G.C.E. (Advanced Level)

Physics

Grade 13
Resource Book
Unit 10

Mechanical Properties of Matter

Department of Science
Faculty of Science and Technology
National Institute of Education
Maharagama
www.nie.lk

© 2020 National Institute of Education. All Rights Reserved.

G.C.E. (Advanced Level)
Physics
Grade 13
Resource Book
ww
Mechanical Properties of Matter

1st Printing - 2021

Department of Science
Faculty of Science and Technology
National Institute of Education
Maharagama
www.nie.lk

Printed by : Press
National Institute of Education
Maharagama

© 2020 National Institute of Education. All Rights Reserved.

Message from the Director General

The National Institute of Education takes opportune steps from time to time for the development of quality in education. Preparation of supplementary resource books for respective subjects is one such initiative.

Supplementary resource books have been composed by a team of curriculum developers of the National Institute of Education, subject experts from the national universities and experienced teachers from the school system. These resource books have been written so that in line with the G. C. E. (A/L) new syllabus implemented in 2017, so that students can broaden their understanding of the subject matter by referring to these books and teachers can refer to them in order to plan more effective learning teaching activities.

I wish to express my sincere gratitude to the staff members of the National Institute of Education and external subject experts who made their academic contribution to make this material available to you.

Dr. Sunil Jayantha Nawarathna
Director General
National Institute of Education
Maharagama.

© 2020 National Institute of Education. All Rights Reserved.

Message from the Director

Since 2017, a rationalized curriculum, which is an updated version of the previous curriculum has been in effect for the G.C.E (A/L) in the general education system of Sri Lanka. In this new curriculum cycle, revisions were made in the subject content, mode of delivery and curricular materials of the G.C.E. (A/L) Physics, Chemistry and Biology. Several alterations in the learning teaching sequence were also made. A new Teachers' Guide was introduced in place of the previous Teacher's Instruction Manual. In concurrence with that, certain changes in the learning teaching methodology, evaluation and assessment are expected. The newly introduced Teachers' Guide provides the expected learning outcomes, a guideline for teachers to mould the learning events, assessment and evaluation.

When implementing the previous curricula, the use of internationally recognized standard textbooks published in English was imperative for the Advanced Level science subjects. Due to the contradictions of facts related to the subject matter among different textbooks and inclusion of the content beyond the limits of the local curriculum, the usage of those books was not convenient for both teachers and students. This book comes to you as an attempt to overcome that issue.

As this book is available in Sinhala, Tamil, and English, the book offers students an opportunity to refer to the relevant subject content in their mother tongue as well as in English within the limits of the local curriculum. It also provides both students and teachers a source of reliable information expected by the curriculum instead of varied information gathered from the other sources.

This book authored by subject experts from the universities and experienced subject teachers is presented to you followed by the approval of the Academic Affairs Board and the Council of the National Institute of Education. Thus, it can be recommended as material of a high standard.

Dr. A. D. A. De Silva
Director
Department of Science

Guidance
Mr. K.R. Pathmasiri
Deputy Director General - Faculty of Science and Technology
National Institute of Education

Supervision
Dr. A.D.A. De Silva
Director, Departmnt of Science
National institute of Education

Editing :

P. Malavipathirana - Senior Lecturer, National Institute of Education
Dr. M.L.S. Piyatissa - Assistant Lecturer, National Institute of Education
Miss. R.A. Amarasinghe - Assistant Lecturer,National Institute of Education
Mrs. R.N.N. Weerasinghe - Assistant Lecturer,National Institute of Education

Subject Guidance

Prof. S.R.D. Rosa - Department of Physics, University of Colombo
Prof. L.R.A.K. Bandara - Department of Physics, University of Peradeniya

Writing

W.A.D. Ratnasuriya - Former Chief Project Officer, National Institute of Education

Translation(from Sinhala to English)

D.S. Vithanachchi - Former Chief Project Officer, National Institute of Education

