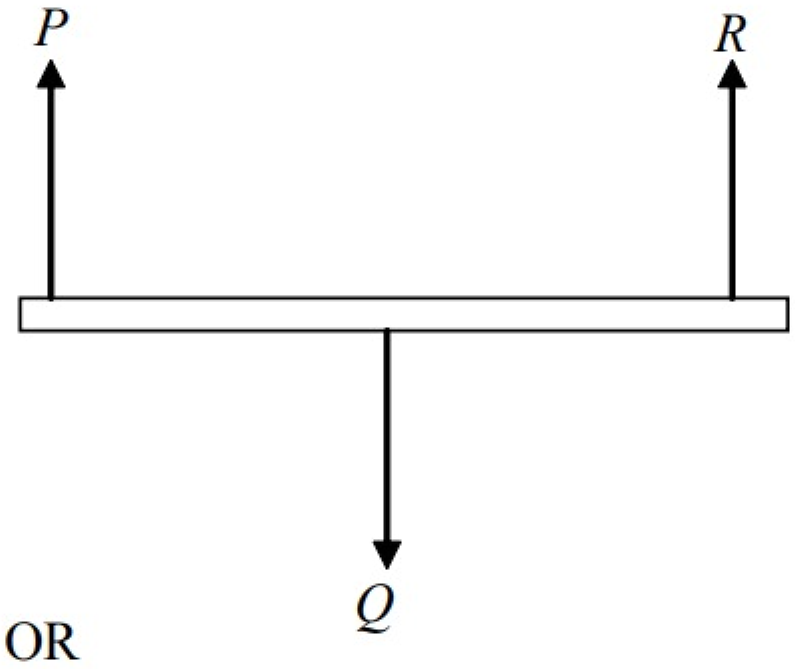


Figure 4.3

1. The three forces should be coplanar.
2. Their lines of action should meet at a point.
3. The three forces should be able to be represented by three sides of a triangle taken in order (this triangle is known as the triangle of forces).

or else



OR

Figure 4.4

1. The three forces should be parallel and coplanar.
2. The algebraic sum of the three forces in their direction should be zero.
3. The algebraic sum of the moments of the three forces about any point in their plane should be zero.

Conditions for the equilibrium of any number of coplanar forces

1. The algebraic sum of the components of all the forces in any direction should be zero
2. The algebraic sum of the components of all the forces in a direction perpendicular to the above mentioned direction should also be zero.
3. The algebraic sum of the moments of all the forces about any point in their plane should be zero.

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Note

Just as three forces in equilibrium can be represented by a triangle of forces, a larger number of coplanar forces in equilibrium can be represented by a polygon of forces.

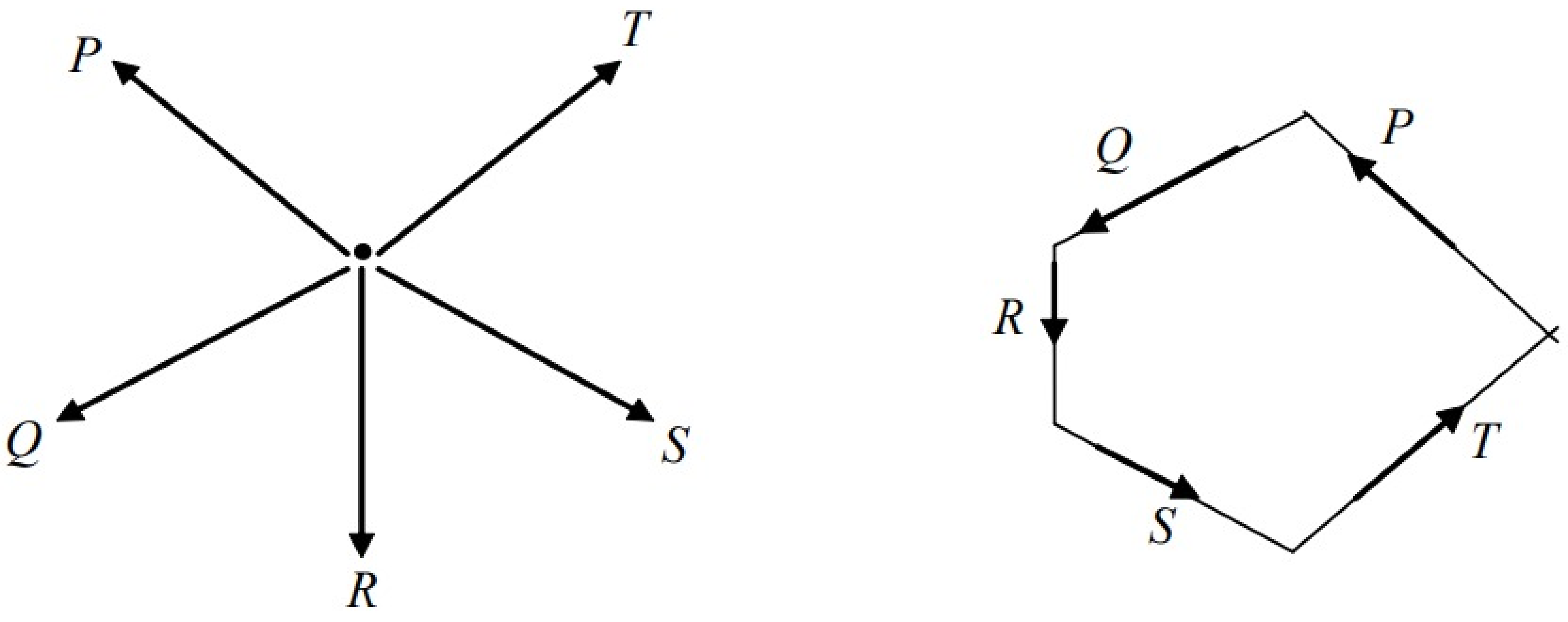

Figure 4.5

States of equilibrium

There could be many ways in which a body can be placed in equilibrium. These are called states of equilibrium of which one would be safe, another moderately safe and yet another unsafe. The three main states of equilibrium are known as

1. Stable equilibrium
2. Unstable equilibrium
3. Neutral equilibrium

Eg :

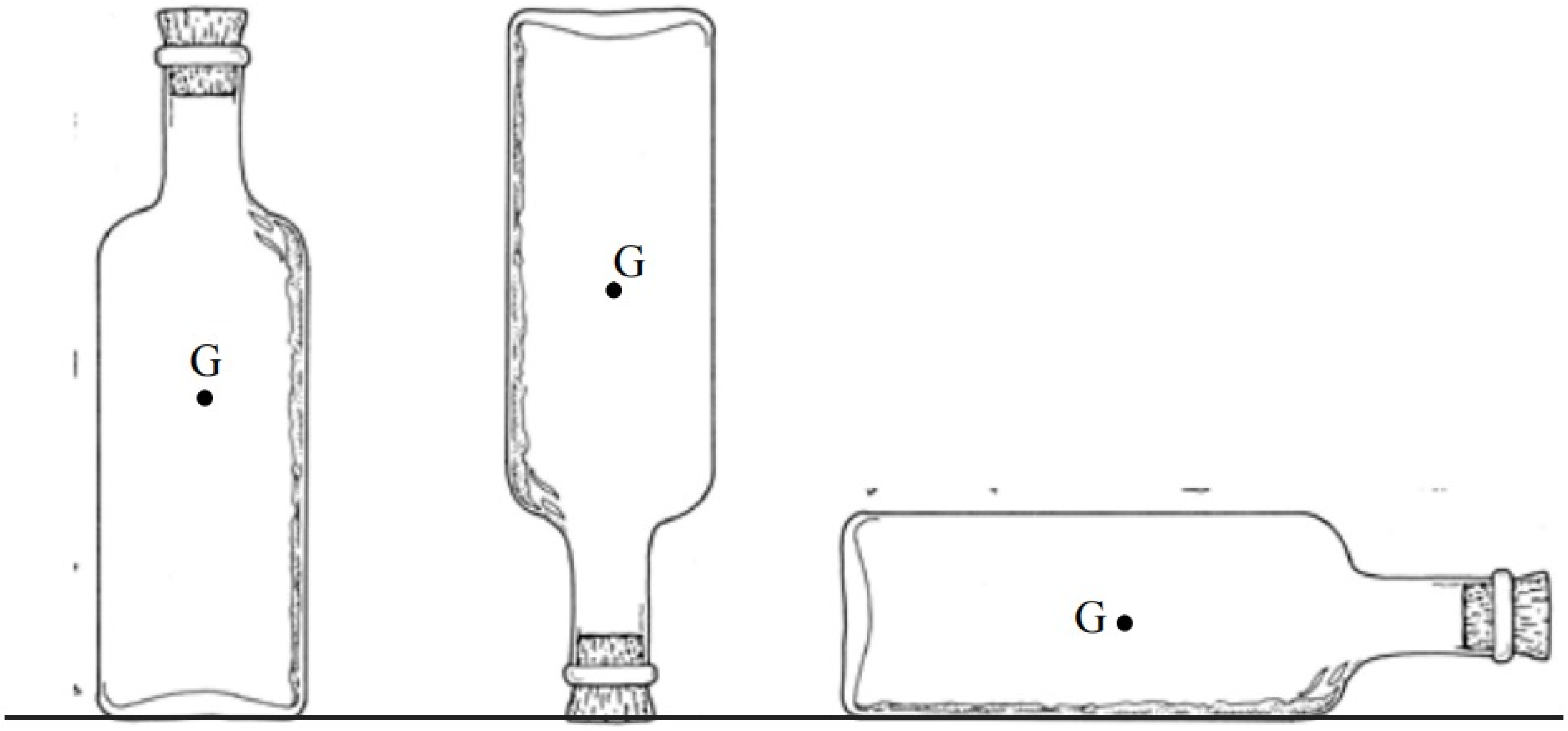

Figure 4.6

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Consider an empty bottle which is placed in three states as follows.

1. Stable equilibrium

   If a body which is in equilibrium, when displaced slightly from its equilibrium position and released, returns to its original equilibrium position, then the body is in stable equilibrium.

2. Unstable equilibrium

   If the body which slightly displaced as above and released does not return to its original equilibrium position, then the body is in unstable equilibrium.

3. Neutral equilibrium

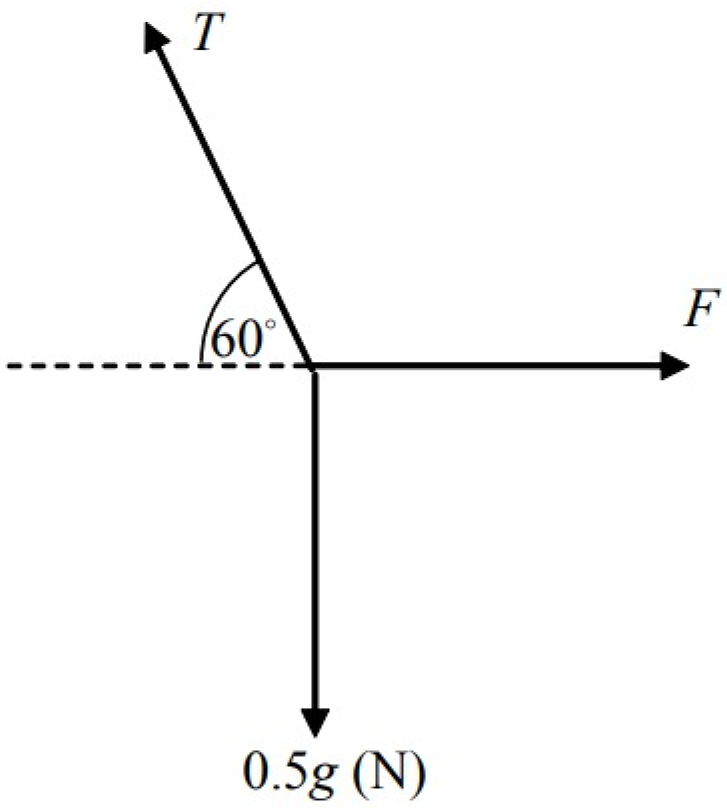
   If the body in equilibrium when displaced slightly as above and released remains to be in its new equilibrium position, then the body is in neutral equilibrium.

From the above three states of equilibrium it can be seen that the unstable equilibrium position is the most unsafe position.

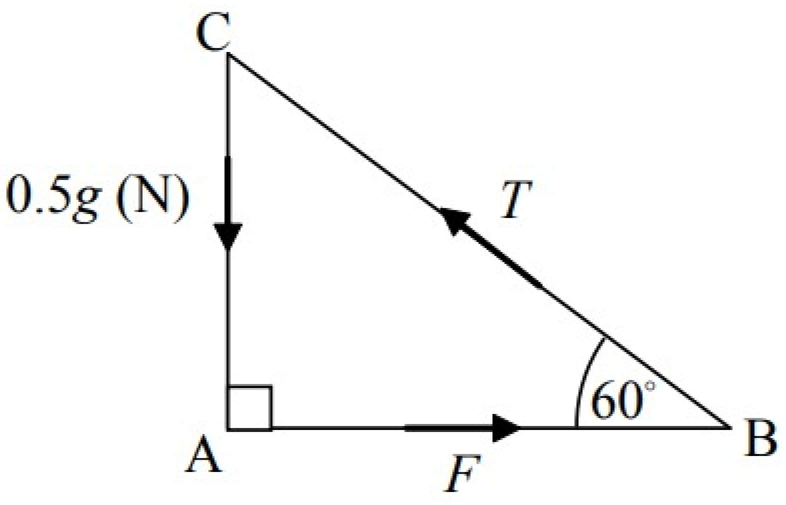
It can also be seen that lower the position of the centre of gravity of a body the more stable or safe the body is. However the main factor of stability of a body is the potential energy possessed by the body. Lesser the potential energy (mgh) of a body, more stable the body is and as the potential energy increases the body becomes more unstable.

Worked example

A body of mass 500 g is hung from a fixed point by means of an inextensible, light string and is then pulled aside by a horizontal force F until the string is inclined at 60° to the horizontal. Find the value of *F*.



Using the triangle of forces



$$\tan 60^{\circ} = \frac{AC}{AB}$$

$$\sqrt{3} = \frac{0.5\ g}{F}$$

$$F = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}\ \text{N}$$

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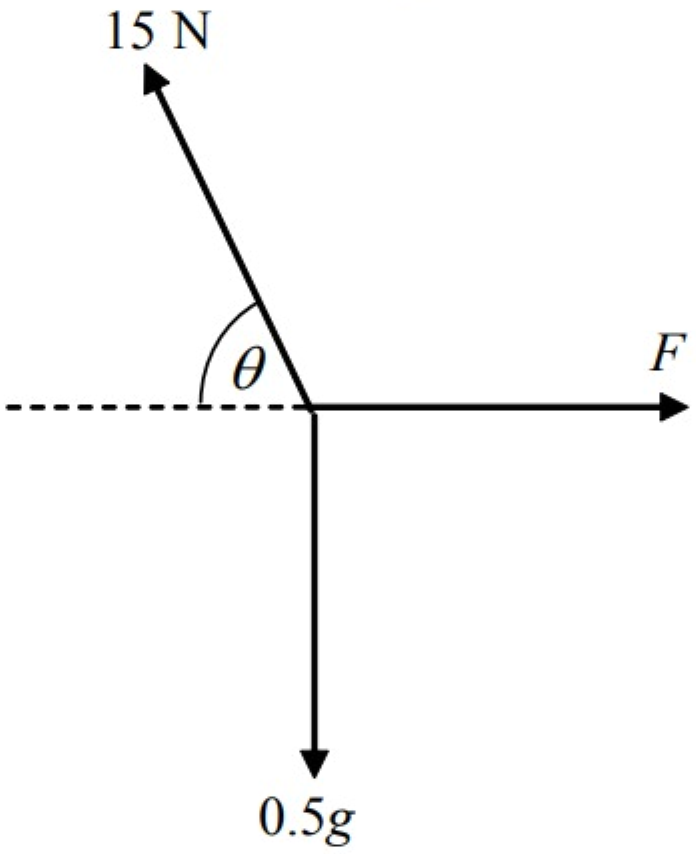
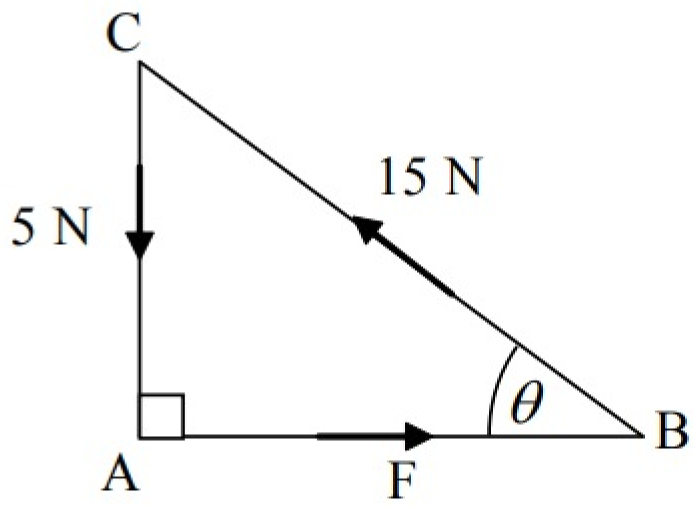
If the above string brakes under a tension of 15 N, find the maximum value of the force *F*.

As *F* increases, the angle of inclination of the string to the horizontal decreases and its tension increases until it reaches the braking value. Let θ be the inclination of the string at the braking tension,

From the triangles of forces ABC,





$$5^2+F^2=15^2$$

$$F^2=225-25=200$$

$$F=\sqrt{200}=10\sqrt{2}\ \text{N}$$

$$=14.14\ \text{N}$$

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Chapter 05

Work, energy and power

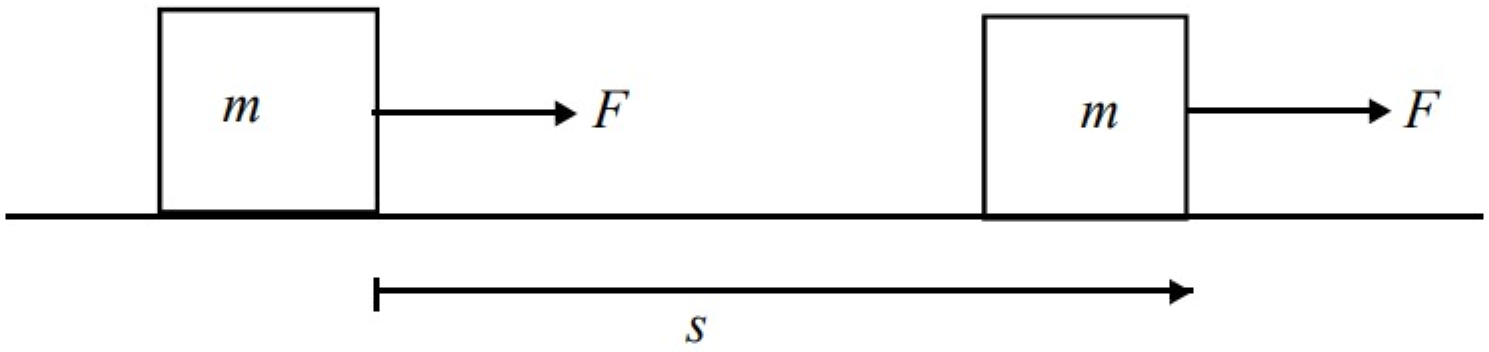

Figure 5.1

If a body gets displaced when an external unbalanced force acts on it, then that force is said to do work.

The amount of work done is given by the product of the displacement of the point of action of the force and the force acting in that direction.

$$\boxed{\text{Work done} = \text{Force x Displacement}}$$

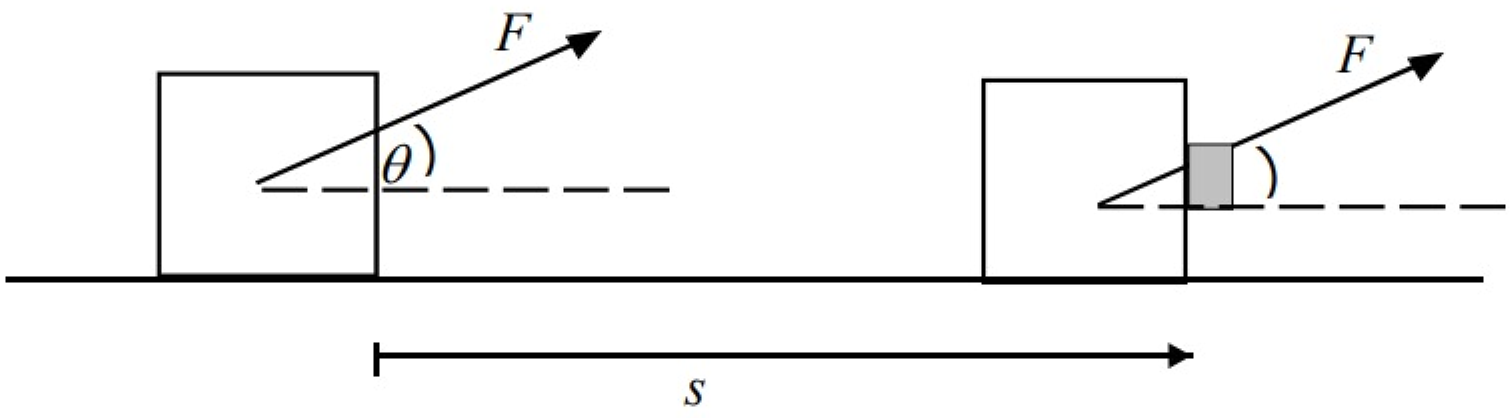

Figure 5.2

If the force is acting at an inclination to the displacement, then the work done is equal to the product of the component of the force in the direction of the displacement and the displacement.

$$\text{work done} = F \cos\theta \,.\, s$$
$$= F.s \cos\theta$$

Energy

Energy is introduced as the capability of doing work.

When a body does work, the body releases an amount of energy equal to it, while when work is done on the body an equal amount of energy is stored in it.

The unit of measurement of energy is the joule (J), the same as the unit of work.

Energy exists in various forms such as light, sound , heat, electricity, chemical, nuclear and mechanical energy, the latter being subdivided into potential energy and kinetic energy.

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Potential energy and kinetic energy

When a body on the ground is raised from the ground to a higher level, a certain amount of work is done on it and this work is stored in it as energy. This energy is referred to as potential energy.

Also work has to be done on a body at rest in order to make it move and this work too is stored in it as energy which is referred to as kinetic energy.

Kinetic energy

As mentioned above kinetic energy is the energy possessed by a moving body showing that a moving body has a capability of doing work unlike a body at rest.

If a body is in linear motion the kinetic energy possessed by it is called translational kinetic energy. The kinetic energy possessed by a rotating body is called rotational kinetic energy.

Translational kinetic energy

When a body moving with a velocity *v* is added upon by an opposing force *F,* let the body comes to rest after moving a distance *s*.