English Language Editing

Dr. (Mrs) C. L. Amarasekara - Consultant, National Institute of Education

Computer Type setting

Mrs. D.K.J.P. Dassanayake - Management Assistant, National Institute of Education

Diagrams

W.A.D. Ratnasuriya - Former Chief Project Officer, National Institute of Education

Jayaruwan Vijayawardana - Computer Graphic Designer (Freelance)

Cover Page

Mrs. R.R.K. Pathirana - Technical Assistant, National Institute of Education

Other Assistance

Mangala Welipitiya - National Institute of Education
Mrs. W.P.P. Weerawardhana - National Institute of Education
Mrs. R.K. Fernando - National Institute of Education
Miss. S.A.S.N. Subasingha - National Institute of Education
Mrs. D. M. Iresha Rangana - National Institute of Education

© 2020 National Institute of Education. All Rights Reserved.

© 2020 National Institute of Education. All Rights Reserved.

Content

© 2020 National Institute of Education. All Rights Reserved.

© 2020 National Institute of Education. All Rights Reserved.

© 2020 National Institute of Education. All Rights Reserved.

Chapter One

Elasticity

1.1 Introduction

In the study of the elasticity of materials, let us first consider a few phenomena which we have witnessed in our day-to-day life and also which can be tested (Figure 1.1).

Figure 1.1

A rubber belt or a helical spring attached to a stand, when pulled down from the lower end and released, returns to its original state. A ping pong ball dropped on to the floor from above is seen to bounce. But a lump of clay dropped on to the ground will stick on to the ground changing its shape. These are happenings well known to us. Let us search for these differences in behaviour of different materials under external influences.

In this unit, the behaviour of mechanical properties of materials under the action of external forces will be considered. Strength, hardness, ductility and stiffness are four important mechanical properties of matter. These are of special importance to engineers when selecting materials for a specific activity.

1.2 Elastic and inelastic substances

When a certain material is acted upon by external forces, up to a certain limit, its shape changes and it is said to have undergone distortion. If it returns to its original shape when the applied force is removed then the material is said to be an elastic material. If it does not return to its original shape when the force is removed, then the material is said to be inelastic. Stresses occurring against distortions in elastic materials are responsible for converting the materials back into the original states.

There are three ways of distorting a material. They are one-dimensional distortion, two-dimensional distortion and three dimensional distortion (Figure 1.2).

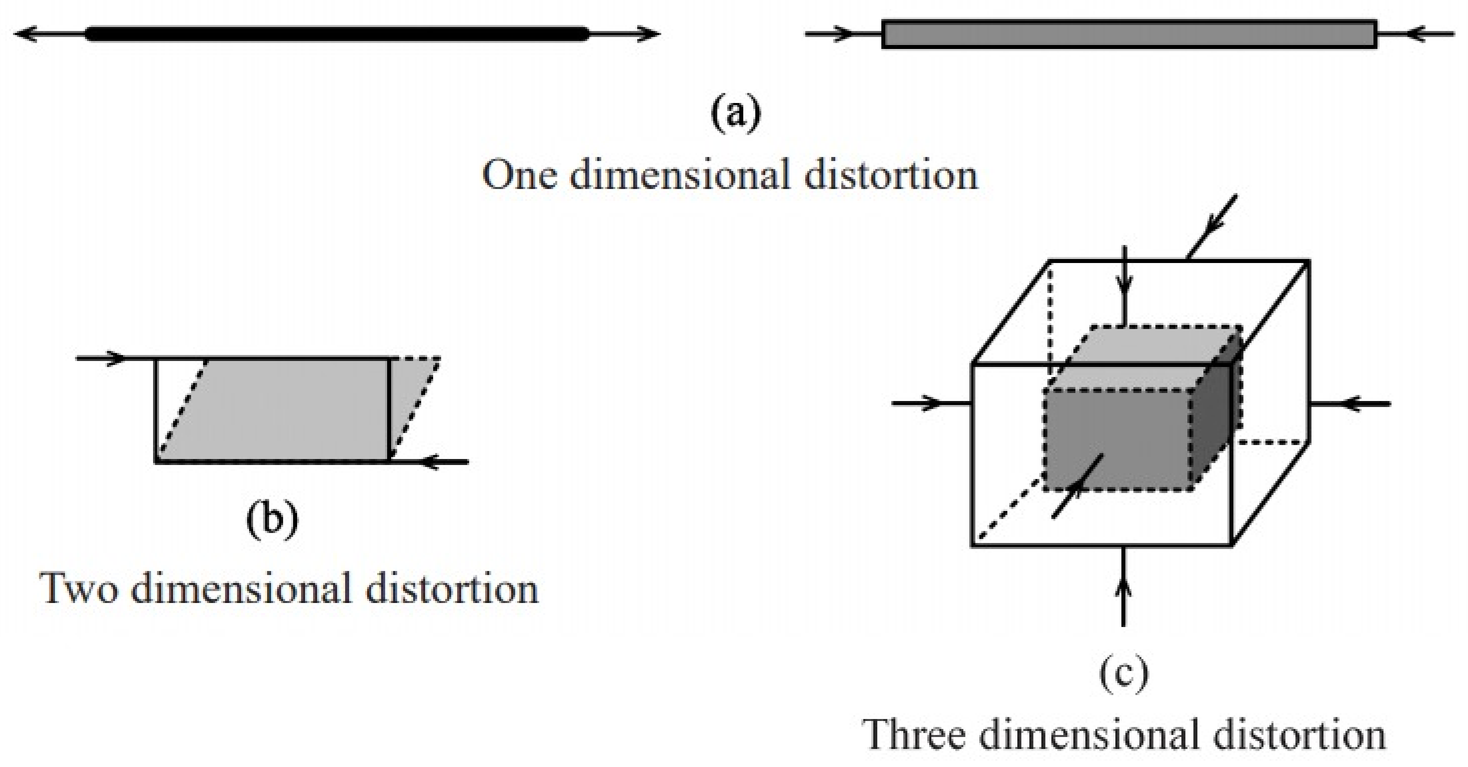


Figure 1.2

A wire or a rod can be subjected to a one dimensional distortion by applying tensile or compressive forces along the same straight line as shown in Figure 1.2 (a).
By applying tangential forces as shown in Figure 1.2 (b) a change of area can be affected. This is two dimensional distortion.
Volume can be distorted by applying forces as in Figure 1.2 (c). This is three dimensional distortion.

In this unit, we mainly consider one-dimensional distortion. For example, mechanical properties of a material can be investigated and found by subjecting a thin wire or a thin belt of the material to a stretching force and observing its behaviour.

1.3 Graphical illustration of change in extension values through a wire for different weights applied

Using an arrangement as shown in figure 1.3 (a), where a thin wire (eg.steel) is hung from a fixed support and by suspending gradually increasing loads from its lower end, the variation of the length increases or extension of the wire with the load, can be studied.