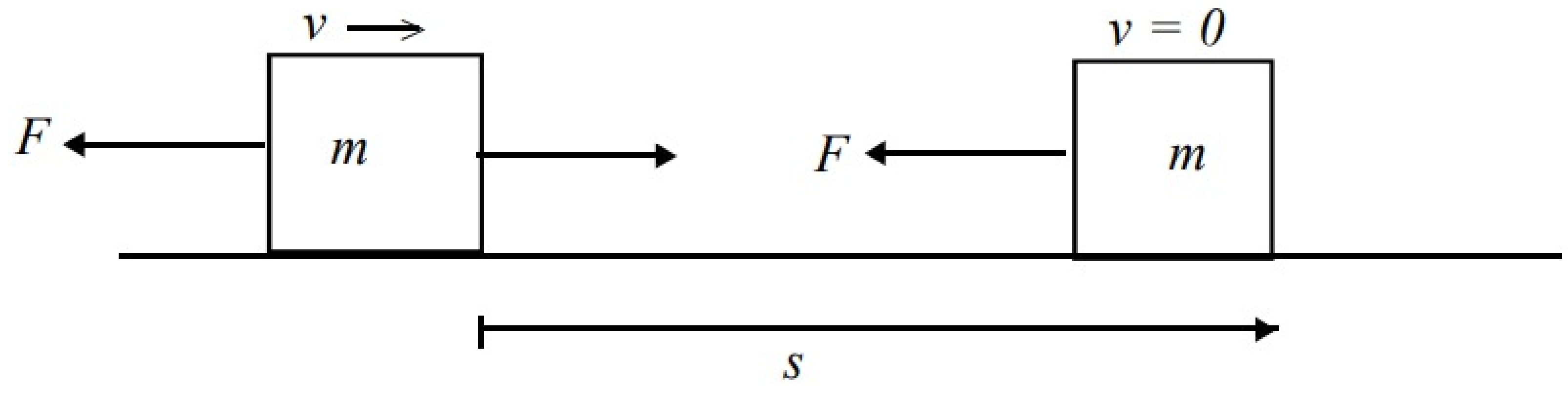

Figure 5.3

Applying $F = ma$ to the body

$$-F = ma$$

$$a = -\frac{F}{m}$$

Using $v^2 = u^2 + 2as$

$$0 = v^2 + 2\left(-\frac{F}{m}\right)s$$

$$0 = v^2 - 2\frac{F.s}{m}$$

$$2\frac{F.s}{m} = v^2$$

$$F.s = \frac{1}{2}mv^2$$

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Work done $W = F.s$

$$\therefore \quad W = \frac{1}{2}mv^2$$

$\therefore$ Translational kinetic energy of the body,

$$\boxed{W = \frac{1}{2}mv^2}$$

Potential energy

Potential energy has two forms namely gravitational potential energy and elastic potential energy.

Gravitational potential energy

The mechanical energy stored in a body due to its position in the gravitational field is known as gravitational potential energy.

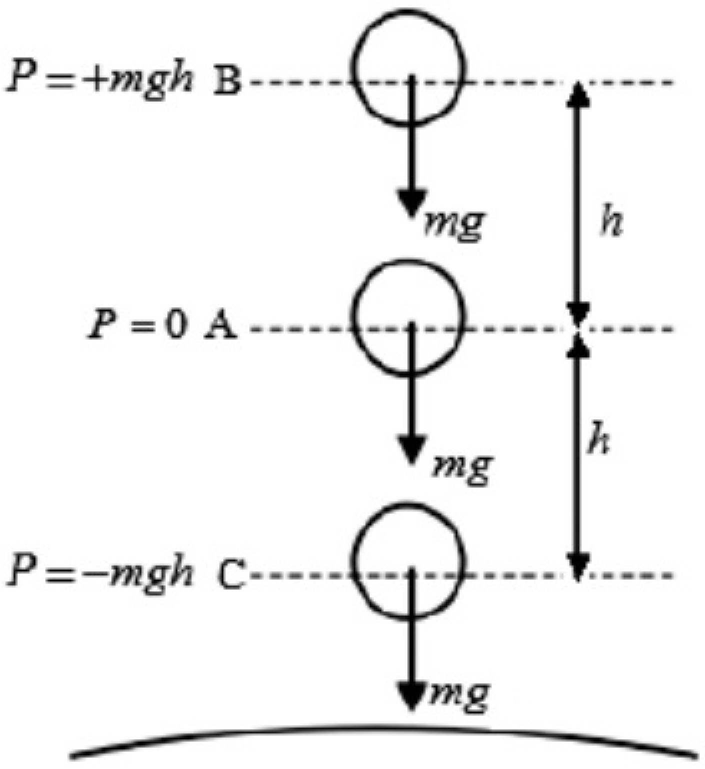

Figure 5.4

Suppose that a body of mass m is raised from position A to a position B at a height h. The force of gravitational attraction acting on the body is $F = mg$ vertically downwards. Work has to be done against this force and hence this work is

$W = F.s \Rightarrow W = mgh$

This work gets stored in the body as gravitational potential energy.

$$\boxed{\text{Gravitational potential energy} = mgh}$$

When the body is lowered from position A to a position C at a height h energy is released. Hence the gravitational potential energy at position C is less than that at position A.

Gravitational potential energy at C $= -mgh$

Accordingly if the gravitational potential energy at some level in a gravitational field is considered zero, the gravitational potential energy at a height h above that level is $+mgh$.

While that at a height h below that level is $-mgh$.

Elastic potential energy

A stretched spring (or a rubber band) or a compressed spring has an ability to do work. This energy possessed by a spring is called elastic potential energy.

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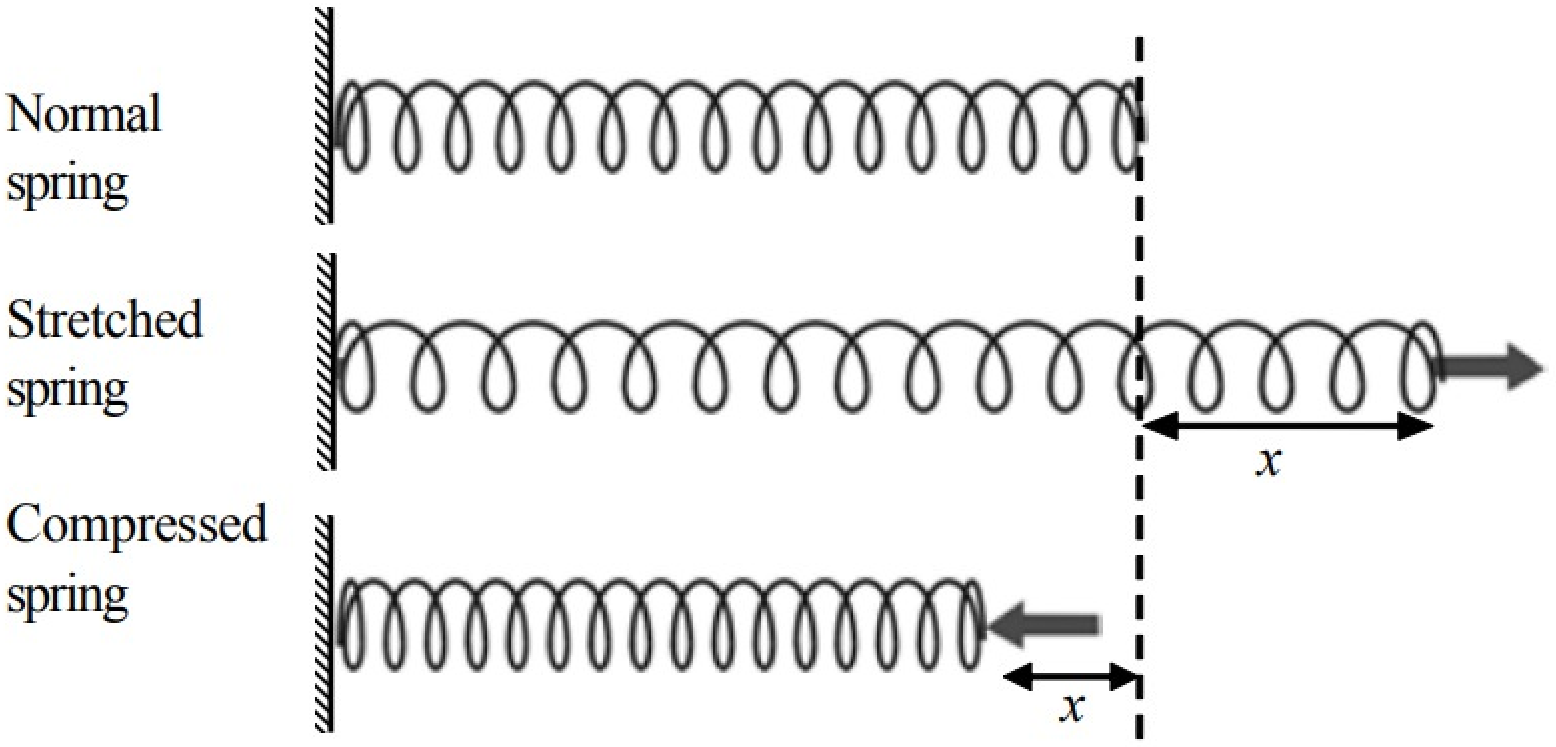

Figure 5.5

If *x* is the extension in a spring acted upon by a force *F*, then this extension is directly proportional to the magnitude of the force.

$$F \propto x$$

$$\boxed{F = kx}$$

The constant of proportionality *k* is known as the spring constant. The energy stored in such a spring is given by

$$W = \frac{1}{2}kx^2$$

$\therefore$ Elastic potential energy $W = \frac{1}{2}kx^2$

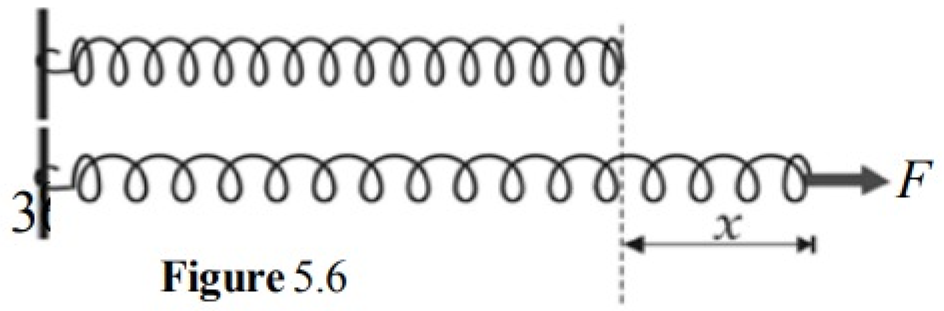

Figure 5.6

When the force increases, the extension increases when the spring is extended by an extension *x*, the average elastic force acting on the spring,

$$F' = \left(\frac{0+F}{2}\right)$$

$\therefore$ The work done by the average elastic force,

$$W = F's = \left(\frac{0+F}{2}\right)x = \frac{F}{2}x \quad \Rightarrow \quad F = kx \text{ , } W = \frac{1}{2}kx.x$$

$\therefore$ Elastic potential energy $\boxed{W = \frac{1}{2}kx^2}$

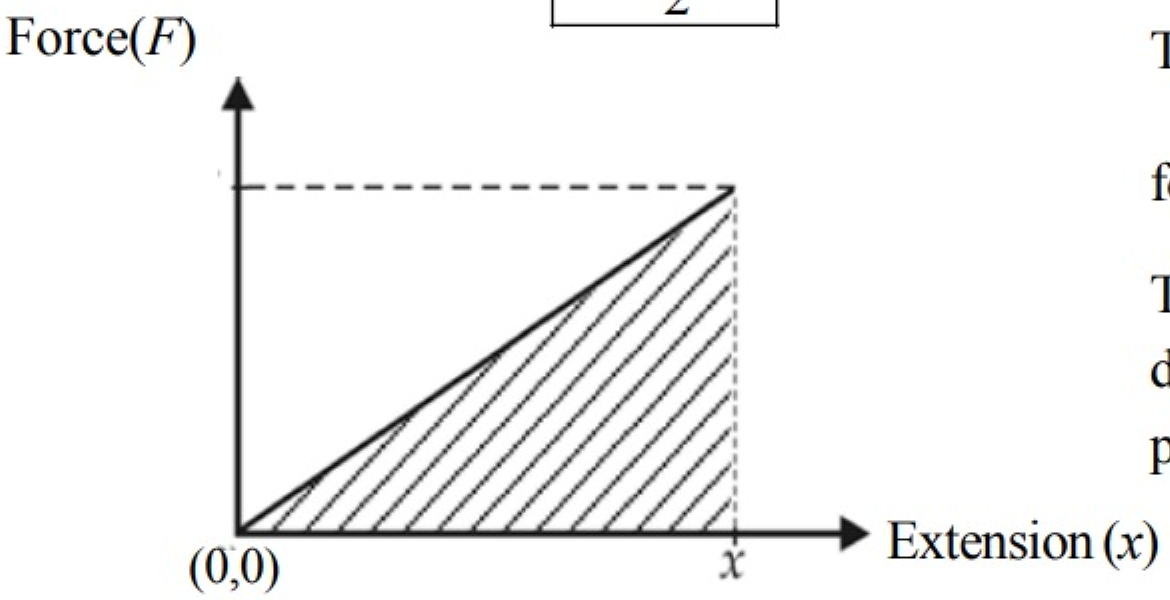

Figure 5.7

The area under the graph; the graph of force (*F*) aganist extension (*x*), is $\frac{1}{2}Fx$.

Therefore this area is equal to the work done by the force and the eleastric potential energy stored in the spring.

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Power

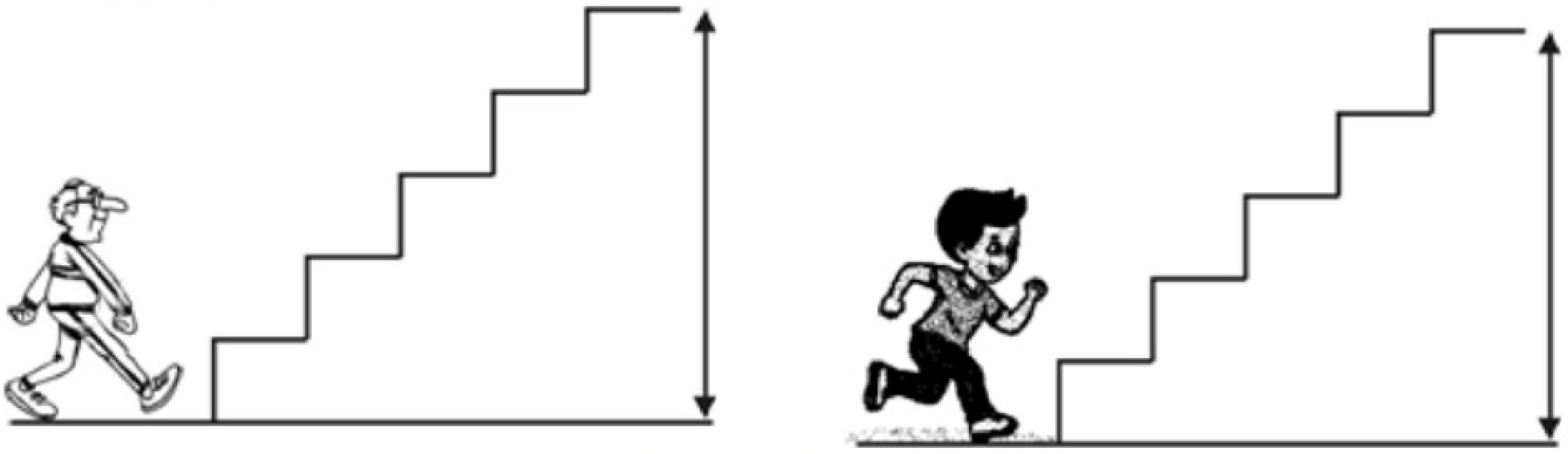

Figure 5.8

A young boy has the ability to climb a set of steps at a much shorter time than an old person. This indicates that the young boy is more powerful than the old person.

The rate at which a body is doing work or the rate at which it releases energy is called power

$$power = \frac{work}{time}$$

$$\boxed{P = \frac{W}{t}}$$

The unit of measurement of power is 'watt' (W)

$$1 \text{ W}=1 \text{ J s}^{-1}$$

Relation between power and velocity

$$power = \frac{work}{time}$$

$$P = \frac{F.s}{t}$$

$$P = F.\frac{s}{t}$$

$$\boxed{P = Fv}$$

$$power = force \times velocity$$

Efficiency

The energy possessed by an equipment cannot be totally converted into useful energy as a part of it gets converted into other forms(eg. heat). Thus a portion of the energy gets wasted.

$$\text{Efficiency}=\frac{\text{useful power output}}{\text{power input}}\times 100\%$$

Principle of conservation of energy

The total energy of a closed system remains constant.

However, within a system one form of energy can get converted into another form of energy.

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Eg.: In a vehicle the energy of the fuel (petrol) gets converted into heat, sound and mechanical energy.

The potential energy possessed by the water at the top of a hydro-power house loses this energy when falling to the bottom and gets converted into kinetic energy on the turbines. This energy is converted into electrical energy by the turbines.

Principle of conservation of mechanical energy

If in a certain process the conservation of energy takes place only between kinetic energy and potential energy, then in that process the sum of the kinetic energy and the potential energy remains a constant. This is the principle of conservation of mechanical energy.

$$\text{kinetic energy} + \text{potential energy} = \text{constant}$$

$$\frac{1}{2}mv^2 \quad + \quad mgh \quad = \quad k$$

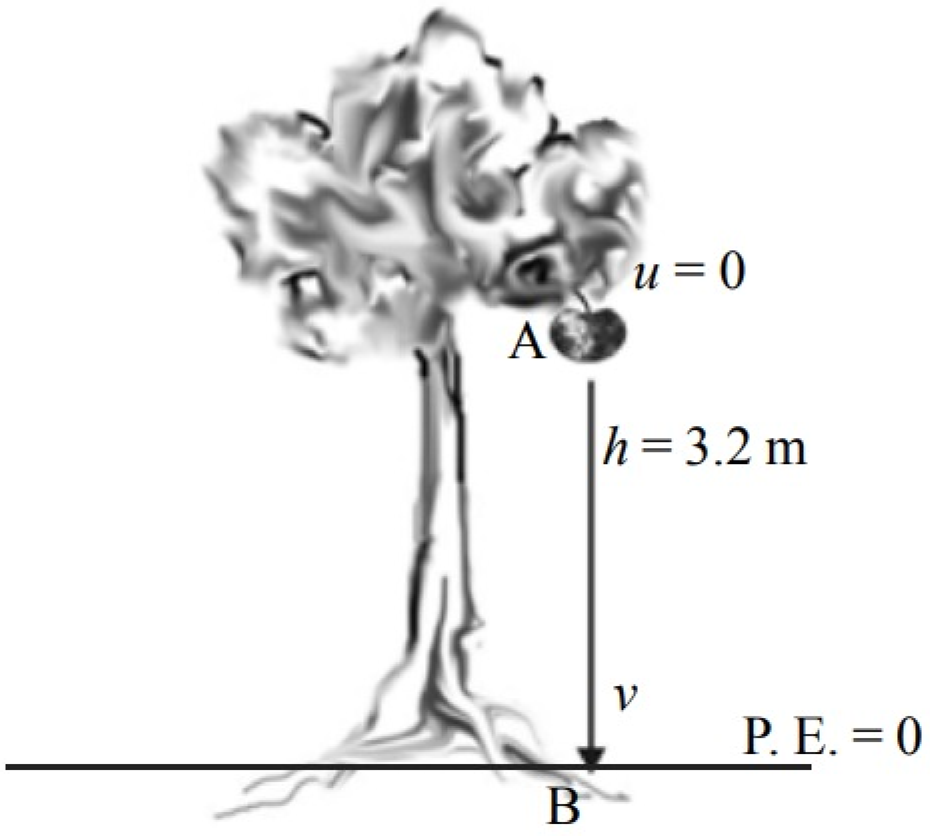

Figure 5.9

In some cases, potential energy may be in the forms of gravitational and elastic potential energy. Kinetic energy may also be in the forms of translational and rotational kinetic energy.

Eg: A mango hanging from a tree at a height of 3.2 m from the ground, drops from rest and falls on to the ground. Find the velocity of the mango when it strikes the ground (disregard the air resistance).

Applying the principle of conservation of energy,

$$\underset{\text{at position A}}{\text{kinetic energy} + \text{potential energy}} = \underset{\text{at position B}}{\text{kinetic energy} + \text{potential energy}}$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$mgh = \frac{1}{2}mv^2$$

$$10 \times 3.2 = \frac{1}{2}v^2$$

$$10 \times 6.4 = v^2$$

$$v = 8 \text{ m s}^{-1}$$

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Chapter 06

Rotational motion and circular motion

Most of the motions in the universe can be explained by straight line and rolational motion. Straight line motion is determined by the straight line distance changes and the rotational motion is determined by the angular changes.

Angular Displacement (θ)

The angular displacement is the rotation of a point or a straight line about a particular axis in a particular sense.

It is a scalar quantity.

If a particle at P, at a distance r from the origin 'O' rotates through an angle θ and if s is the are length,

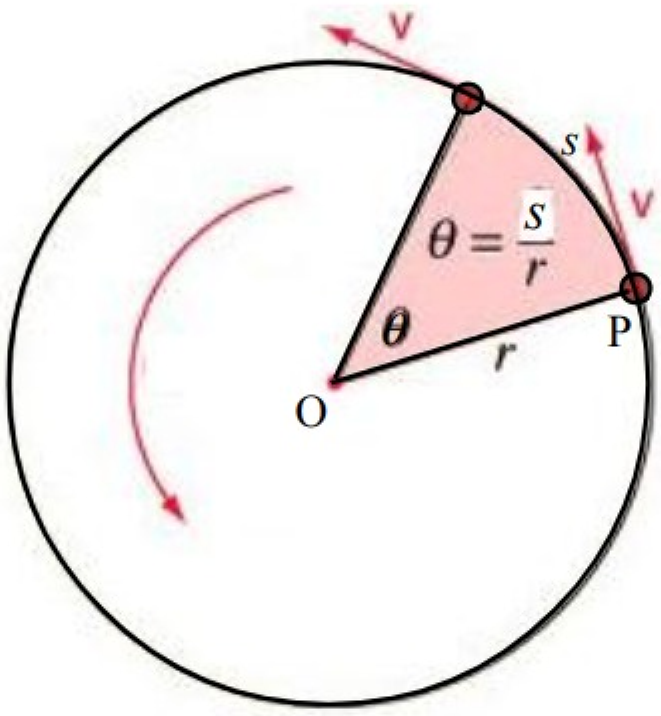

Figure 6.1

Then $$\theta = \frac{s}{r}$$

$$\boxed{s = r\theta}$$

Unit of θ is radian which is denoted by the symbol 'rad'.