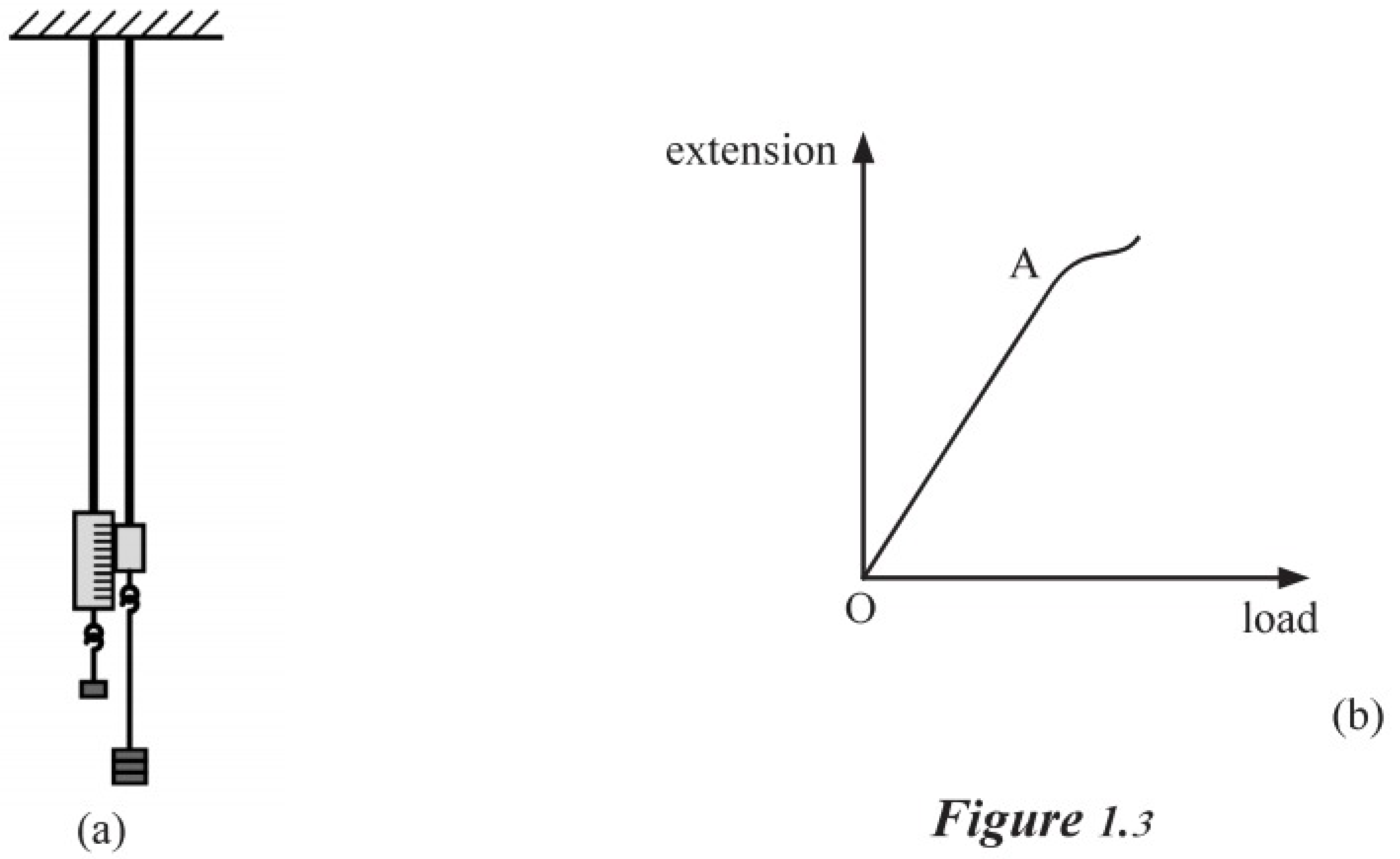


Figure 1.3

© 2020 National Institute of Education. All Rights Reserved.

Figure 1.3 (b) is a graphical illustration of extension against load thus obtained. In the portion OA of the graph, when the load is removed the wire returns to its original length. The relationship between the load and the extension was first found by the British scientist Robert Hooke in 1676.

1.4 Hooke's law

The extension of a wire is directly proportional to the load or the tension of the wire with in the proportional limit.

This means that if 'e' is the extension of a wire when subjected to a tension caused by a force of magnitude F, then,

$$F \propto e, \quad F = ke$$

where k is the constant of proportionality and is called the force constant of the wire. The SI unit of k is N m^{-1}.

The extension occurs when a force is applied to the strip or wire or any material with elastic property.

The extension thus occurring in a wire depends on a few factors such as,

(i) The material of the wire
(ii) The force acting on the wire
(iii) The area of cross-section of the wire
(iv) The original length of the wire

The force acting on the given elastic material and the extension occurs due to the stress on it, referred to as tensile stress and tensile strain repectively. A rod can be subjected to compressive forces, the relevant terms are called compressive stress and compressive strain.