Radian

1 rad is equivalent to the angle subtended at the centre of a circle by an arc equal in length to the radius.

Since the circumference of a circle of radius r is $2\pi r$, the angle subtended at the centre of the circle by the circumference is $\frac{1}{r} \times 2\pi r = 2\pi$

Thus 360° is equal to 2π rad,

Angular velocity (ω)

The angular velocity is the rate of change of angular displacement. It is a vector quantity. Direction is given by righthand cork screw rule.

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

Unit of ω is rad s^{-1}

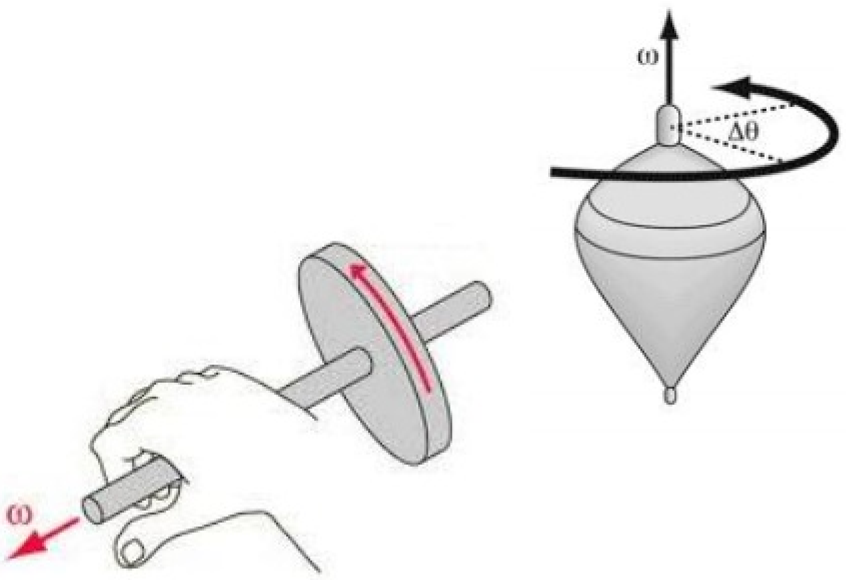

Figure 6.2

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Angular accleration (α)

The rate of change of angular velocity of a moving body at a moment is the angular accederation at that moment.

$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$$

Unit of α is rad s^{-2}

If angular velocity varies from ω_1 to ω_2 during a time period of *t*,

then, angular acceleration $\alpha = \dfrac{\omega_2 - \omega_1}{t}$

Worked example

A fan was rotating at a speed of 10 rounds per second. Due to a power failure fan became still with a constant angular deceleration after 10 seconds. Find the angular deceleration of the fan.

Initial angular velocity $\omega_1 = 2\pi \times 10$

Final angular velocity $\omega_0 = 0$

Time taken for the deceleration $t = 10$

Angular acceleration $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{0 - 2\pi \times 10}{10} = -2\pi \text{ rad s}^{-2}$

$\therefore$ Angular deceleration $\alpha = 2\pi \text{ rad s}^{-2}$

Note :- The angular acceleration can occur in three ways.

1. Change of magnitude without changing the direction of the angular velocity.
2. Change of direction of angular velocity without changing the magnitude.
3. Both the magnitude and direction changes of angular velocity.

If only the magnitude of the angular velocity changes without changing the rotating plane, then the direction of the angular acceleration is along the direction of the angular velocity.

If the rotating plane changes, angular acceleration will be along the direction of the change of angular velocity.

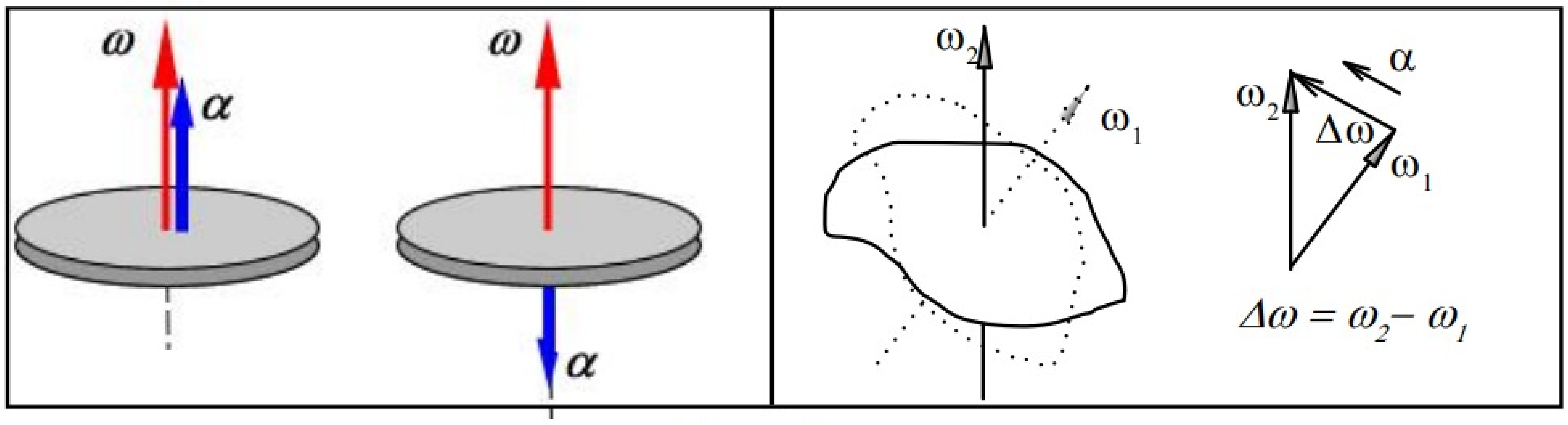

Figure 6.3

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Period of rotation (*T*)

Period of rotation is the time taken for one complete rotation.

$$\boxed{T = \frac{2\pi}{\omega}}$$

Frequency of rotation (*f*)

The number of rotations executed in one second is the frequency of rotation

$$f = \frac{\omega}{2\pi} \qquad \text{or} \qquad \omega = 2\pi f$$

Relation between rectlinear motion and angular motion

(1) From $s = r\theta$

$$\Delta s = r.\Delta\theta$$

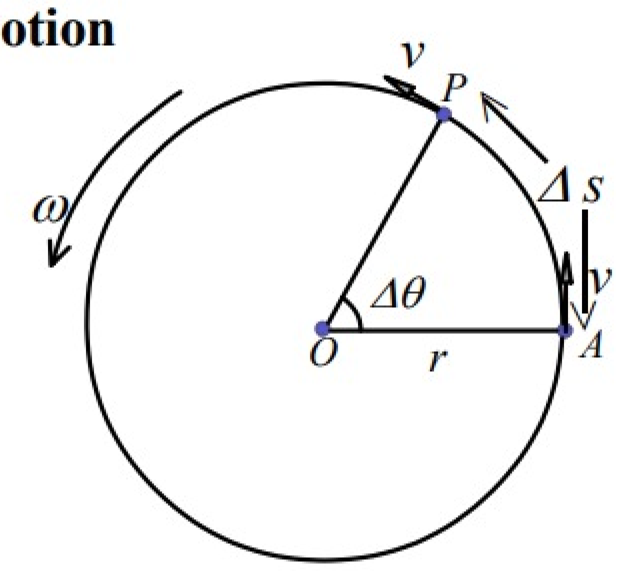

Figure 6.4

If Δt is the time taken from A to P,

$$\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

$$\boxed{\therefore v = r\omega}$$

v is perpendicular to the radius OA. Keeping ω, a constant, if $\Delta\theta$ varies the variation of v is given as,

$$v_A > v_B > v_C$$

$$v = r\omega$$

$$\Delta v = r\Delta\omega$$

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

$$\therefore \boxed{a_t = r\alpha}$$

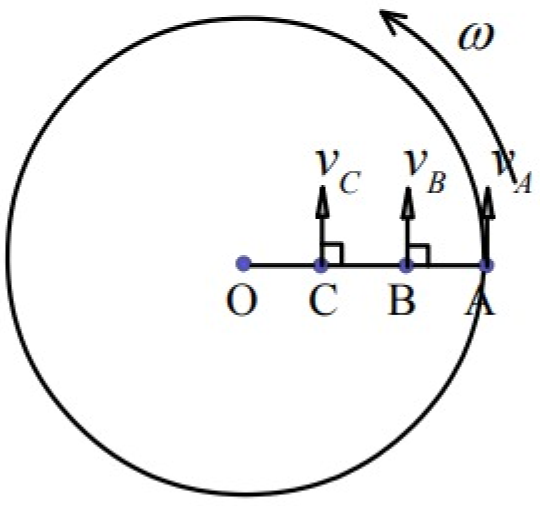

Figure 6.5

where a_t the acceleration along the tangent.

If particle P moves with a uniform angular velocity

Then $\Delta\omega = 0$

$$\alpha = 0$$

$$a_t = 0$$

Therefore, for a particle moving along a circular path with uniform angular vdocity, the acceleration along the tongent is zero. But there is an accaleration towards the centre, normal to the tangent. If a_c is the acceleration towards the centre of the circular path,

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$$a_c = \frac{v^2}{r} = r\omega^2 = v.\omega$$

This acceleration is the centripetal acceleration. Since the direction of motion of the particle changes with time, the particle has this acceleration.

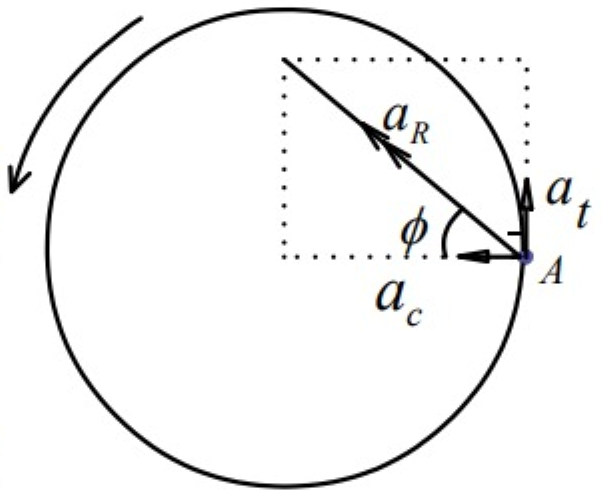


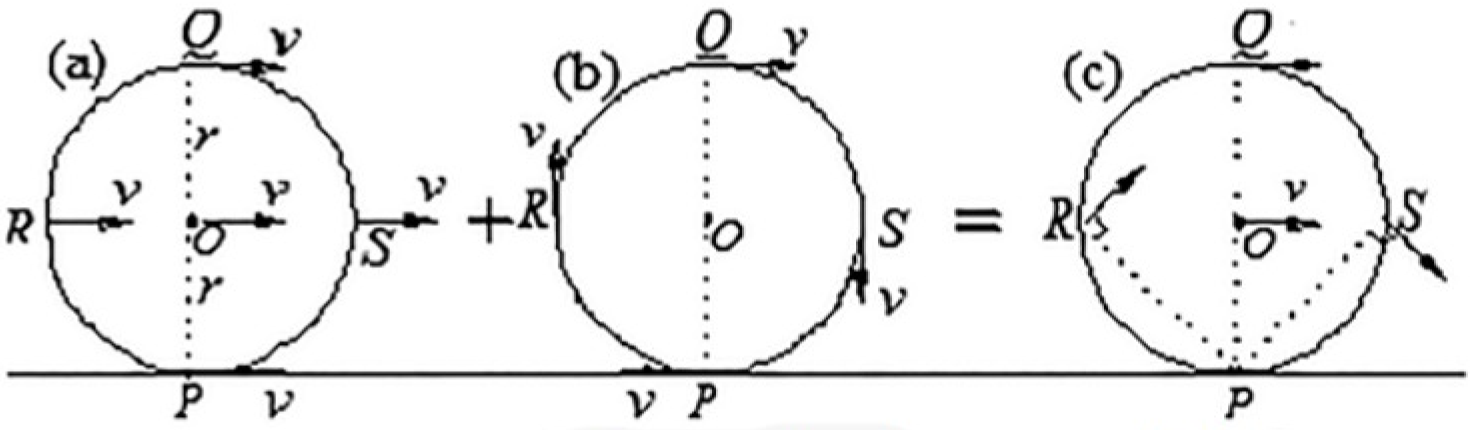
Figure 6.6

If the particle rotates with nonuniform angular velocity, it will have an acceleration along the tangent also.
Then the resultant acceleration will not be towards the centre.The resultant accaeleration a_R is given by

$$a_R = \sqrt{a_t^2 + a_c^{\ 2}}$$

$$\tan\phi = \frac{a_t}{a_c}$$

Note



Pure translational motion

Pure rotational motion

Rolling

(a)

(b)

(c)

Note: P will always be momentarily at rest.

In the pure translatory motion [(Figure 6.7 (a)] of every point of the wheel and the centre as well moves to the righthand side with the same linear speed.

In the pure rotational moting [Figure 6.7 (b)] every point of the wheel rotates with the same angular velocity ω about the centre. The points in the outer edge of the wheel move with the same linear speed *v.*

The rolling motion Figure 6.7 (c)] is a combination of the motions given by (a) and (b) above.

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Motion with Uniform angular acceleration

Consider a body rotating about an axis.

Let initial angular velocity ω_0

Angular velocity after time *t* is ω

Angular displacement θ

If α is the angular acceleration

1. $\omega = \omega_0 + \alpha t$

2. $\theta = \left(\frac{\omega + \omega_o}{2}\right) t$

3. $\theta = \omega_o t + \frac{1}{2} \alpha t^2$

4. $\omega^2 = \omega_o^2 + 2\alpha\theta$

Moment of Inertia (*I*)

The mass of a body is a measure of its reluctance to any change of linear motion. The corresponding property for rotational motion is called moment of inertia.

If a particle *P* of mass *m* is revolving (rotating) in a circular path of radius *r*, then the moment of Inertia of P about the centre O is given by

$$\boxed{I = mr^2}$$

Unit of *I* is kg m^2

Figure 6.8

Moment of Inertia of a system of particles

If a system of particles of masses $m_1, m_2, m_3, \ldots, m_n$ are situated at perpendicular distances $r_1, r_2, r_3, \ldots, r_n$ from the axis l, Then the moment of inertia of the system of particles I_l is given by

$$I_l = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$$

$$I_l = \sum_{i=1}^{n} m_i r_i^2$$

Note: Moment of inertia is a scalar quantity

Examples

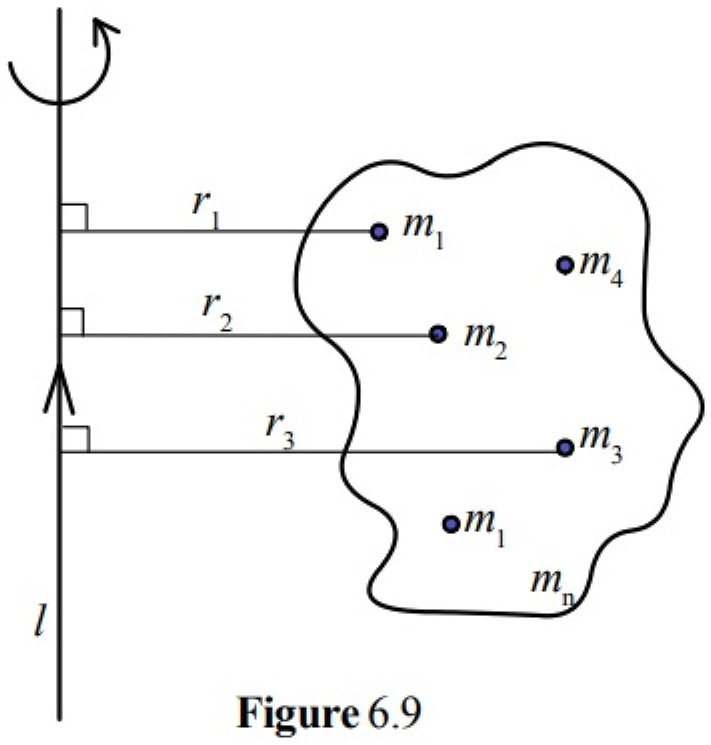

Figure 6.9

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1) Moment of inertia of a uniform rod of length *L*, mass *M* about an axis passing normally through the end of the rod is given by

$$I = \frac{1}{3} ML^2$$

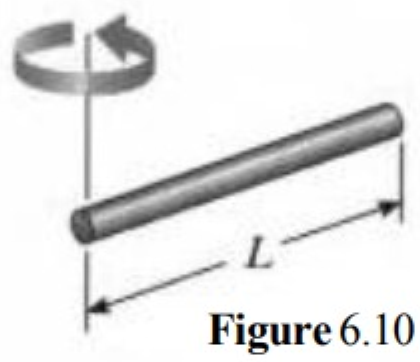

Figure 6.10

2) Moment of inertia of a uniform rod of length *L*, mass *M* about an axis passing normally through the midpoint of the rod is given by

$$I = \frac{1}{12} ML^2$$

Figure 6.11

(3) Moment of inertia of a uniform circular disc or solid cylinder of radius *R*, mass *M* about an axis normal to the plane passing through the centre is given by

$$I = \frac{1}{2} MR^2$$

Figure 6.12

(4) Moment of Inertia of a uniform solid sphere about an axis through the centre is given by

$$I = \frac{2}{5} MR^2$$

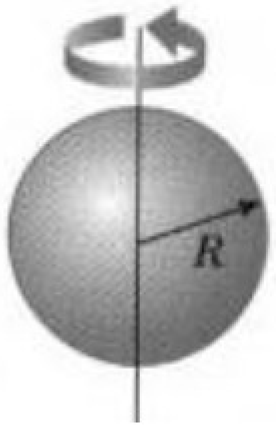

Figure 6.13

Rotational kinetic energy

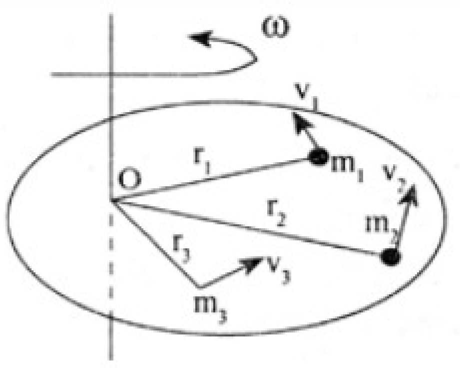

Figure 6.14

Consider an object rotating about an axis through a fixed point O on the object, with a uniform angular velocity ω. Let particles of masses $m_1, m_2, m_3, \ldots, m_n$ situated at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from point O be moving with speeds $v_1, v_2, v_3, \ldots, v_n$ respectively.

Then the rotational kinetic energy of the body,

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$$E = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 + \frac{1}{2}m_3{v_3}^2 + \ldots + \frac{1}{2}m_n{v_n}^2$$

$$= \frac{1}{2}\left[m_1(r_1\omega)^2 + m_2(r_2\omega)^2 + m_3(r_3\omega)^2 + \ldots + m_n(r_n\omega)^2\right]$$

$$= \frac{1}{2}\left[m_1{r_1}^2 + m_2{r_2}^2 + m_3{r_3}^2 + \ldots + m_n{r_n}^2\right]\omega^2$$

$$= \frac{1}{2}\left[\sum m_i{r_i}^2\right]\omega^2$$

$$= \frac{1}{2}I\omega^2 \qquad \text{where } I = \sum m_i r_i^2$$

The quantity I is called the "moment of inertia" of the object about the axis of rotation. "I" depends on the position of the axis of rotation and how the mass of the body is distributed about it.

Angular momentum (L)

Angular momentum of a rigid body rotating about an axis

The angular momentum (L) of a lamina moving with an angular velocity ω about an axis passing through O is given by

$$L = p.r$$

$$= \Sigma(m_i v_i) r_i$$

$$= \Sigma(m_i v_i \omega) r_i$$

$$= \Sigma(m_i\, r_i^2)\omega$$

$$\boxed{L = I\omega}$$

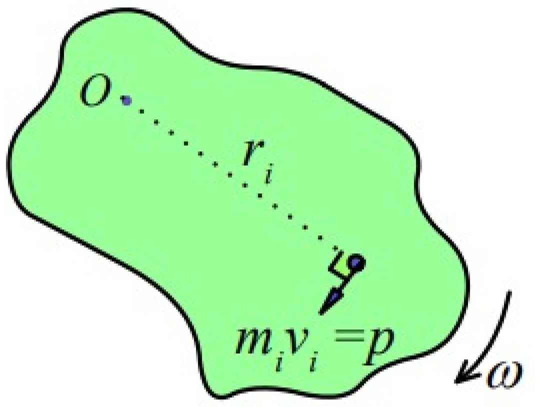

Figure 6.15

Unit of L is kg m^2 s^{-1}

$$\boxed{\begin{array}{ccccc}\text{(Angular momentum)} & = & \text{(Moment of inertia)} & \times & \text{(Angular velocity)} \\ L & = & I & \times & \omega\end{array}}$$

Note : Angular momentum is a vector quantity.

Its direction is given by the right hand cork screw rule.

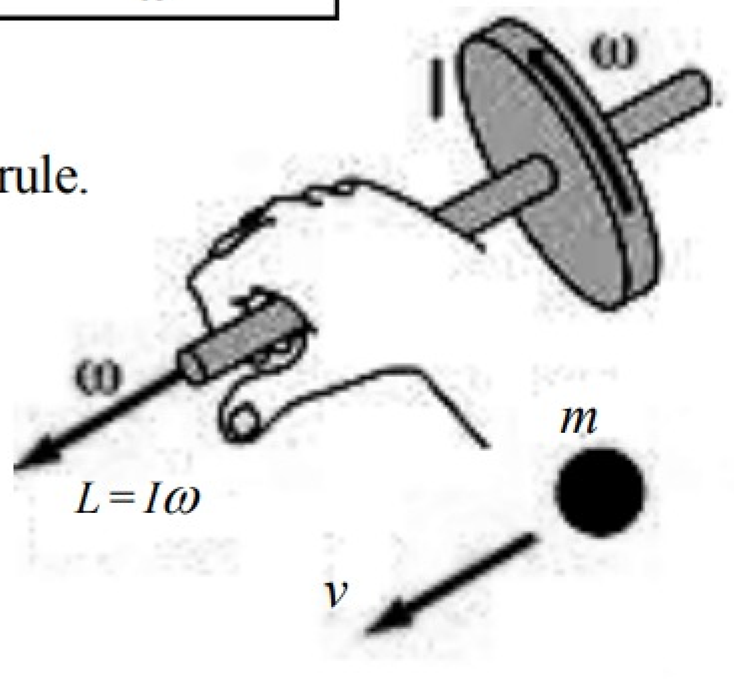

Figure 6.16

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Torque (τ)

In translatory motion, force is defined as the rate of change of linear momentum. Likewise in rotational motion, torque or moment of the force is defined as the rate of change of angular momentum.

$$L = p.r$$

$$\frac{\Delta L}{\Delta t} = \frac{\Delta p}{\Delta t}.r$$

$$\boxed{\tau = F \times r} \text{ ....(1)}$$

From the relation $L = I\omega$

$$\frac{\Delta L}{\Delta t} = \frac{\Delta I\omega}{\Delta t}, I \text{ does not change with time}$$

$$\therefore \frac{\Delta L}{\Delta t} = I\frac{\Delta\omega}{\Delta t}$$

$$\boxed{\tau = I\alpha} \text{ ....(2)}$$

Equation (2) is similar to Newton's 2nd law, $F = m\,a$

Direction of torque can be obtained by the same way as the direction of angular momentum is obtained

Principle of conservation of angular momentum

The total angular momentum of a system remains constant provided no external torque acts on the system.