1.5 Modulus of elasticity

In the earlier experiment, up to a certain limit of the load, the wire returns to its original length when the load is removed. This limit is known as the elastic limit. Within this elastic limit, the tensile stress is directly proportional to the tensile strain.

$$\text{Stress} \propto \text{strain}$$

$$\text{Stress} = E.\text{strain}$$

E is known as the modulus of elasticity. In the distortions under tensile and compressive forces, E is referred to as Young's modulus (Y), under sheer stresses and strains, E is the rigidity modulus (n) while under volume distortions, E is the bulk modulus (k).

1.6 Tensile stress, tensile strain and Young's modulus

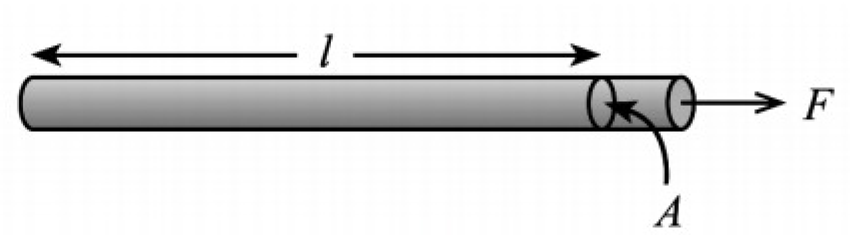


Figure 1.4

Consider a wire of initial length l and area of cross-section A being clamped at one end and subjected to a tensile force F at the other end causing an extension e in the wire (Figure 1.4).

The tensile force acting on a unit area of cross section of the wire is known as the tensile stress.

$$\text{Tensile stress} = \frac{\text{Tensile force}}{\text{Area of cross - section}}$$

$$= \frac{F}{A}$$

$$\text{Unit of tensile stress} = \frac{\text{N}}{\text{m}^2} = \text{N m}^{-2}\ \text{(Pa)}$$

The extension per unit length of the wire is known as the tensile strain.

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Initial length}}$$

$$= \frac{e}{l}$$

Since the strain is a ratio of similar measurements, it has no units.

The ratio of tensile stress to tensile strain is called the Young's modulus of the material of the wire.

$$\text{Young's modulus} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

Symbol of Young's modulus is Y

$$Y = \frac{F/A}{e/l} \qquad \text{———} \quad (1.1)$$

$$\text{Units of Young's modulus} = \text{N m}^{-2}\ \text{(Pa)}$$

$$\text{Dimensions of Young's modulus} = \frac{\text{M L T}^{-2}}{\text{L}^2}$$

$$= \text{M L}^{-1}\ \text{T}^{-2}$$

© 2020 National Institute of Education. All Rights Reserved.

Table 1.1: Young's modulus of a few materials at 20 ºC

Table 1.1

Material	Young's modulus ($\times 10^{11}$ N m^{-2})
Steel	2.0
Copper	1.2
Brass	0.9
Aluminum	0.7
Glass	0.5

The equation 1.1 above based on the definition of Young's modulus can be considered as another way of stating Hooke's law.

It can be written as $F = \left(\frac{A\,y}{l}\right) e$

as $F = k\,e$; $k = \frac{A\,y}{l}$ k is known as the force constant.