Initial angular momentum = Final angular momentum

$$I\omega = I'\omega'$$

Comparison of translatory motion and rotational motion

Translatory motion	Rotational motion
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = \left(\frac{u+v}{2}\right)t$	$\theta = \left(\frac{\omega_0 + \omega}{2}\right)t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0{}^2 + 2\alpha\theta$

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Applications of the principle of angular momentum

(1) An ice skator is playing on ice revolving slowly with her hands extended.

After a small time if she bend her hands with the body she will revolve faster. According to the law of conservation of angular momentum.

$$I\omega = I'\omega'$$

But $I' < I \Rightarrow \omega' > \omega$

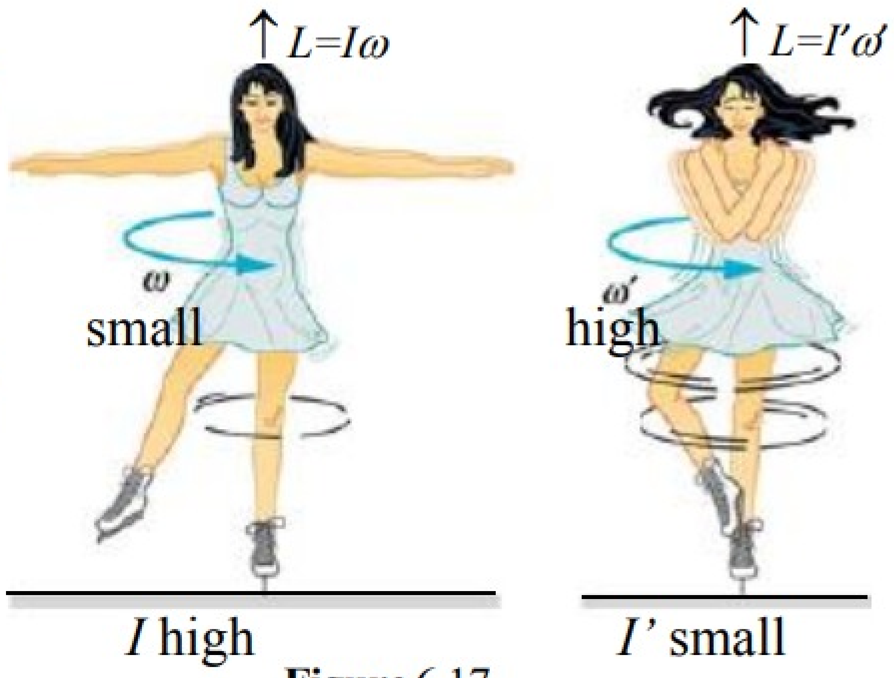

Figure 6.17

(2) A student sitting on a freely rotating stool holds two heavy weights in his extended arms. When he bring the weights in, the speed of rotation increases. When he extends his arms, the speed of rotation decreases. According to the Law of conservation of angular momentum, with the changes of moment of inertia, angular velocity of the rotating system changes.

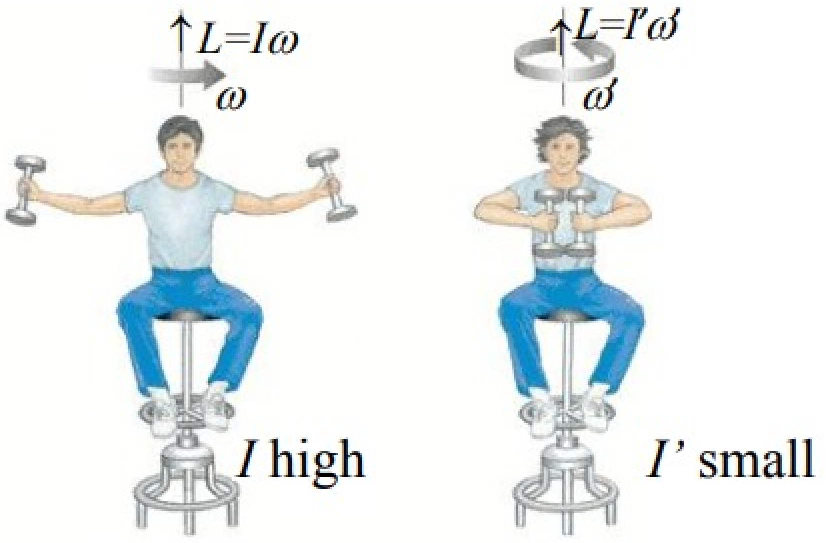

Figure 6.18

(3) The diver leaves the high diving board above the swimming pool, gets the vertical velocity by pulling the board. He then bends and pulls his arms and legs (Figure 6.19) so that the moment of Inertia is to decrease. Hence the angular velocity will increase. So he will revolve very fast at height. When he gets closer the water level he extends his legs and arms as shown and gets his angular velocity reduced.

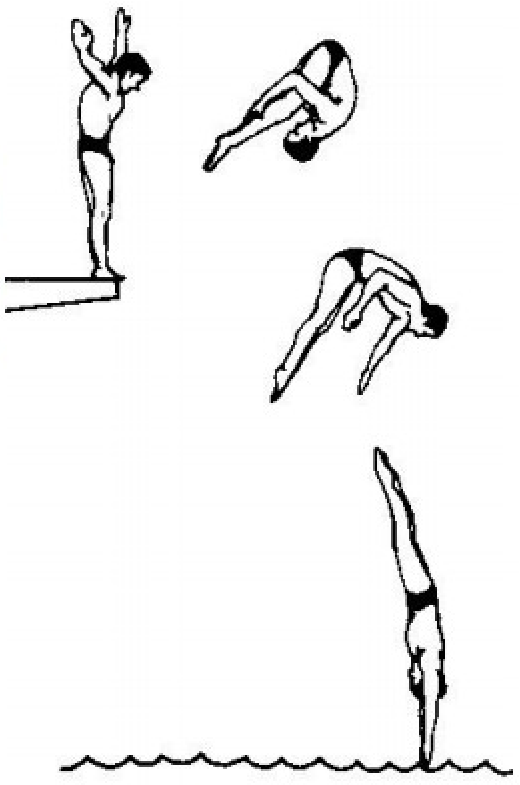

Figure 6.19

(4) When the string inside the tube is pulled downwards. The angular velocity of the particle revolving on the table will increase.

As r decreases I will decrease. To keep $I\omega$ constant, ω will increase.

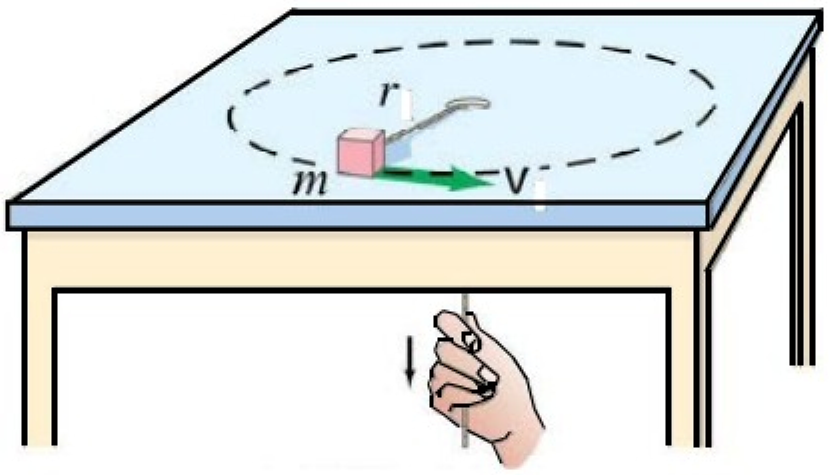

Figure 6.20

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(5) Since angular momentum is a vector quantity, keeping the magnitude constant, if only the direction is changed that is enough to affect the system. This can be shown using a cycle wheel. A person sitting on a rotating chair is keeping the rotating cycle wheel as shown in Figure 6.21 (i). The direction of the angular moment is shown in the figure. If he turns the wheel upside down as shown in Figure 6.21 (ii). Then the chair will start to rotate in the opposite direction to which the wheel is rotating.

According to the law of conservation of angular momentum,

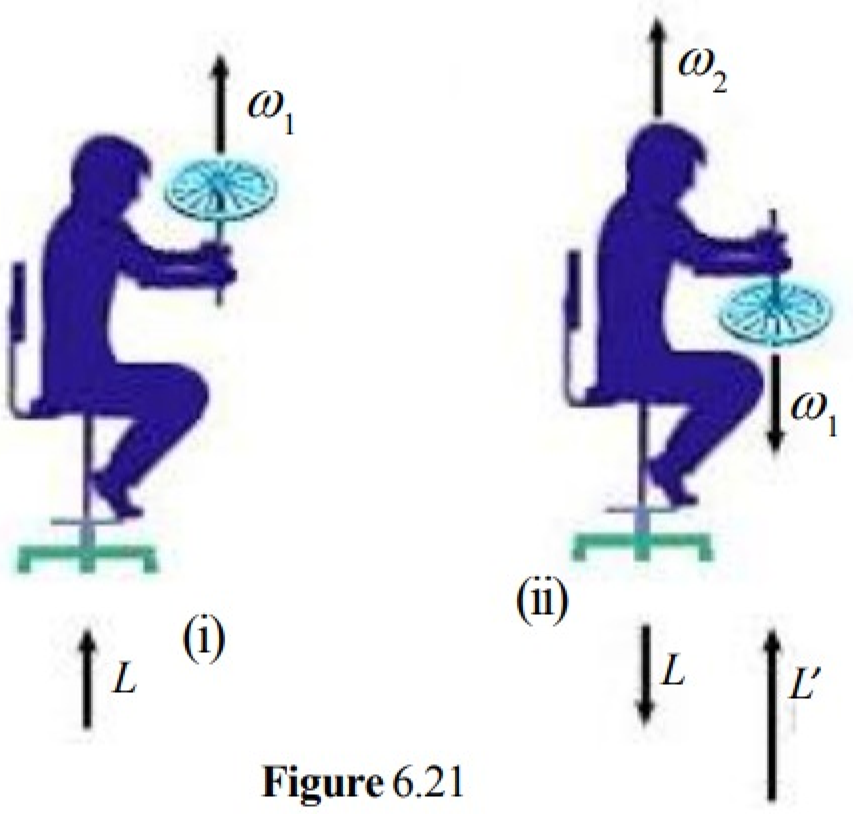

Figure 6.21

Initial angular momentum of the wheel	+	Initial angular momentum of the man and chair	=	Angular momentum of the wheel after it has turned down	+	Angular momentum of man and chair after the wheel turned down
L	+	0	=	$-L$	+	L'

$$L' = 2L$$

Here even though the magnitude of the angular momentum of the wheel is not changing the direction has changed.

Hence the chair will rotate with double the angular momentum of the initial value.

Applications related to reluctance to change of angular momentum

$$\tau \times \Delta t = \Delta L$$

Torque × Time = Change of angular momentum

The above equation shows that a torque should be applied to change the angular momentum.

(1) When a bycycle is driven it will be in equilibrium by itself withou cycle is not moving, it is difficult to keep in equilibrium.

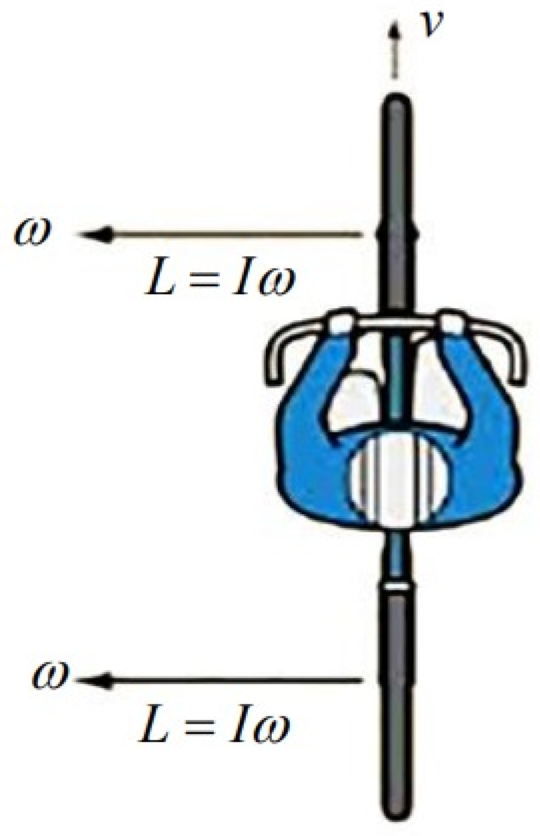

Figure 6.22

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(2) Spinning top will be in equilibrium when it is rotating. It will fall when the spin stops.

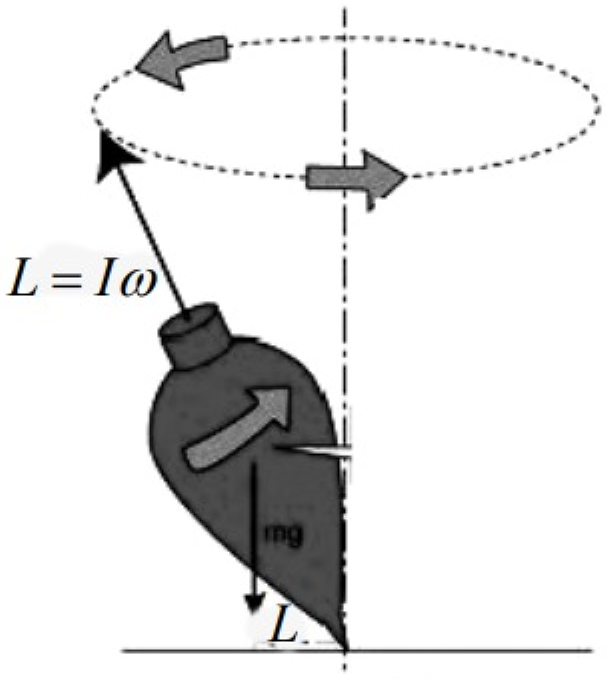

Figure 6.23

(3) The bullet from the gun will attack the target correctly only if the bullet is moving with rotation. If there is no rotation it will follow a path of a projectile.

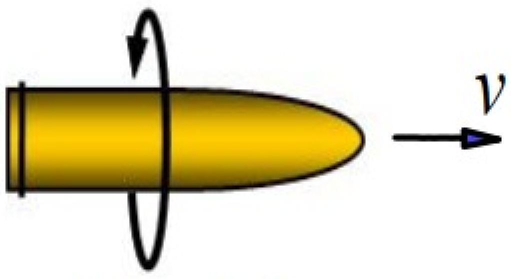

Figure 6.24

(4) The young coconut will not burst when it falls while rotating. Its angular momentum is in the vertical downward direction. Therefore, it hits the ground vertically and minimizes the possibility of bursting.

Figure 6.25

(5) When rabaana is rotated by a man, it will be in equilibrium. But it cannot be kept in equilibrium if it is not rotating.

Figure 6.26

Circular motion

When a body moves along a circular path around a fixed point or an axis outside it as the centre, the body is said to perform circular motion.

Eg. :- 1. A stone attached to the end of a string being whirled around a circular path.
2. Earth orbiting around the Sun along an approximately circular path.
3. A motorcycle or a bicycle taking a circular bend.

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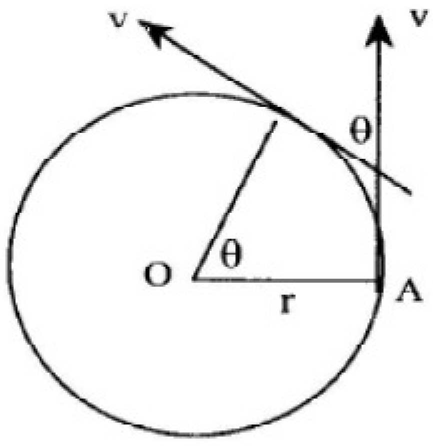

Figure 6.27

To study circular motion consider a body attached to a string being whirled around a circular path with a constant speed.
When the velocity of the body is considered its magnitude is constant since the speed is constant. However along a circular path the direction of the velocity is continuously changing.

When any one of magnitude or direction is changing it means that the velocity is changing. Changing velocity means acceleration.
Hence it can be concluded that any body performing circular motion is doing so with an acceleration, even if the speed is constant.

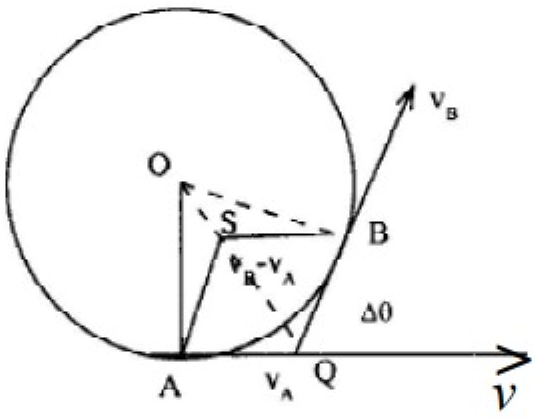

Figure 6.28

Let v_A and v_B be the velocities of the body at two points A and B seperated by the time interval $\Delta\, t$

It can be seen that the vector

$v_B - v_A = v_B + (-v_A)$ passes through the centre O

of the circle. Hence the acceleration $a = \dfrac{v_B - v_A}{\Delta t}$

is directed towards the centre of the circular path.

Thus the acceleration of a body performing circular motion is directed towards the centre of the circle and is hence known as "centripetal acceleration".
It can be proved that if v is the speed and r is the radius of the circle.

Centripetal acceleration 'a' $= \dfrac{v^2}{r}$

Also if ω is the angular velocity

$$v = r\omega$$

$$\therefore a = \frac{(r\omega)^2}{r} = r\omega^2$$

$\therefore$ Centrpetal acceleration $\boxed{a = \dfrac{v^2}{r} = r\omega^2}$

Since a body is moving with an acceleration only under the action of an external force, a body performing circular motion is doing so under the action of an external force acting in the direction of the centripetal acceleration. This force is thus known as "centripetal force" and is given by.

$$F = ma$$

$$\boxed{F = \frac{mv^2}{r} = mr\omega^2}$$

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Centripetal force on several objects undergoing circular motion.

Object performing circular motion	Centripetal force
1. Stone attached to a string	Tension in the string
2.Earth revolving around the Sun	Gravitational attraction of the Sun
3. Motorcycle taking a circular bend	Friction of the road

Applications of centripetal force

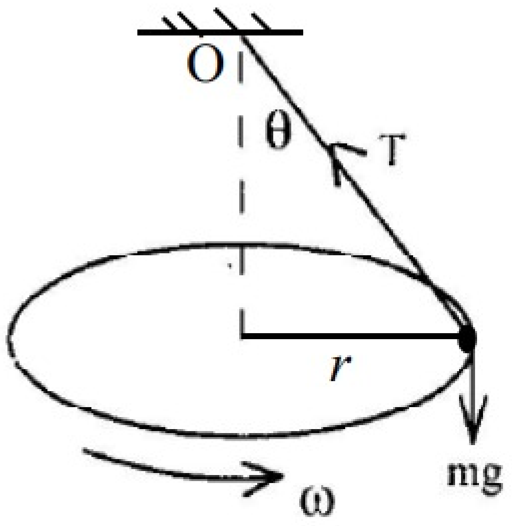

Figure 6.29

1.The conical pendulum

Consider a body performing circular motion in a horizontal plane, while being hung from a fixed point O above by means of a string.

The centripetal force $F = \frac{mv^2}{r} = mr\omega^2$ needed for this circular motion is provided by the horizontal component $T\sin\theta$ of the tension in the string, while its vertical component $T\cos\theta$ balances the weight of the body.

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

2. A motorcycle taking a circular bend

Considering a motorcycle (or a bicycle) moving along a circular bend of radius '*r*' with a speed '*v*'. The centripetal force $F = \frac{mv^2}{r}$ needed for this motion is provided by the frictional force.

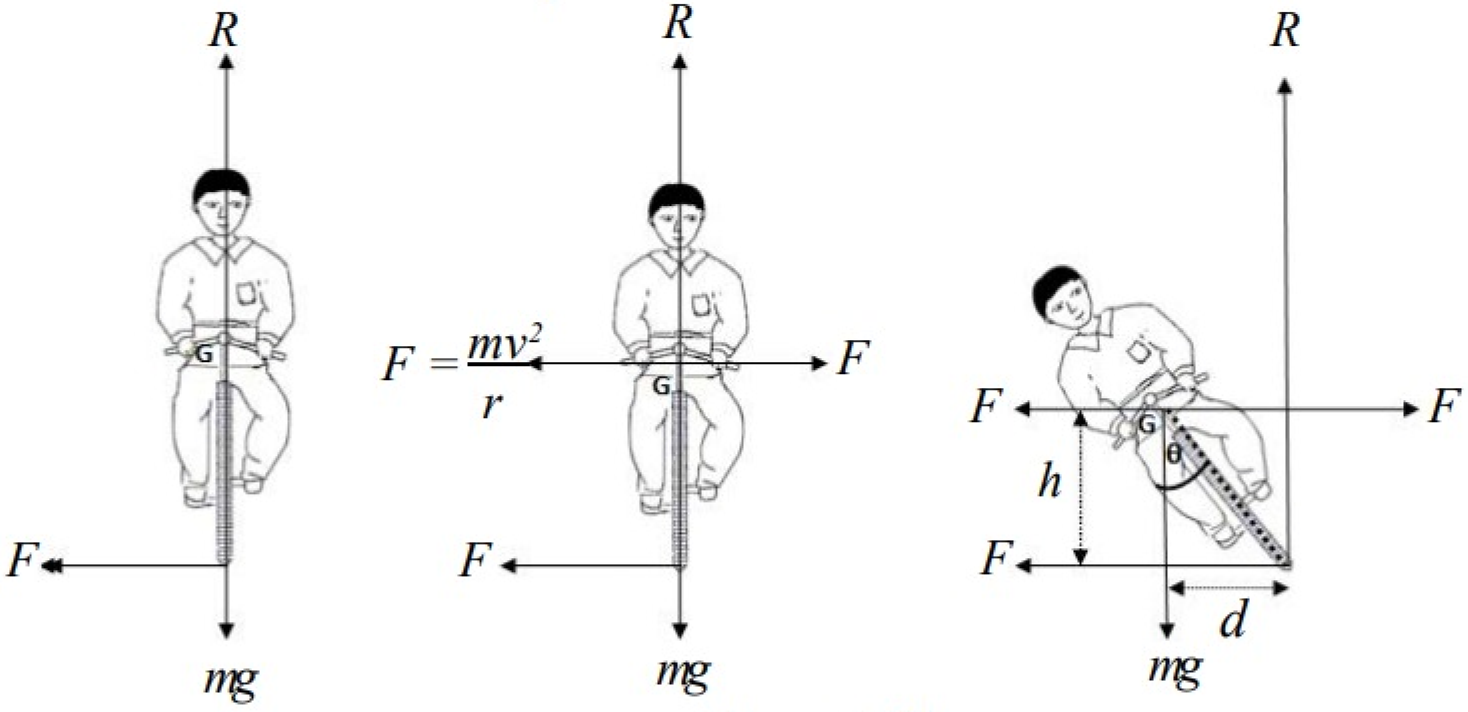

Figure 6.30

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The maximum frictional force is limiting friction $F' = \mu R$
Hence for circular motion of the cycle without slipping,

$$F \le F'$$
$$\frac{mv^2}{r} \le \mu R$$
$$\mu R = \mu mg \Rightarrow R = mg$$
$$\therefore v^2 \le \mu rg \Rightarrow v \le \sqrt{\mu rg}$$

However since this centripetal force has to act at the centre of gravity G of the system, a couple of forces tending to topple the system begins to act on it as shown in Figure 6.29.