1.7 Graph of stress against strain

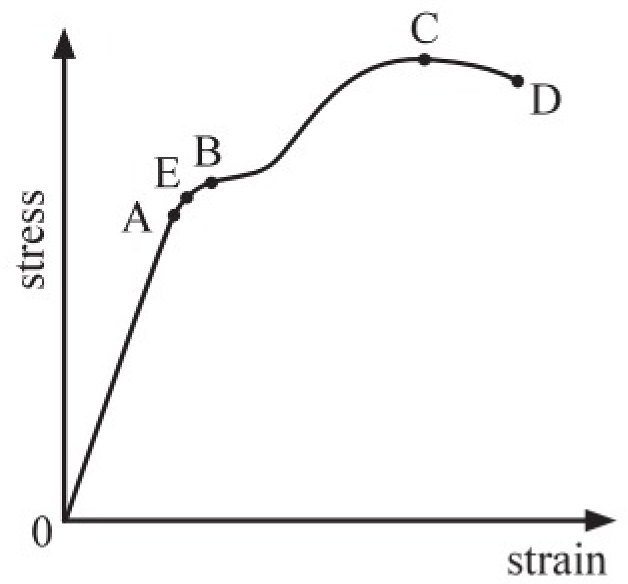


Figure 1.5

A = Proportional limit
E = Elastic limit
B = Yield point
BC = Plastic deformation
C = Breaking stress
OE = Elastic deformation
D = Breaking point

When the stress in the material in the form of a wire is plotted against strain a graph as shown in figure 1.5 will be obtained. The graph consists of a straight line portion OA and a curved portion AEBCD. Straight line OA represents a proportional relationship between the stress and the strain. A is known as the elastic limit. From A to E the stress is not proportional to the strain. However, in the region from O to E in the curve when the stress is gradually decreasing the strain too decreases along the curve itself. When the load is completely removed the wire attains its exact original length. In this region OE, the wire behaves elastically. The point E is known as the elastic limit. In the region OE, wire is supposed to undergo an elastic deformation with the introduction of the load. Beyond the elastic limit E, exists the yield point. Between these two points, the extension occurs according to the force applied due to the elastic property in the wire as well as due to its plastic property. This plastic property is a result of the sliding of the internal molecule layers of the wire between each other. Beyond the yield point the extension in the wire is mainly due to this plastic property. Thus in this portion of the graph, a low increase in tension or the load shows a large change in the extension.

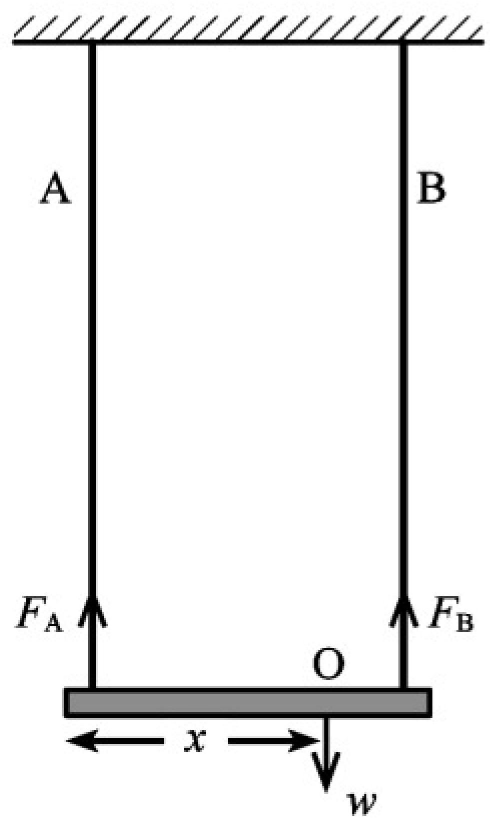
Accordingly, a plastic deformation is said to occur in the wire in the portion of the graph beyond the yield point, when a load exceeding that relevant to the yield point is applied to the wire and is then removed, a small permanent extension would remain in the wire and the load when gradually decreased would not follow the path of the graph as it happened within the elastic limit. When the load applied to the wire is gradually increased this graph indicates the maximum load it can hold at point C. This point is referred to as the breaking load and there after the wire gets thinner and breaks at a point such as D. The point D is called the breaking point.

Solved exercise:

(1) A light rod 100 cm long is suspended by two wires A and B of equal lengths as shown. The area of cross-section of A is 1 mm^2 while that of B is 2 mm^2. At what point on the rod should a load w be hung in order to make equal strains on both A and B?