In order to prevent the toppling, the system, motorcycle and the rider bends inwards to form a counter couple of forces from the weight (mg) and the reaction (R) on the system.

Hence to prevent toppling,

$$mg \times d = \frac{mv^2}{r} \times h$$
$$\frac{d}{h} = \frac{v^2}{rg}$$

$$\tan\theta = \frac{v^2}{rg} \qquad \text{for no toppling}$$

According to the above expression the angle of bending depends on the speed of motion. Higher the speed of motion more should be the bending towards the ground.

Worked examples

1. A sound record rotates at a speed of $33\frac{1}{3}$ r.p.m. (revolutions per minute). A coin rests on the rotating record wothout being thrown away from it. What is the maximum distance from the centre of the record the coin can be placed without slipping, if the coefficient of friction between the coin and the record is $\frac{1}{3}$ (Take $\pi^2 = 10$).

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Answer : For no slipping of the coin

Centripeal force $\leq$ Limiting frictional force

$$mr\omega^2 \leq \mu R \qquad \text{(mass of the coin is taken as } m\text{)}$$

$$mr\left(\frac{100}{3\times 60}\times 2\pi\right)^2 \leq \frac{1}{3}mg$$

$$r\left(\frac{100^2\times 4\pi^2}{9\times 3600}\right) \leq \frac{1}{3}\times 10$$

$$r \leq 0.27 \text{ m}$$

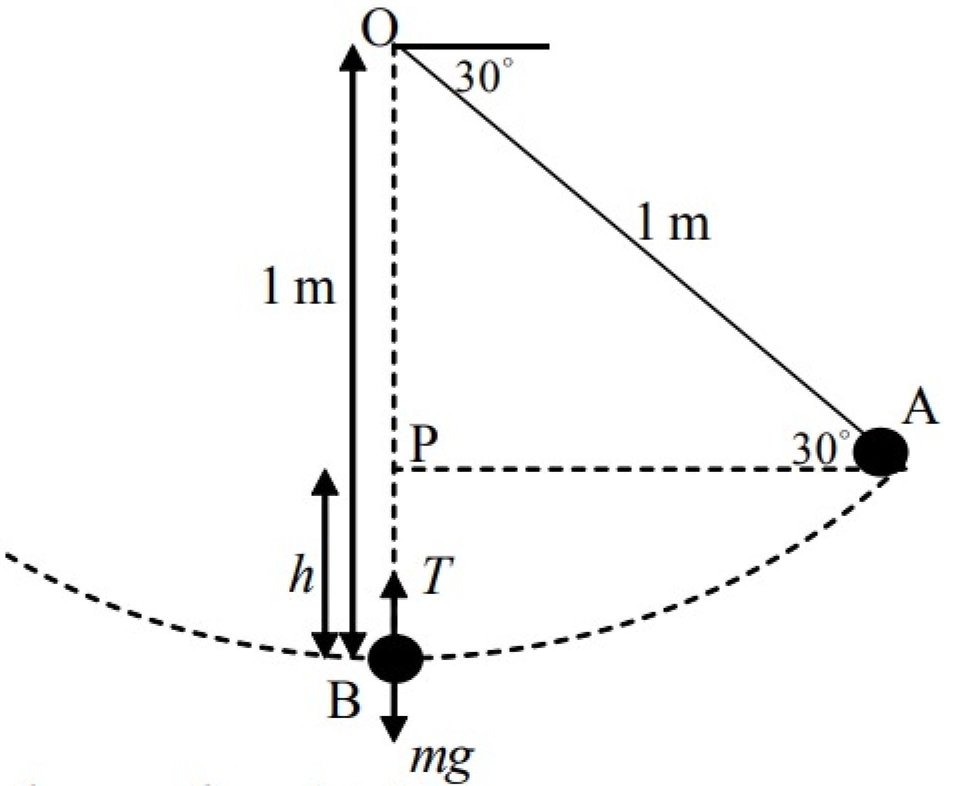
Maximum distance from the centre= 27 cm

2. A body of mass 0.5 kg is hung by a light inextensible string of length 1 m from a fixed point. The body is then pulled aside until the string is inclined at 30° to the horizontal and then released from rest. When the body passes through the lowest point along its path, find

(i) its velocity

(ii) the centripetal force

(iii) the tension in the string



Answer :

$$OP = 1\times \sin\ 30^\circ = \frac{1}{2}$$

$$\therefore PB = OB - OP = 1 - \frac{1}{2} = \frac{1}{2}$$

(i) Applying the principle of conservation of energy from A to B,

$$mgh = \frac{1}{2}mv^2 \quad \text{(potentioal energy at the level B is taken as zero)}$$

$$v = \sqrt{2gh} = \sqrt{2\times 10\times \frac{1}{2}}$$

$$= \sqrt{10} = 3.16 \text{ m s}^{-1}$$

(ii) Centripetal force $F = \dfrac{mv^2}{r} = \dfrac{0.5\times 10}{1} = 5 \text{ N}$

(iii) Since the string has to provide the centripetal force and also to bear the weight of the body,

$$T = mg + \frac{mv^2}{r}$$

$$T = 5 + 5 = 10 \text{ N}$$

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Chapter 07

Hydrostatics

It is known to us that matter exists in 3 states which are called solids, liquids and gases. Although a solid has a defined shape, liquids and gases do not have definite shapes and hence their behaviour is different to that of solids. Thus a study regarding liquids and gases becomes more important technologically as well as scientifically.

The field of study on fluids is referred to as fluid machines. Fluids at rest and fluids in motion exhibit different properties. Hence it is possible to explain this field under two headings namely

1. Hydrostatics
2. Fluid dynamics

There are a number of physical quantities of importance when identifying fluids from one another. Density takes prominence one of these qualities

Density

The mass of a substance changes according its volume. The mass of a unit volume of a substance is termed its density.

$$\text{Density} = = \frac{\text{Mass}}{\text{Volume}}$$

Symbols such as ρ and d are used to represent density. Since we are used to symbolize mass with m and volume with V, the equation for density can be written as,

$$\rho = \frac{m}{V}$$

The standard unit of measuring density is kg m^{-3} Sometimes the unit g cm^{-3} is also used practically.

$$1 \text{ g cm}^{-3} = 1000 \text{ kg m}^{-3}$$

The density of a fluid changes according to temperature and pressure. Generally the density decreases with increasing temperature . But there are instances where it in not so. The table below shows the densities of few fluids at a pressure of 1 atm and a temperature of 4 °C.

Liquid	**Density** (kg m^{-3})	**Gas**	**Density** (kg m^{-3})
Mercury	13.6×10^3	Oxygen	1.43
Glycerine	1.23×10^3	Air	1.29
Milk	1.03×10^3	Nitrogen	1.25
Sea water	1.03×10^3	Helium	0.17
Water	1.00×10^3	Hydrogen	0.09
Coconut Oil	0.8×10^3		
Alcohol	0.8×10^3		

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When the pressure of certain fluids increase their densities increase. This happens if the fluid is compressible. Our study in this field is confided to incompressible fluids.

Incompressible fluids

"Those fluids which do not undergo any changes in volume when subjected to pressure are known as incompressible fluids"

Lequids used by us do not undergo appreciable volume changes when subjected to pressure and hence are incompressible fluids.

Homogeneous fluids

Those fluids at all points of which the density has the same value are known as homogeneous fluids.

When the pressure of a gas at rest is increased its value changes largely and hence gases are called compressible fluids. As only the liquids are incompressible fluids the study of fluids at rest is named Hydrostatics.

Relative density

The density of substance relative to that of water is called the relative density of the substance

$$\boxed{\text{Relative density} = \frac{\text{density of substance}}{\text{density of water}}}$$

When the density of the substance is denoted by ρ and the density of water by ρ_w, then

$$\text{Relative density} = \frac{\rho}{\rho_w}$$

Since relative density is a ratio of equal quantities it has no units . When the relative density of a substance is given, its density can be obtained in SI units by multiplying the relative density by 1000

Relative density can also be expressed as the ratio between the masses of equal volumes of substance and water.

$$\text{Relative density} = \frac{\text{mass of a given volume of subsance}}{\text{mass of an equal volume of water}}$$

Hydrostatic pressure

When an air bubble released from the bottom of a deep vessel of water is observed it can be seen that the bubble becomes larger when rising through the water. Figure 7.1 illustrates this. The reason for the bubble to be small at the bottom is that the water column above it exerts a pressure on it.

Figure 7.1

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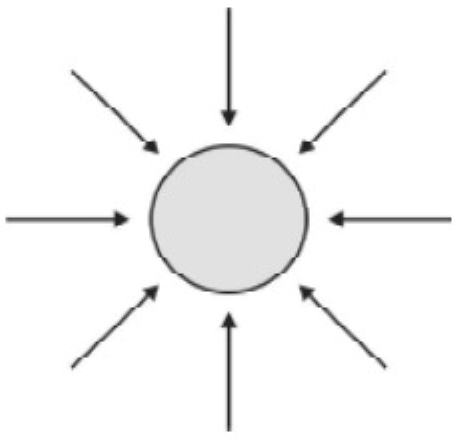

Figure 7.2

Figure 7.2 illustrates how forces act on the bubble. When the bubble ascends the height of the water column above it decreases, thus decreasing the pressure on it. Hence the bubble becomes larger.

Pressure is defined as the force on a unit area of a surfaces, normal to that surface. Accordingly, if *F* is the force acting normally on an area *A* of a surface, then the pressure there is.

$$p = \frac{F}{A}$$

The S.I. unit of pressure is N m^{-2} and this unit is known as "pascal" (Pa).

$$1 \text{ Pa} = 1 \text{ N m}^{-2}$$

Pressure has no definite direction. Hence pressure is considered as a scalar quantity. A few other units are also in use to measure pressure. They are atmospheres (atm), mm Hg, bar and torr. Pressure of one millimetre of mercury (1 mm Hg) means the pressure exerted by a mercury column of height 1mm. The value of each of these units in pascals is given below.

1 atm = 1 x 10^5 Pa

1 bar = 1 x 10^5 Pa

1 torr = 1mm Hg = 133.3 Pa

In order to develop an expression for hydrostatic pressure, consider a horizontal surface of area *A* at a depth *h* below the surface of a liquid of density ρ .

Since the volume of the liquid above the horizontal surface considered is *Ah*,

Weight of the liquid column = $Ah\rho g$

As pressure is the force acting normally on a unit area,

$$p = \frac{Ah\rho g}{A} = h\rho g$$

This is the expression for the hydrostatic pressure at O.

But in addition to the hydrostatic pressure offered by the liquid column, the air above the liquid surface too exerts a pressure. This pressure, exerted by the atmospheric air, is known as atmospheric pressure.

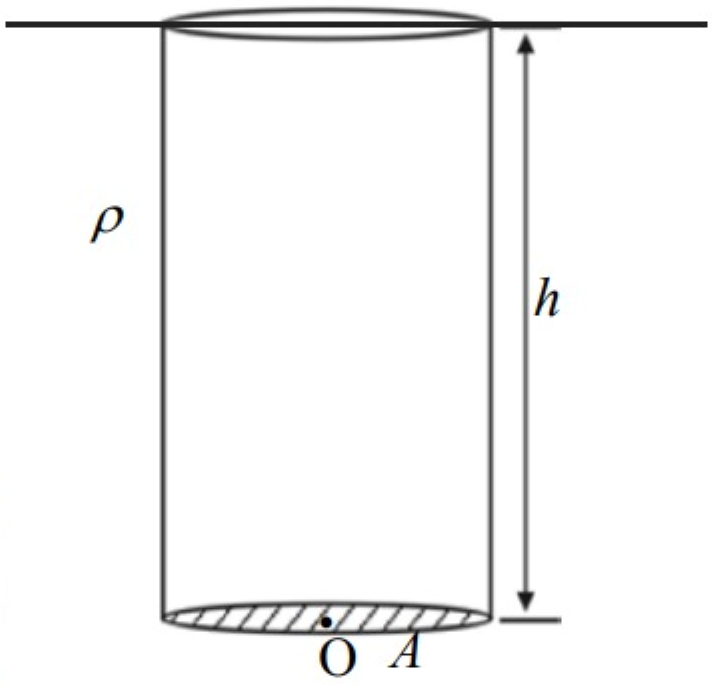

Figure 7.3

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Atmospheric pressure

The atmosphere exists to a large height from the earth surface. The density of the air in the atmosphere gradually decreases with the increase of height; Any object in the environment is acted upon by a pressure due to the weight of the air above it. This pressure is atmospheric pressure . The atmospheric pressure gradually decreases upwards from the sea level . At a height of about 5600 m above the sea level the atmospheric pressure is about a half of the pressure at the sea level.

Measurement of atmospheric pressure

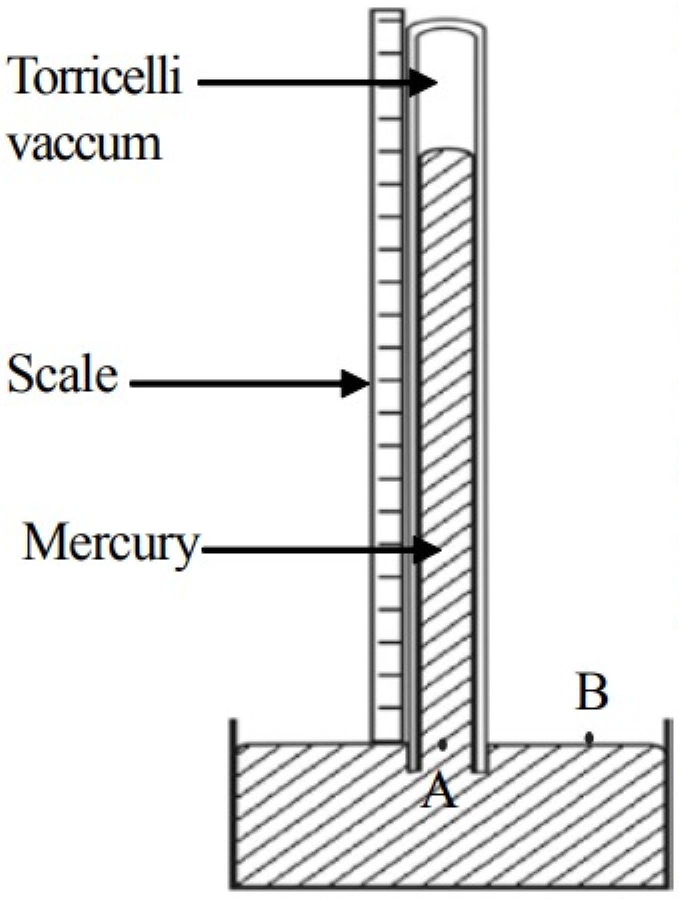

Figure 7.4

Figure 7.4 shows the apparatus constructed by the Italian scientist Torricelli to measure atmospheric pressure. It is known as the mercury barometer. When a tube of about 1 m long and closed at one end is filled with mercury and inverted into a trough of mercury, the mercury will be found to remain as a colum in the tube and Torricelli revealed that this height measures the atmospheric pressure. Since a vacuum exists above the mercury column in the tube, the pressure at the point A is caused by the weight of the mercury column above.

Hence $p_A = h\rho g$

The pressure at the point B which is at the same level, is also the same. It is caused by the atmospheric pressure. Hence the atmospheric pressure is given by *hpg*. ρ is the density of mercury, while h is the height of the mercury column and g is the acceleration due to gravity. Torricelli has found that the mercury barometric height at the sea level is 760 mm. It is this pressure that has been standardized as 1 atmosphere.

According to $p = h\rho g$

$$p = 760 \text{ x } 10^{-3} \text{ x } 13600 \text{ x } 10$$

$$= 1.01325 \text{ x} 10^{5} \text{ Pa}$$

For the convenience of calculations this value is used as 1 x 10^5 Pa. However, 760 mm Hg is also used as atmospheric pressure on many occasions.

Why mercury is being used as the barometer liquid is due to its high density. If water were used as the barometer liquid, the height of the Barometer tube to be used for a pressure of a atmosphere would be as high as 10 m.

According to meters that we can see at filling stations and service stations, the units used to measure pressure of air inside vehicle tyres are “pounds per square inch (PSI)” and “ kilograms per square centimeter (kg cm^{-2})”.

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Pressure at a point in a liquid exposed to atmosphere.

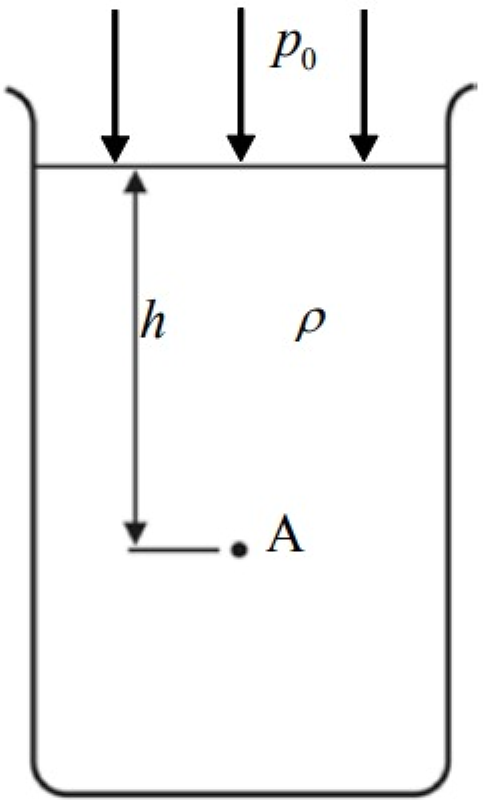

Figure 7.5

Consider a point A at a depth h below the surface of a liquid exposed to the atmosphere. If p is the pressure at the point A,

$$p = p_0 + h\rho g$$

where p_0 is the atmospheric pressure and ρ is the density of the liquid.

Pascal's principle on transmission of pressure

Distribution of pressure in an incompressible fluid contained in a closed vessel when subjected to an external pressure was investigated by the scientist Pascal and his findings are included in Pascal's principle which is stated below.

"The pressure applied at a certain point of an incompressible enclosed fluid at rest is transmitted undiminished to all the other points of the fluid as well as to the walls of the container containing the fluid".

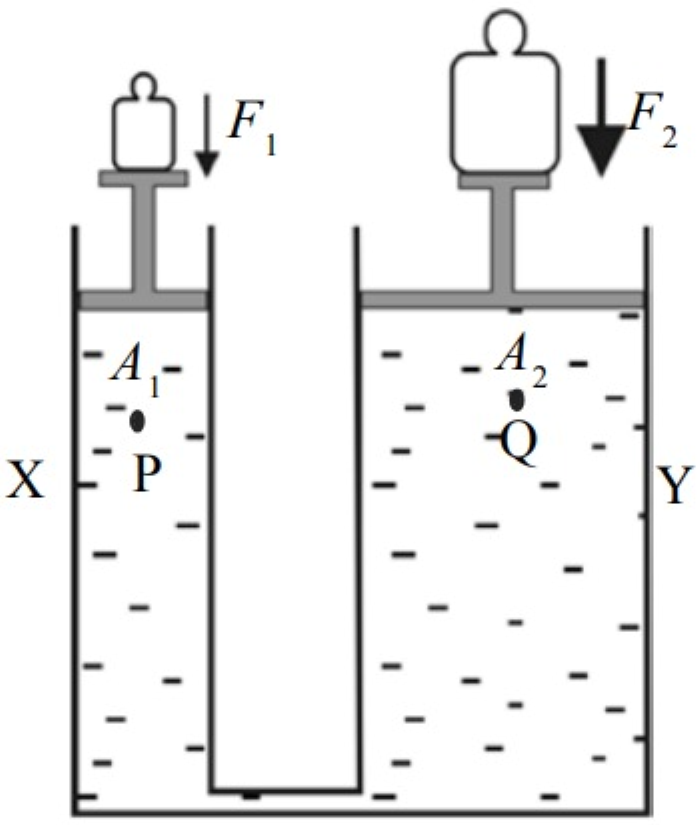

Figure 7.6

This principle is technically very important and has many applications, one of which is the hydraulic press.

The Figure 7.6 shows a simple setup of the hydraulic press. This set up consists of a vertical cylindrical tube X of narrow cross – section and another vertical cylindrical tube Y of wide cross – section. These are connected by a horizontal tube and the apparatus is filled with a liquid such as oil. Two movable pistons are placed on the liquid surfaces in the two vertical tubes and on the two pistons are placed two loads.

By placing a certain load on the smaller piston a heavier load can be raised by placing it on the larger piston. The relationship between the forces can be obtained as follows.

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If the area of cross – section of the smaller piston is A_1 and the force on it is F_1,

the pressure at point $P = \frac{F_1}{A_1}$

Similarly if the area of cross- section of the larger piston is $A_2$and the force obtained from it *is* F_2,

the pressure at point $Q = \frac{F_2}{A_2}$

According to the principle of transmission of pressure

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2 F_1}{A_1}$$

It can be seen from the above equation that by increasing the ratio $\left(\frac{A_2}{A_1}\right)$ between the cross sections of the pistons, the ratio between the forces $\left(\frac{F_2}{F_1}\right)$ will also get increased.

There are many applications of this type. They are,

- Hydraulic jack
- Hydraulic brakes
- Hydraulic lifts used in service stations
- Patients chair used in dental clinics
- Heavy vehicles such as backaw engines

Hydraulic jack

The hydraulic jack is used to raise vehicles for purposes such as servicing of vehicles. By operating the lever attached to the small piston, it is possible to raise the vehicle by the larger piston. By increasing the AC and AB length ratio, the force required to raise the vehicle can be further decreased.

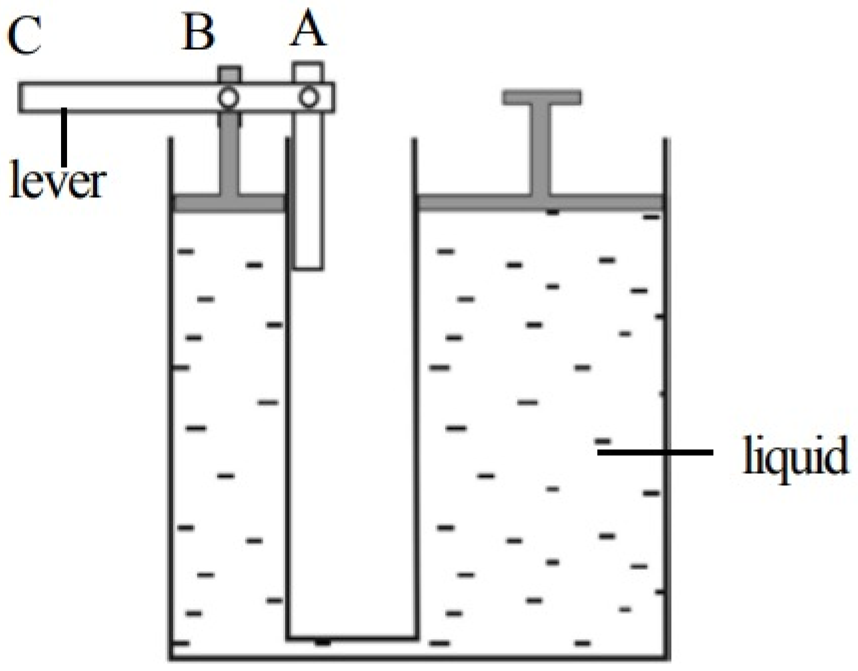

Figure 7.7(a)

Figure 7.7(b)

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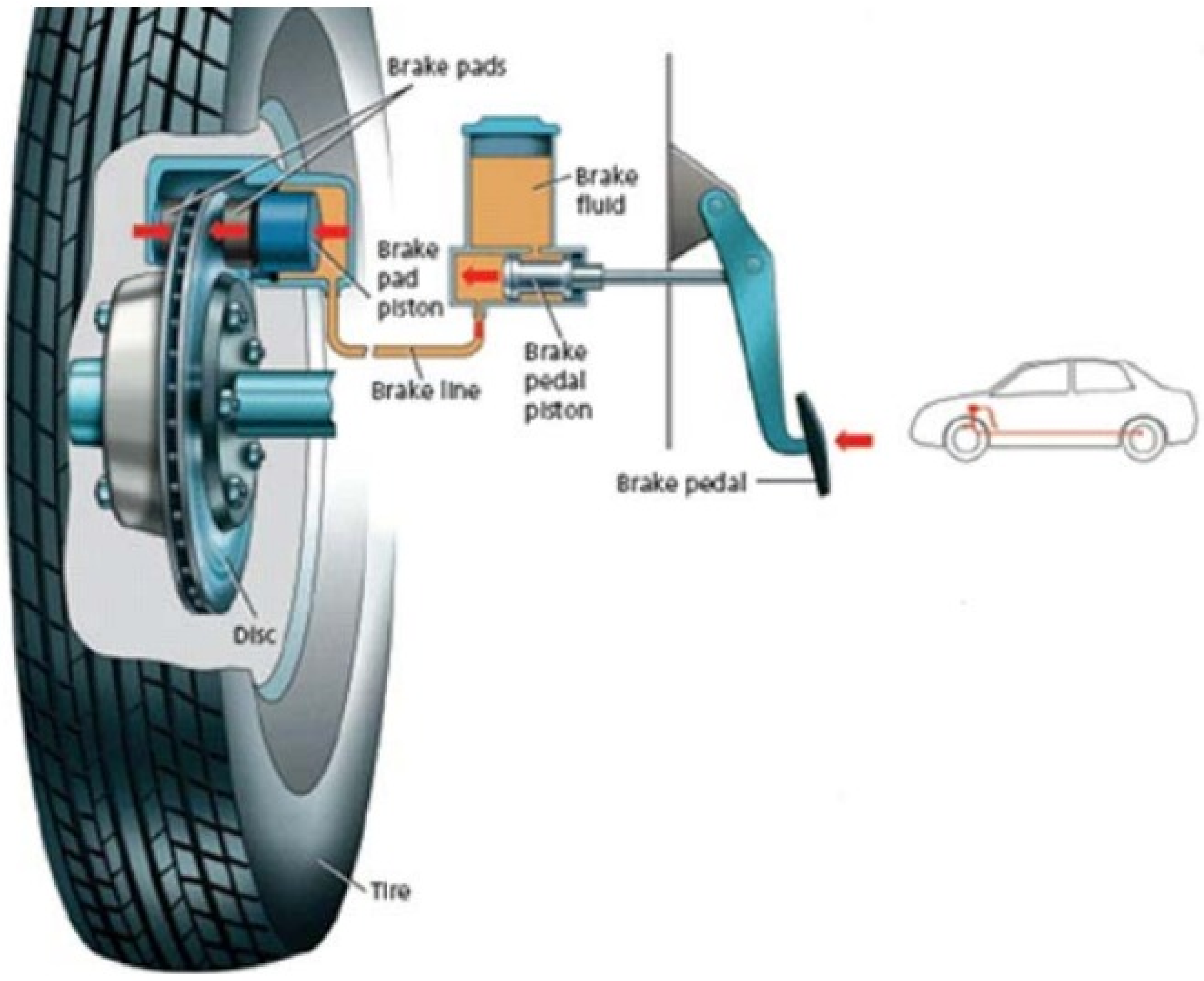


Figure 7.8

Hydraulic brakes

Figure 7.8 shows a hydraulic brake system used in vehicles. In this the lever set up with the brake pedal is fitted to apply a force on the piston in the main cylinder. The pressure applied to the liquid contained in the main cylinder is transmitted to the pistons with large tags crosssection. These pistons offer considerably large force to spot the vehicle.

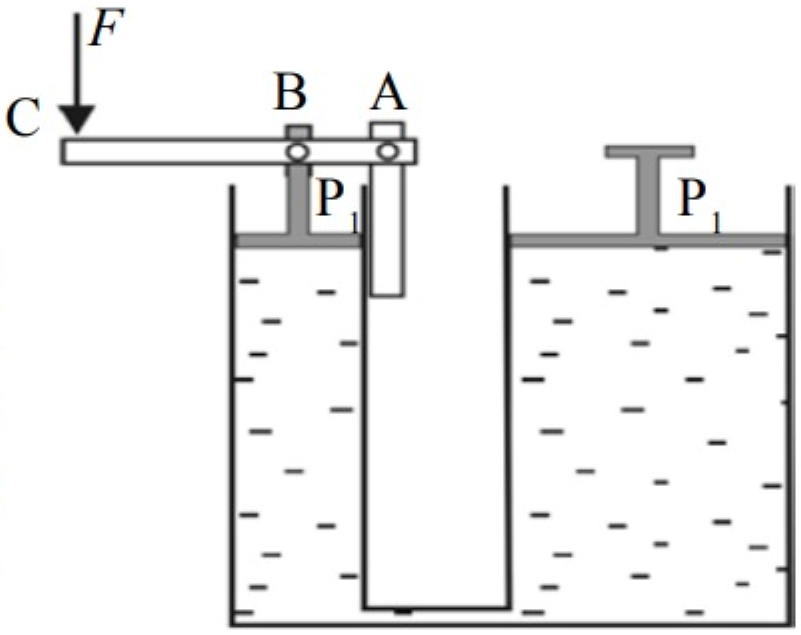
Operation of components of heavy vehicles

Figure 7.9

Figure7.9 shows a backaw engine. Pistons that are used to offer the force required to operate various components have different cross sections. The pressure caused by the force applied to the main piston is transmitted to the other pistons to create the required force.

Example

Figure 7.10 shows a simple set up of a hydraulic jack . The radii of cross – sections of the pistons P_1 and P_2 are 4 cm and 20 cm respectively. A force of 6400 N is required to be applied by the large piston to to raise a motor car to change a wheel. The lever bar is pivoted at A.



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(i). Calculate the force required to be applied on the small piston.

(ii) If, in the lever bar AB = 4 cm and BC = 16 cm find the force *F* that should be applied at C in order to supply the force calculated in (i)

(iii) What modification should be done in the above equipment in order to reduce the force *F* further ?

Solution

(i)

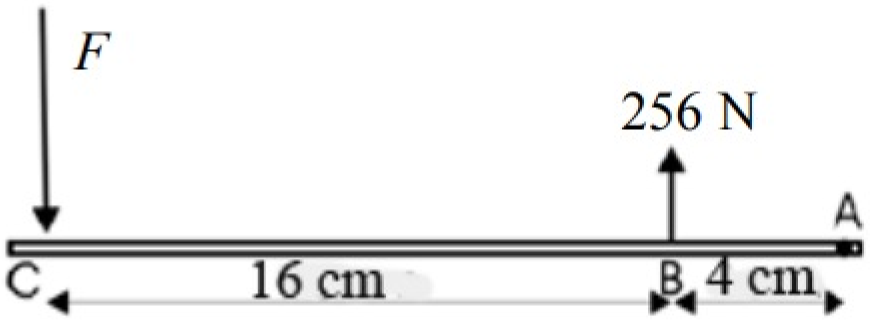
$$\frac{F_1}{A_1}=\frac{F_2}{A_2}$$

$$\therefore F_1=\frac{F_2A_1}{A_2}$$

$$\therefore F=\frac{6400\times\pi\left(4\times10^{-2}\right)^2}{\pi\times\left(20\times10^{-2}\right)^2}$$

$$=\underline{\underline{256N}}$$

(ii)



A ↻ BA × 256 = F × AC

$4 \times 10^{-2} \times 256 = F \times 20 \times 10^{-2}$

$F = 51.2$ N

(iii) In order to reduce the value of *F* the length BC should be increased further. That is, the length of the lever bar should furher be increased.

Upthrust

When a plastic ball is dropped to water it floats, When it is pressed into water by hand the hand feels a force acting upwards in it. When the hand releases the ball rises to the surface. This happens due to an upward force acting on the ball. This force is known as the 'upthrust' or the force of buoyancy.

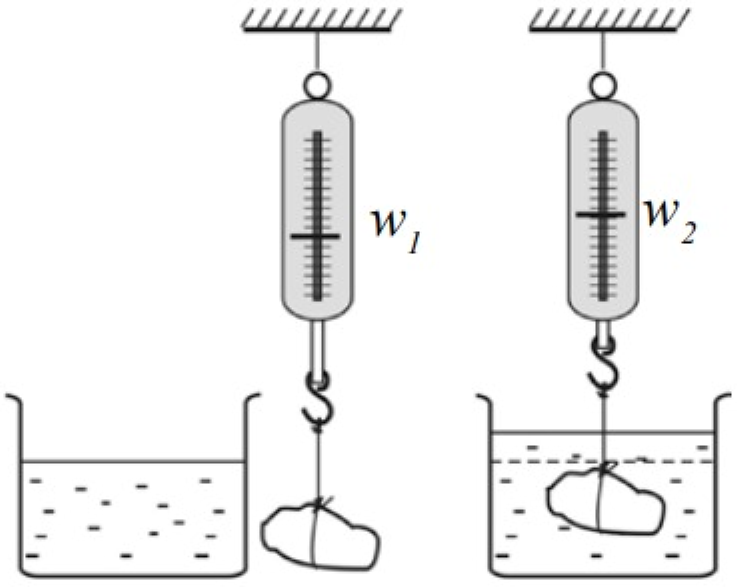


Figure 7.10

Activity in Figure 7.12 can be used to find the up-thrust.

If w_1 is the reading in a newton spring balance when a small stone is suspended from it and w_2 the reading when the same stone is totally immersed in water. Then apparent loss in weight $= w_1 - w_2$

Since this loss in weight is due to the upthrust,

Upthrust $U = w_1 - w_2$

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When a body is immersed in a liquid the body displaces a volume of liquid. Greek astronomer Archimedes (287-212 BC) succeeded in determining a relationship between the volume of the liquid displaced and the upthrust. This relationship introduced by him is known as Archimedes' principle.

Archimedes' Principle

When a body is completely or partially immersed in a fluid at rest, it experiences an upthrust equal to the weight of the fluid displaced by the body.

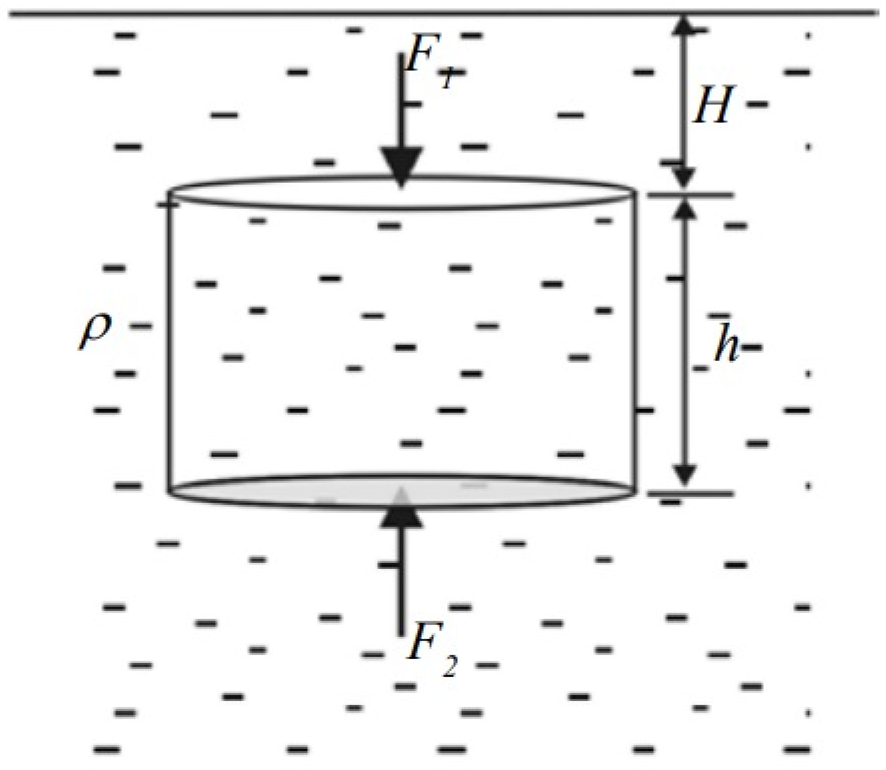
If a volume V of a body is immersed in a fluid of density ρ , the upthrust according to Archimedes principle, is

$$\boxed{U = V\rho g}$$

In order to verify Archimedes' principle theoretically, consider a cylindrical object of height h immersed with its axis vertical in a liquid of density ρ .

If A is the area of cross – section of the cylinder and H the height of the liquid surface from the top surface of the cylinder then the pressure of liquid on the top surface of the cylinder is $H\rho g$

Hence $F_1 = AH\rho g$



$= Ah\rho g$

Figure 7.11

Since the pressure on the bottom surface of the cylinder is $F_2 = (H + h)\,\rho g$

$$\text{Upthrust} \quad = F_2 - F_1$$

$$= A\,(h\ +\ h)\,\rho g - AH\,\rho g$$

$$= Ah\rho g$$

$$U = V\rho g$$

Above V is the volume of the cylinder. Since the displaced volume is the same, Archimedes' principle is verified theoretically.

Shown in the Figure 7.14 below is a set up which can be used to verify Archimedes' principle practically.

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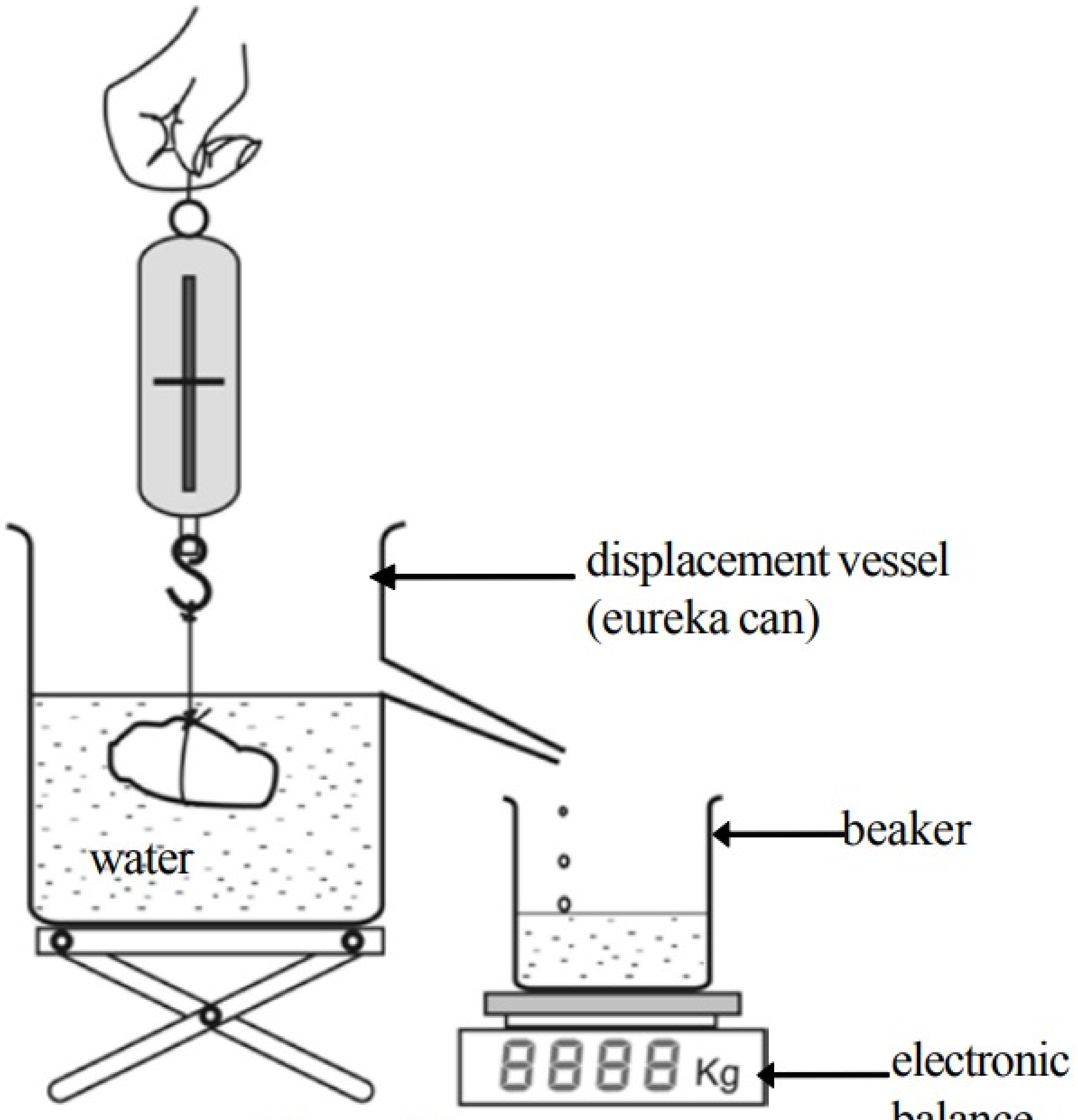

Figure 7.12

Place a beaker on an electronic balance and adjust the balance to read zero. Direct the exit tube of a displacement vessel filled with water into the beaker. Suspend a small stone from a spring balance to record its reading (w_1) and lower the stone into the displacement vessel to immerse it completely in the water. Record the reading (w_2) again.

If the difference between these two reading (w_1 - w_2) equals the reading of electronic balance, Archimedes' principle will be verified.

Determination of mean density of a body using Archimedes' principle

Archimedes' principle can be used to determine the mean density of a body as follows.

As show in the Figure 7.15 the body is hung from a balance and the reading w_1 of the balance is recorded. The body is next immersed completely in water and the reading w_2 of the balance is recorded again.

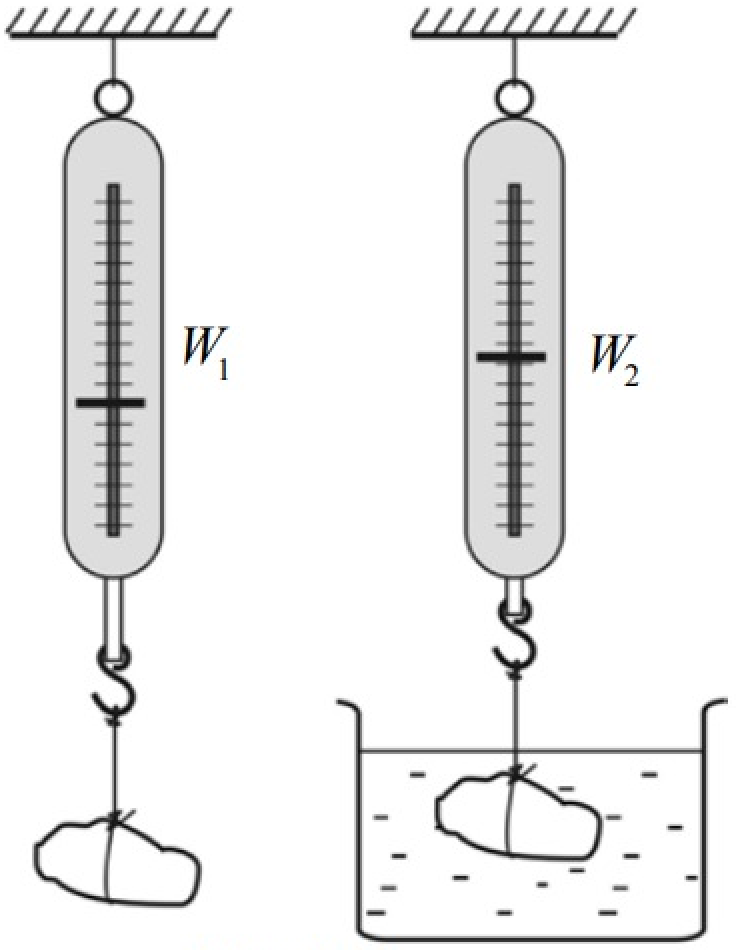

Figure 7.13

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Then the upthrust acting on the body, $U = w_1 - w_2$

According to Archimedes' principle, the weight of an equal volume of water is equal to the upthrust.

$$\text{Hence, relative density} = \frac{w_1}{w_1 - w_2}$$

$$\therefore \text{Mean density} = \left(\frac{w_1}{w_1 - w_2}\right)\rho_w$$

where ρ_w is the density of water.

Example

When a block of metal is suspended from a spring balance and weighed in air the reading was 12 N. When the block is completely immersed in water the reading in the balance was 8 N. Find the mean density of the metal in the block.

$$\text{Relative density} = \frac{\text{weight of the body}}{\text{weight of an equal volume of water}}$$

$$\text{Relative density} = \frac{\text{weight of the body}}{\text{upthrust}}$$

$$= \frac{w_1}{w_1 - w_2}$$

$$= \frac{12}{12 - 8} = 3$$

$$\therefore \text{Density} = 3 \times 1000 \text{ kg m}^{-3} = 3000 \text{ kg m}^{-3}$$

Floatation

If a body is to float in a liquid, the forces on it should be in equilibrium. That is, the upthrust should be equal to the weight of the body.

$$m\,g = U$$

This relation is known as the principle of floatation.

Shown below are two states of floatation of a body.

Figure 7.16 shows the completely immersed state of flotation of a body

For equilibrium of the body.

$$mg = U$$

$$Vdg = V\rho_w g \quad (d \text{ is the mean density of the body})$$

$$d = \rho_w$$

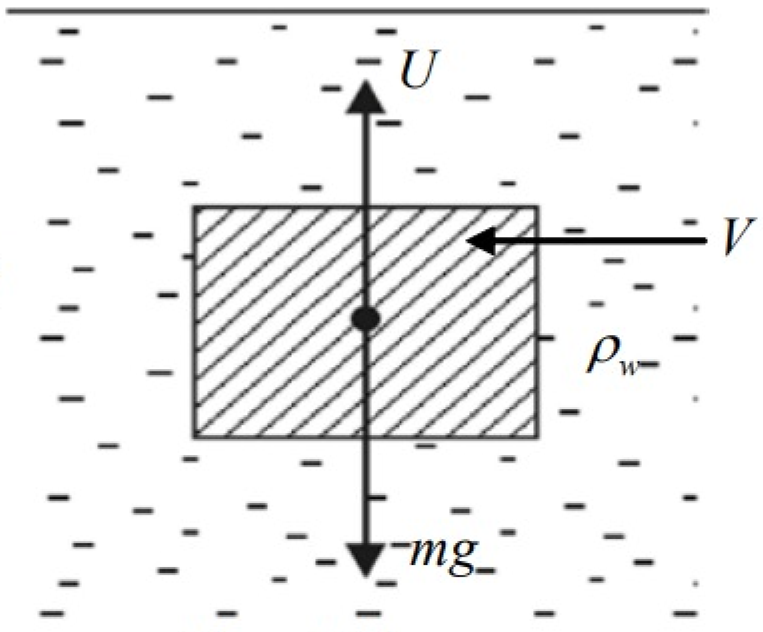

Figure 7.14

The mean density of the body is equal to the density of water.

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Figure 7.17 shows a partially immersed state of flotation of a body.

For equilibrium of the body,

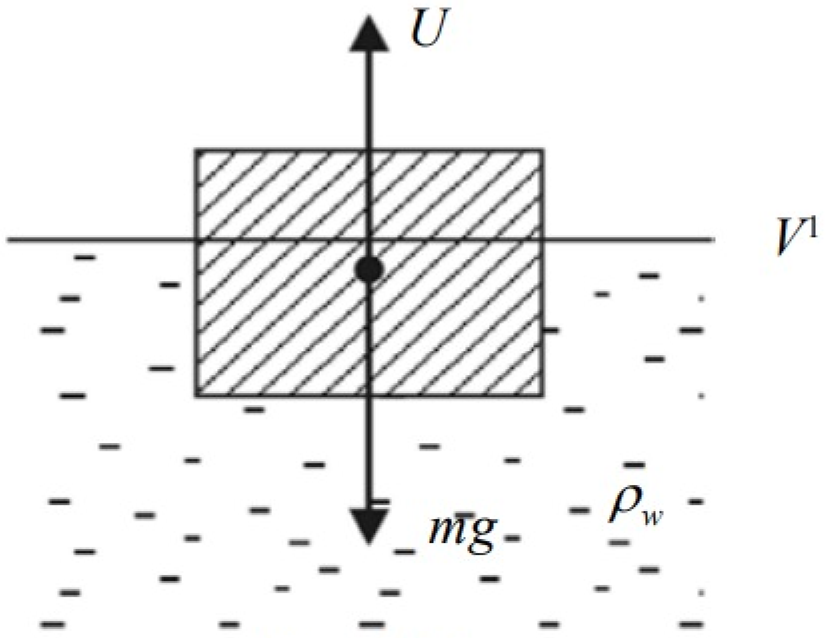

Figure 7.15

$$mg = U$$

$$Vd\cancel{g} = V'\rho_w\cancel{g}$$

Since $V > V'$

$$d < \rho_w$$

The mean density of the body is less than the density of water .

To increase the amount immersed, the mean density of the body should be made less. For this purpose the body can be formed with a cavity to increase the volume . Vessels moving in water, are made using this principle.

Centre of buoyancy

The point where the up-thrust of a floating body acts is called the centre of buoyancy. This point is situated at the geometrical centre of the portion immersed or the centre of gravity of the displaced volume of the liquid.

Consider an instance when a pencil is dipped in water. The weight of the pencil acts at its centre of gravity(G) while the upthrust acts at the centre of buoyancy (G_1). Although mg and U are equal in magnitude, they are not collinear as a result of which a moment is formed. Hence the pencil will set horizontally.

If a ball of clay is attached at the end of the pencil it will begin to float vertically, the reason being that because of the clay ball the centre of gravity of the pencil has shifted to a point below the centre of buoyancy. Then due to the moment, the pencil would set vertical even if the pencil is placed in an inclind position.

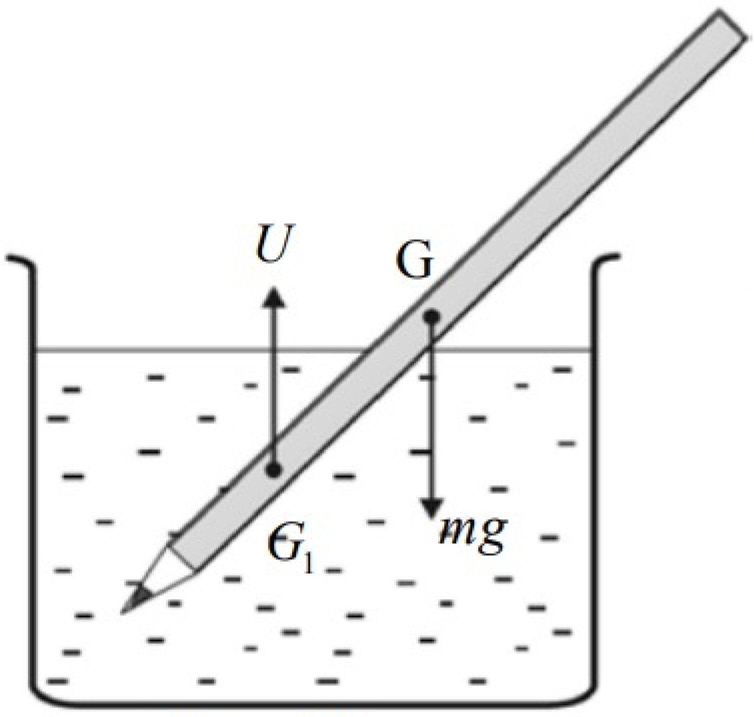

Figure 7.16

This can be used to compare the densities of liquids. The depth that is immersed varies according to the density of the liquid.

The pencil would immerse to a large depth in liquids of low densities while it would immerse to a lesser depth in liquids of high densities.

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Hydrometer

The hydrometer can be introduced as a simple instrument that can be used to compare densities of two liquids.

The Figure 7.20 shows the state of floatation of a hydrometer in a liquid. The lower bulb loaded with lead lowers the centre of gravity to make the hydrometer float vertically In order to obtain a high up-thrust when floating in any liquid the bulb is constructed with a large volume.

The thin long stem increases the sensitivity of the instrument.

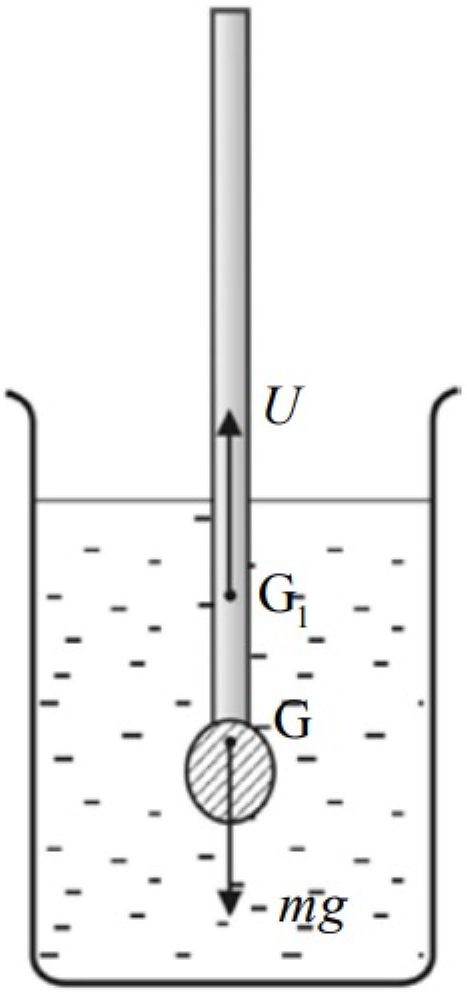

Figure 7.17

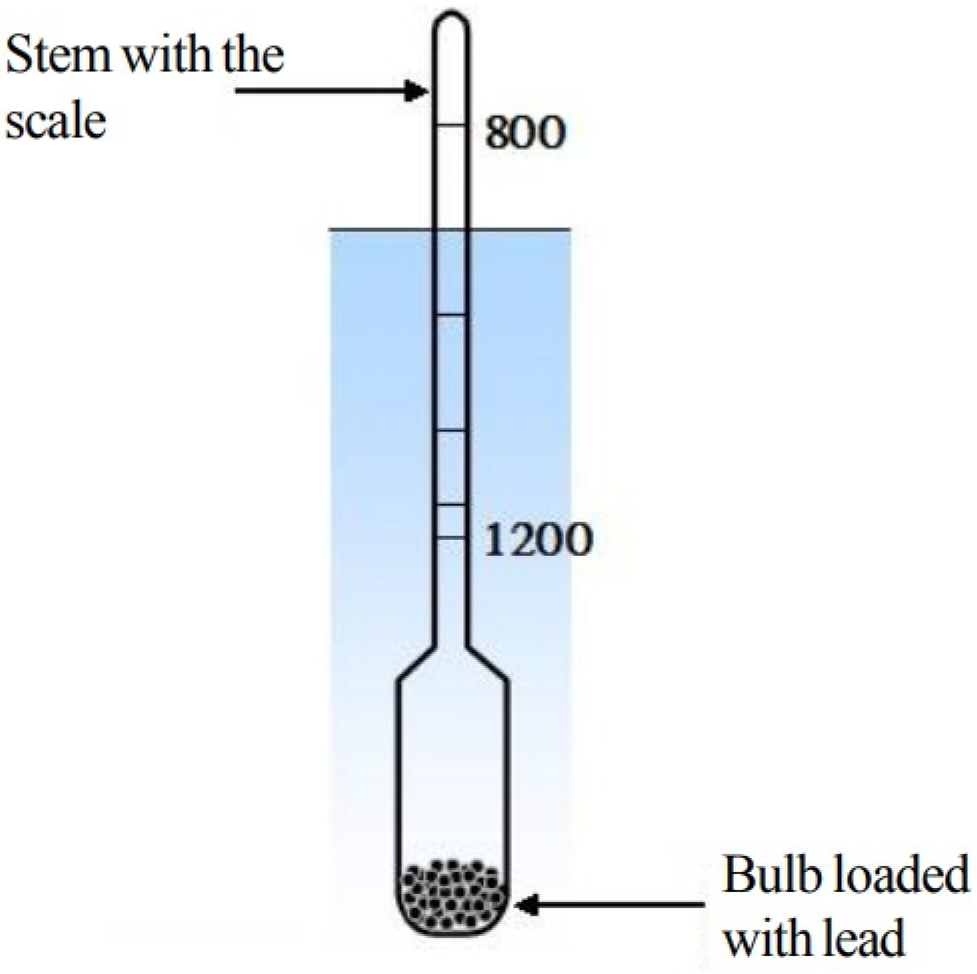

Figure 7.18

The gaps between consecutive markings of the hydrometer scale are not equal to each other (or scale is not linear). This is shown by the following expression.For equilibrium of floating hydrometer,

$$m\cancel{g} = (V + Ah)\,\rho\cancel{g}$$ (*m* is the mass of hydrometer)

$$\frac{m}{\rho} = V + Ah$$

Hence it is seen that there is no linear relation between h and ρ .

In this above expression V is the volume of the bulb and the loaded bulb. A is the area of cross–section of the stem. h is the height of the immersed portion of the stem.

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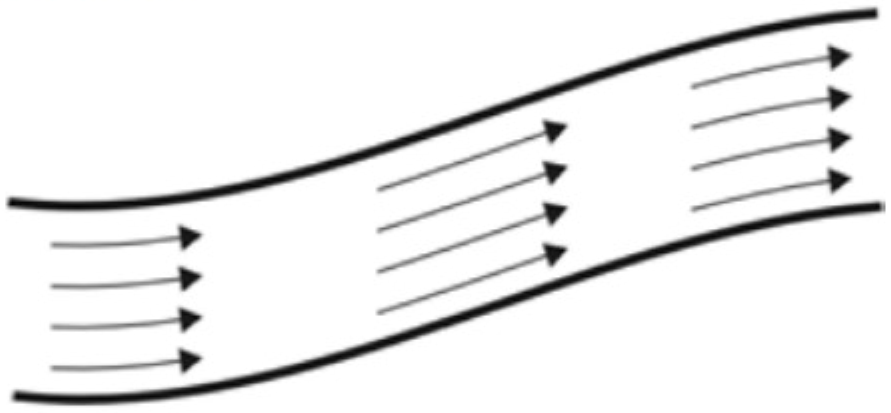
Chapter 08

Fluid dynamics

Fluid dynamics is the study of the flow of fluids In order to identify the characteristic of fluid flow the flow, of water from a tap can be considered. When the tap is opened slightly, as that the flow takes place slowly. It is clearly seen that there is no turbulence nature among the water particles. Such a flow is referred to as a streamline flow.

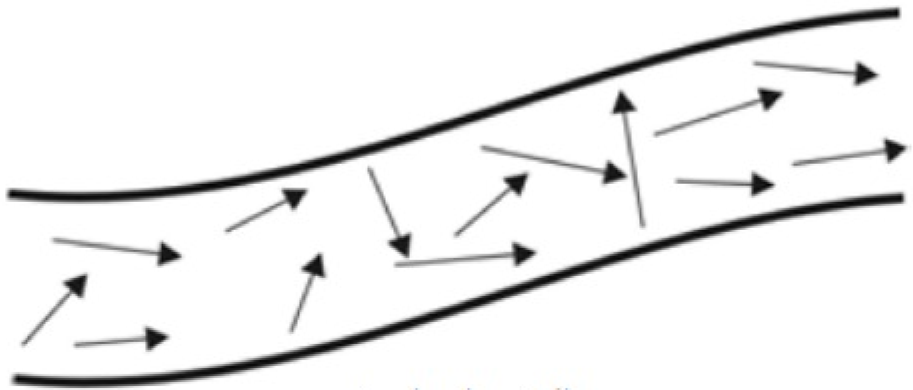
When the tap is fully opened allowing the water to flow fast the flow becomes disorderly and the water particles do not travel along specific directions. Such a flow can be recognized and is called a turbulent flow.

Considering its importance this chapter deals with streamline flow of fluids which can be introduced as follows.



streamline flow

Figure 8.1



turbulent flow

Figure 8.2

Steady floor

In a fluid flow if the velocity of a fluid particle passing through any point does not change with time the flow is called a steady or a continuous flow.

In a streamline flow the fluid flows as layers and hence is also known as lamina flow. Due to the relative motion between these layers frictional forces come into play between them and these frictional forces are known as viscous forces. In the study of the unit the effects due to viscous forces are considered negligible.

The streamline

In a streamline flow, the line which indicates the path of a fluid particle is known as a streamline The tangent drawn at a certain point on a streamline indicates the direction of motion of particles passing through that point. Streamlines never intersect each other.

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Tube of flow

The region through which flows a collection of streamlines is a tube of flow.

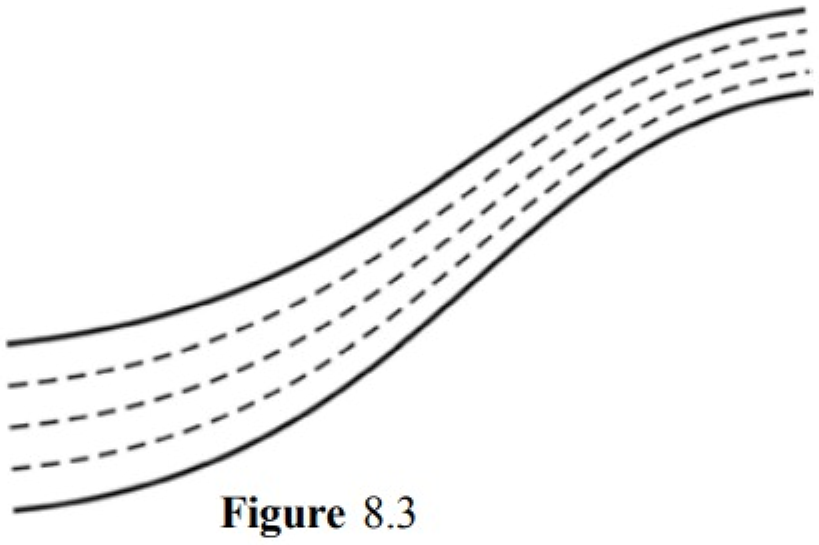

Figure 8.3

Equation of continuity

Consider the thinning of a tube of flow due to the streamlines getting closer to each other.

Let v_1 be the speed of flow at the entrance of the tube where the area of cross- section is A_1.

Volume of fluid entering the tube through area A_1.

per unit time = A

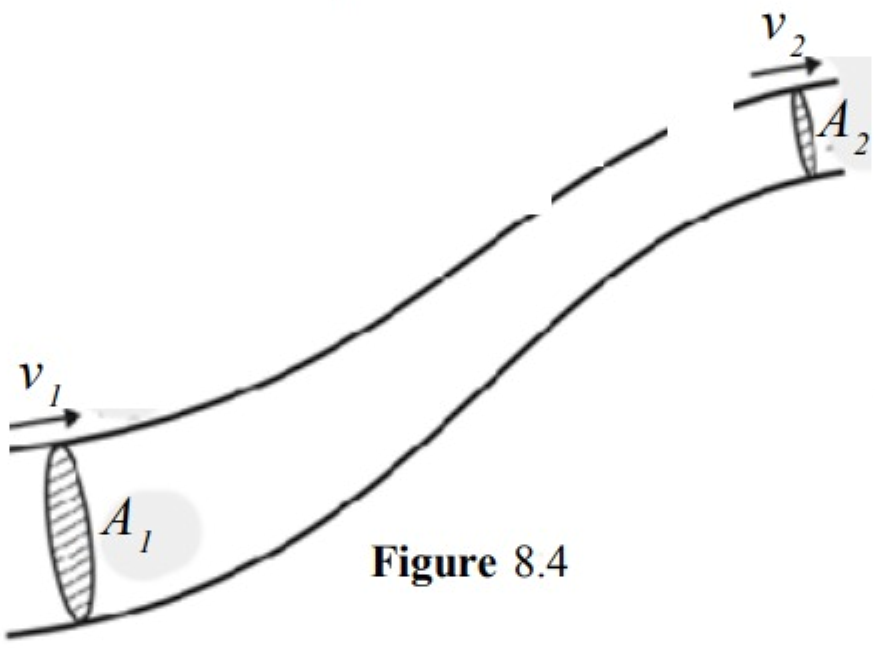

Figure 8.4

Since the flow is continuous, the volume of fluid flowing past the cross section A_2 per unit is the same. If v_2 is the velocity of the fluid flowing past A_2,

The volume of fluid emerging per unit time = A_2v_2

$$\therefore\ A_1v_1 = A_2v_2$$

This equation which explains the variation of velocity of flow with the variation of cross section of the tube of flow, is referred to as the equation of continuous flow or the equation of continuity. There are many situations in which this equation is technically made use of.

For example, when spraying water from a rubber tube connected to a water tap, the mouth of the tube is squeezed to reduce is cross section and increase the emerging velocity of water.

The shower in the bathroom and the insecticides sprayer are other devices which use this principle to increase the speed of flow.

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The diameters of water pipes leading from the water tank on top of a house down to the taps gradually reduce in diameters in order to increase the speed of flow of the water.

Figure 8.5 shows a water pipe system consisting of two pipes with different cross sections connected at a junction. The diameter of pipe A is 60 mm while that of B is 20 mm. If water enters at A with a velocity of 0.2 m s^{-1} and flows steadily through the pipe to emit from B, find the emerging velocity of water at B.

According to equation of continuity.

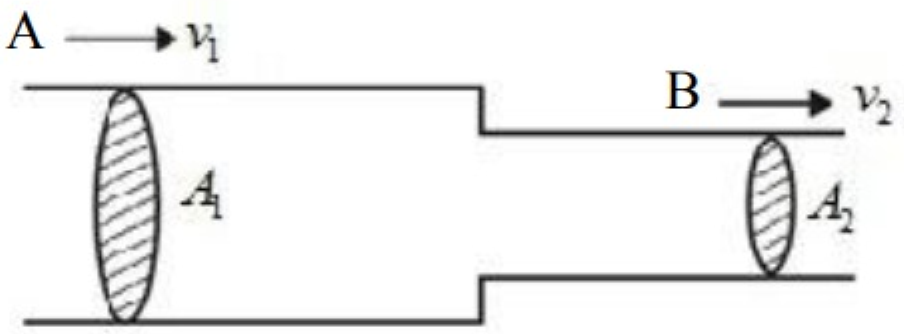

Figure 8.5

$$A_1v_1 = A_2v_2$$

$$\pi\left(\frac{d_1}{2}\right)^2 v_1 = \pi\left(\frac{d_2}{2}\right)^2 v_2$$

$$d_1^{\,2}v_1 = d_2^{\,2}v_2$$

$$v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{60}{20}\right)^2 \times 0.2$$

$$v_2 = \underline{\underline{1.8 \text{ m s}^{-1}}}$$

Incompressible fluids

When a fluid is subjected to pressure if its density remains unchanged, than the fluid is said to be an incompressible fluid.

Although gases at rest are considered as compressible fluids, a gas flow can be considered incompressible. Hence in fluid dynamics motions of liquids and gases are being explained.

Bernoulli's principle

Well known Swiss scientist Johann Bernoulli put forward a principle on the flow of fluids which comes to be known as Bernoulli's principle.

He has shown that in a streamline flow, the useful work done by pressures on a unit volume of the fluid is equal to the sum of the increase in its potential energy and the increase in kinetic energy.

Accordingly, Bernoulli's equation can be obtained as follows.

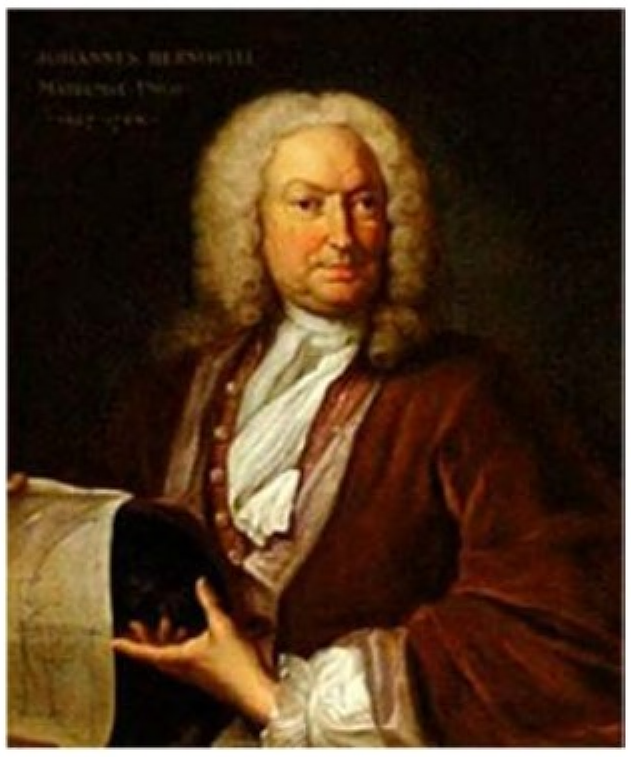

Figure 8.6 - Johann Bernoulli

Work = force x displacement

= pressure $\times$ area of crosssection $\times$ displacement

= pressure $\times$ volume

Work for a unit volume = pressure

Consider the streamline flow through a water pipe as shown in Figure 8.7. Let p_1 and p_2 be the pressures at two places where the areas of cross section A_1 and A_2 respectively.

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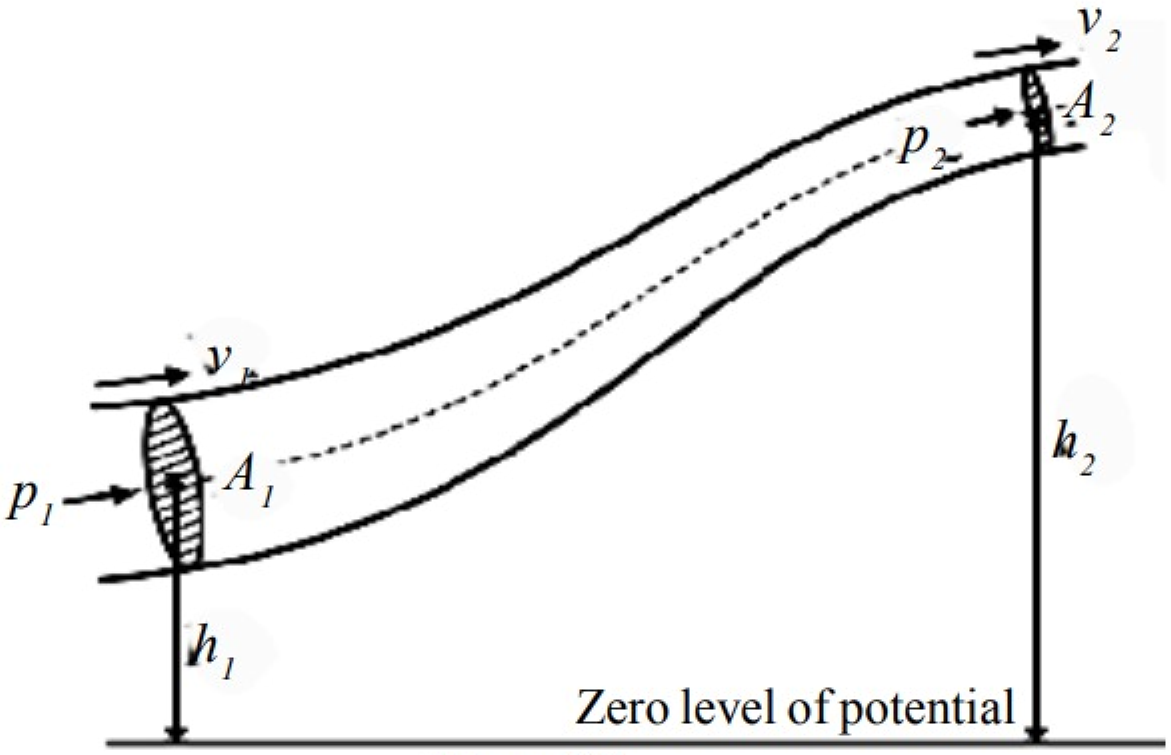

Figure 8.7

Work done by pressure when a unit volume of fluid enters through the cross- section A_1 $\} = p_1$

Work done by pressure when a unit volume of fluid enters through the cross – section A_2 $\} = p_2$

Hence, the useful work done by pressure $= p_1 - p_2$

Potential energy $= mgh$

Also since the mass per unit volume of the fluid is the density ρ,

Increase of potential energy per unit volume $= \rho gh_2 - \rho gh_1$

The kinetic energy is $\frac{1}{2}mv^2$. Hence, the kinetic energy per unit volume is $\frac{1}{2}\rho v^2$.

Thus, the increase of kinetic energy per unit volume $= \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$

Work done by the pressure = Increase potential energy + Increase of kinetic energy.

$$p_1 - p_2 = (\rho gh_2 - \rho gh_1) + \left(\tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2\right)$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$p + \frac{1}{2}\rho v^2 + \rho gh = K \quad (K \text{ is a constant})$$

In accordance with the above equation, Bernoulli's principle can be stated as follows.

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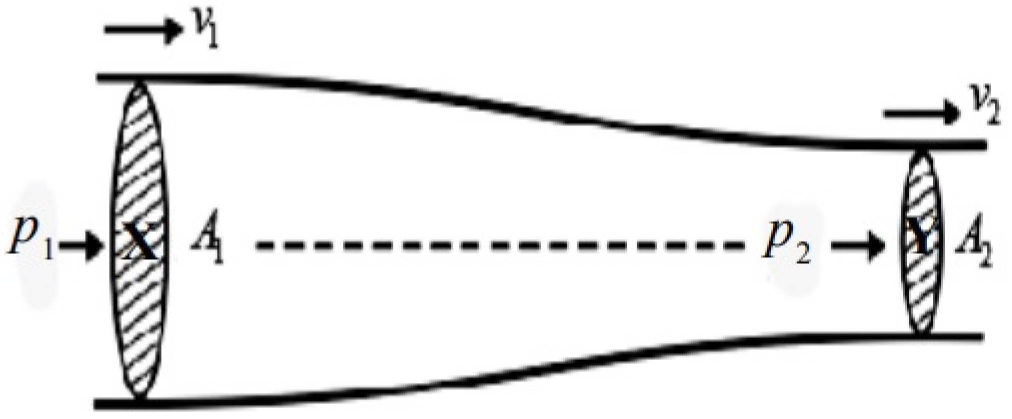
On any point along the same streamline of a steady flow of an incompressible fluid of negligible viscosity, the sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume is a constant.

Consider two places of different cross sections of same potential level along a streamline Applying Bernoulli's principle to points X and Y along the same streamline,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since the potential energies are equal,



$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

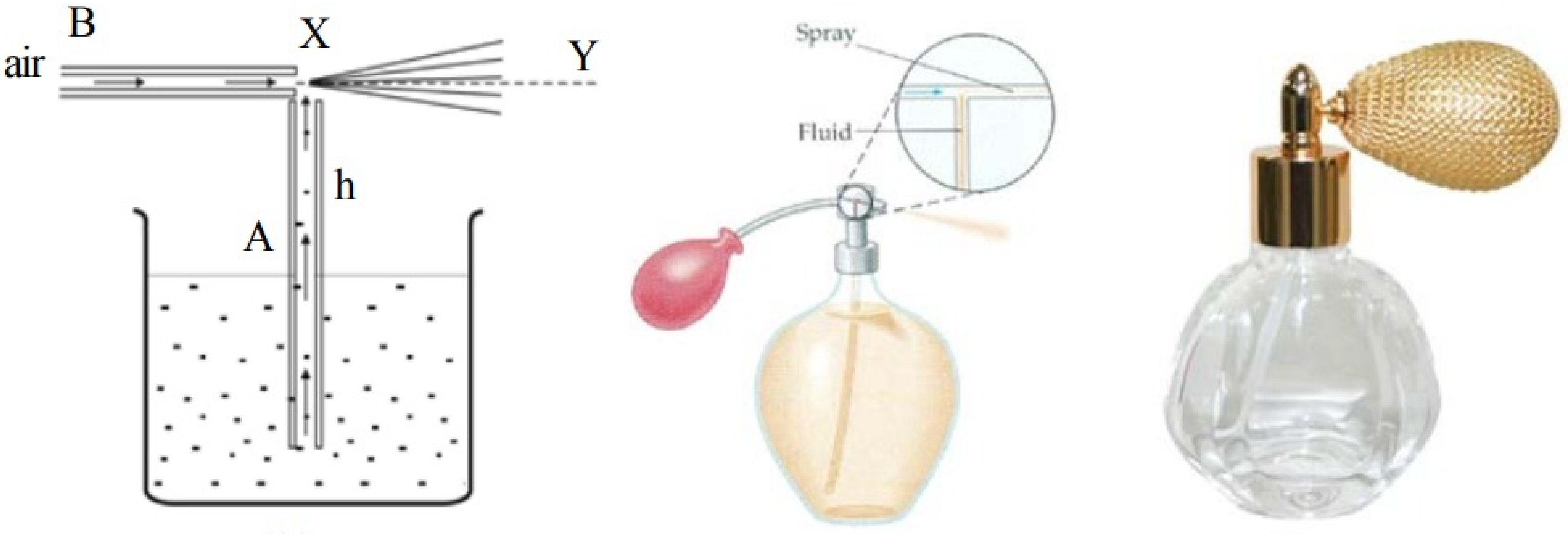
$$p_1 - p_2 = \frac{1}{2}\rho\,(v_2^2\text{-}v_1^2)$$

Figure 8.8

We know from the equation of continuity that when the cross – section gets narrow the velocity of flow increases. According to the above equation it is clear that when the speed increases the pressure decreases. There are many occasions in which this result is utilized technologically. The spray pumps, the shaping of the air plane wing, changing the direction of a moving ball by spinning it are some examples.

Spray tube

As shown in Figure 8.9 (a) a tube A is placed vertically in a vessel containing water. When air is blown over A by another end B, water rises in A to mix with the air and get sprayed. This is the action of a spray pump. Figure 8.9(b) and Figure 8.9(c) shows spray pumps used for various purposes.



(a) (b) (c)

Figure 8.9

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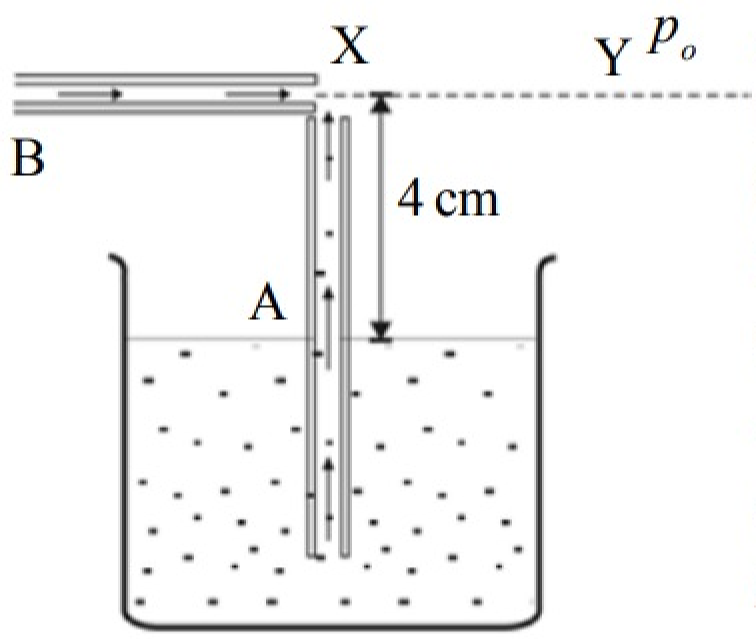
In the set up shown in 8.9 (a) the reason for water rising in it is the lowering of pressure at its upper end below atmospheric pressure. This is due to the high velocity of air which is blown above A across X. By applying Bernoulli principle form X to Y (which are in the same streamline) a relationship between the rise of the water in the tube and the velocity of blow of air can be obtained. In chemical spray pumps, paint sprayers and in service stations the action of the spray pump is made use of.

Worked example



The vessel shown in the figure 8.10 contains a liquid of density1000 kg m$^{-3.}$ Find the velocity with which air should be blown through tube B in order to make the liquid in the tube A to rise by 4 cm to the top of tube A and spray (the density of air can be taken as 1kg m^{-3}) it.

Solution :-

Consider two points X and Y along a streamline in the same horizontal plane. Let p be the pressure at X and p_o the pressure at Y. As v is the velocity of air at X and zero is the velocity at Y,

$$p+\frac{1}{2}\rho v^2 = p_0 + 0 \Rightarrow p_0 - p = \frac{1}{2}\rho v^2$$, where ρ is the density of the air.

Since the liquid in A is rising due to the pressure difference of p and p_0

$p_0 - p = hdg$,where d is the density of liquid.

$$\therefore hdg = \frac{1}{2}\rho v^2$$

$$v^2 = \frac{2hdg}{\rho} = \frac{2\times 4\times 10^{-2}\times 10^3\times 10}{1}$$

$$v^2 = 800 = \underline{20\sqrt{2}\ \text{m s}^{-1}}$$

$$\therefore v = \underline{\underline{20\sqrt{2}\ \text{m s}^{-1}}}$$

Changing of the direction of motion of a ball by spinning

Figure 8.10 can be used to explain how the direction of motion of a projected ball in a game of cricket can be changed by spinning it.

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Consider a ball which is spinning clockwise and projected towards the right with a velocity v. The air passes with a velocity v towards the left relative to the ball. The tangential velocity of the rotating ball causes the velocity of air to decrease at point A and to increase at point

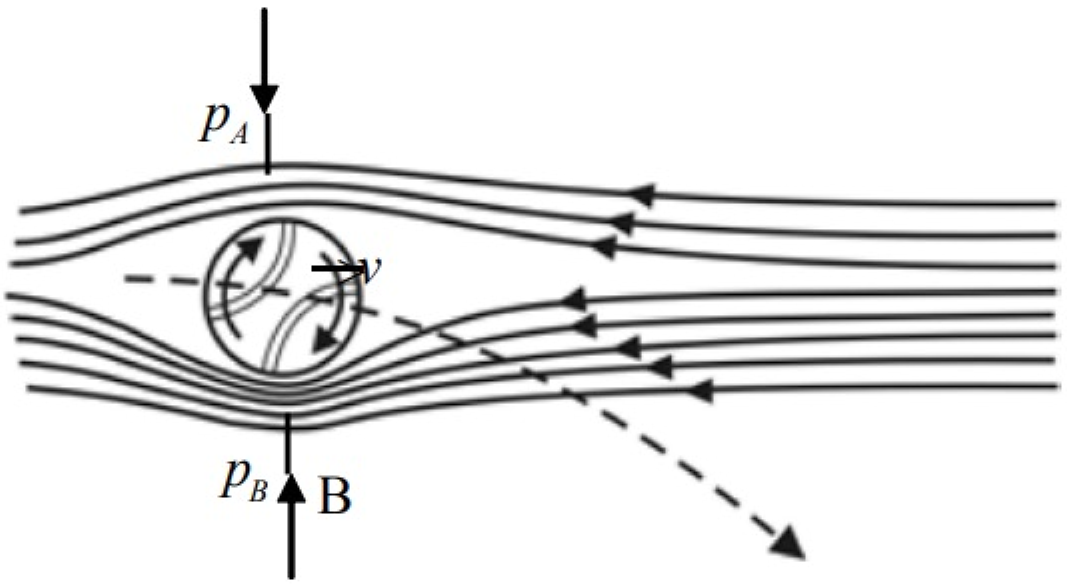

Figure 8.10

$$\therefore v_A < v_B$$

Hence according to Bernoulli's principal , it can be shown that $p_A > p_B$. As a result, when the ball moves forward it gets deviated towards the low pressure region there by setting the ball along a curved path.

Construction of the wing of an airplane

The shape of the cross- section of the wing of an airplane is so constructed that the air streams passing across the wing flow as closer streamlines above the wing, but it is not so with the streams passing below the wing.

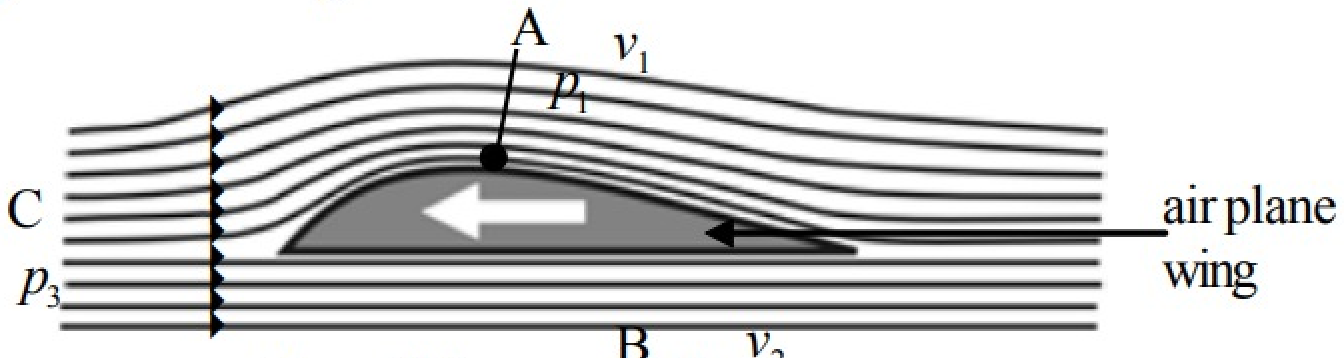

Figure 8.11

Therefore ,the speed of the air flow at A above the wing is greater than that at B. Thus the pressure at A becomes less than the pressure at B and the excess pressure $p_2 - p_1$ creates an up thrust on the wing

If A is the surface area of the wings,

The upthrust = $A(p_2 - p_1)$

If the above value exceeds the weight of the plane it will be lifted

By applying Bernoulli's principle between two points such as A and C along the same streamline the required equitation for the above expression can be obtained.

Many other phenomena occur according to Bernoulli's principle. Some examples are,

Roofs of certain closed houses are blown off during gales.

When an express train passes through a station people standing by the side can get pulled towards the train due to an unbalanced force. When a boat is moving fast in water, the fish on either side of the boat experience an unbalanced force towards the boat and get pulled towards it. When a wind is blowing a half closed door closes fully.

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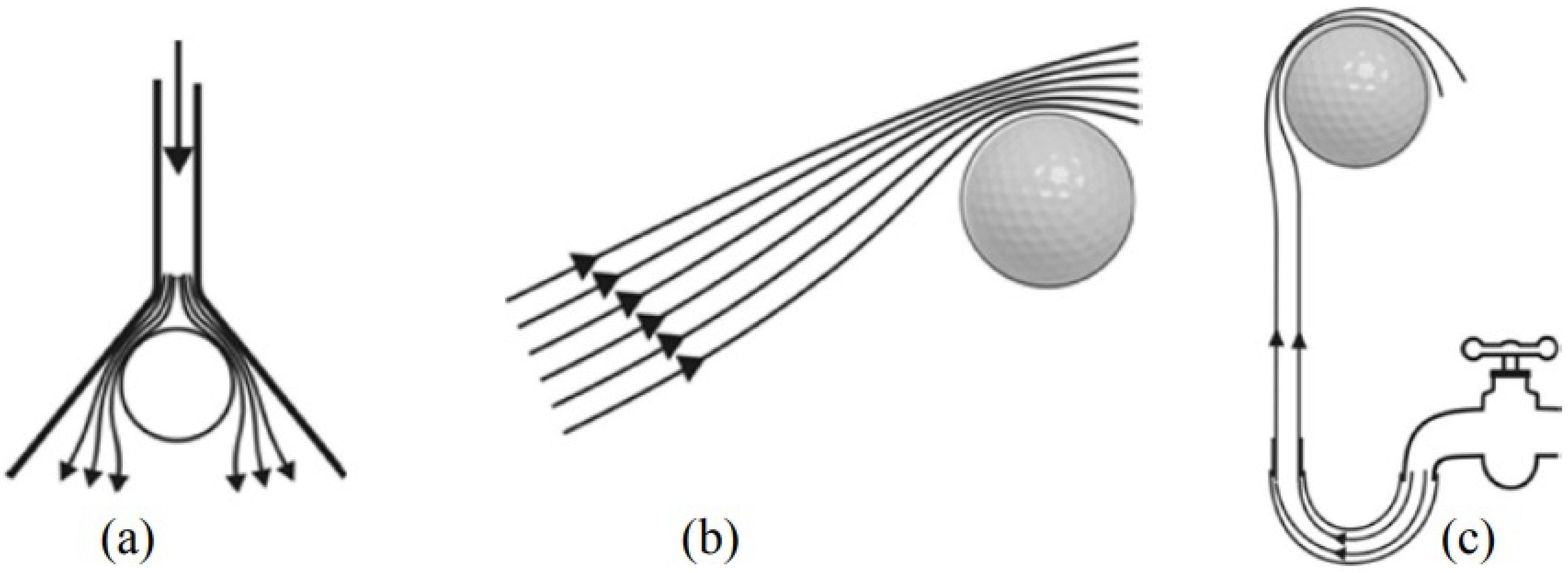

Figure 8.13

Balancing of a balloon by a stream of pressurized air and balancing of ping-pong balls in a stream of water are some items displayed in exhibitions which can be explained by Bernoulli's principle.

Figure 8.12 (a) shows a funnel attached to the end of a tube connected to a water tap. When a ping-pong ball is placed close to the mouth of the inverted funnel through which water is flowing, the ball stays there while rotating

Figure 8.12 (b) shows a balloon placed close to a stream of air coming from a compressor. The stream gets thinner above the ball and the upward force created would bear the weight of the balloon.

Figure 8.12 (c) shows a vertical stream of water formed in a thin tube and a ping- pong ball being kept turning there. In the same way it can be explained the way of air circulation in mounds built by termites.

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