Young's Modulus of wire A = 2.0×10^{11} N m^{-2}

Young's Modulus of wire B = 1.6×10^{11} N m^{-2}



Solution:

Let F_A and F_B be the tensions in the two wires, A and B respectively. X is the distance between the end where wire A is attached and the point (O) where the load w is hung on the rod.

If S is the strain in the two wires,

For wire A, $\dfrac{F_A}{1 \times 10^{-6}} = 2.0 \times 10^{11} \times s$ ——— (1)

For wire B, $\dfrac{F_B}{2 \times 10^{-6}} = 1.6 \times 10^{11} \times s$ ——— (2)

$$\frac{(1)}{(2)} \rightarrow \frac{2F_A}{F_B} = \frac{2.0}{1.6}$$

$$\therefore \frac{F_A}{F_B} = \frac{5}{8}$$

Taking moments about O,

$$F_A \times x = F_B \times (100 - x)$$

$$\therefore \quad \frac{F_A}{F_B} = \frac{(100 - x)}{x} = \frac{5}{8}$$

$$\therefore \quad x = \underline{\underline{61.5}}$$

Length x is 61.5 cm

(2) A cylindrical copper wire and a cylindrical steel wire of 1.5 m in length and 2 mm in diameter are connected to each other at one end to form a composite wire of length 3 m. Loads are hung from it until the length of the wire becomes 3.003 m. Calculate the strains in the copper and steel wires and the force applied to the wire.

Young's modulus of copper $= 1.2 \times 10^{11}$ N m^{-2}

Young's modulus of steel $= 2.0 \times 10^{11}$ N m^{-2}

Assume π as 3.14

Solution:

Extension $e = \dfrac{l F}{E A}$

where l = Initial length
F = Applied force
E = Young's modulus
and A = Area of cross-section

$$\text{Extension of the copper wire } e_{Cu} = \frac{1.5\, F}{1.2 \times 10^{11} \times 3.14 \times (1 \times 10^{-3})^2}$$

$$= \frac{1.5\, F}{1.2 \times 3.14 \times 10^5}$$

$$\text{Extension of the steel wire } e_{Fe} = \frac{1.5\, F}{2 \times 10^{11} \times 3.14 \times (1 \times 10^{-3})^2}$$

$$= \frac{1.5\, F}{1.2 \times 3.14 \times 10^5}$$

$$\text{Extension of the composite wire} = e_{Cu} + e_{Fe}$$

$$0.003 = \frac{1.5\, F}{1.2 \times 3.14 \times 10^5} + \frac{1.5\, F}{2 \times 3.14 \times 10^5}$$

$$= \frac{1.5\, F}{3.14 \times 10^5}\left(\frac{1}{1.2} + \frac{1}{2}\right)$$

$$F = \frac{0.003 \times 3.14 \times 10^5 \times 1.2 \times 2}{1.5 \times 3.2}$$

$$= \underline{\underline{471 \text{ N}}}$$

© 2020 National Institute of Education. All Rights Reserved.

The magnitue of the force is 471 N.

$$\text{Strain in the copper wire} = \frac{471\ \text{N}}{(1.2 \times 10^{11}\ \text{N m}^{-2}) \times 3.14 \times [(1 \times 10^{-3})^2\ \text{m}^2]}$$

$$= \frac{471}{1.2 \times 3.14 \times 10^5} = \underline{\underline{1.25 \times 10^{-3}}}$$

$$\text{Strain in the steel wire} = \frac{471\ \text{N}}{(2.0 \times 10^{11}\ \text{N m}^{-2}) \times (3.14 \times [1 \times 10^{-3}]^2)\ \text{m}^2}$$

$$= \frac{471}{2 \times 3.14 \times 10^5} = \underline{\underline{0.75 \times 10^{-3}}}$$

1.8 Experimental determination of Young's modulus of a metal wire

A steel wire is used in an experimental set up as shown in Figure 1.6 . Care should be taken regarding the matters stated below in setting up and carrying out the experiment in order to obtain more accurate and successful results.

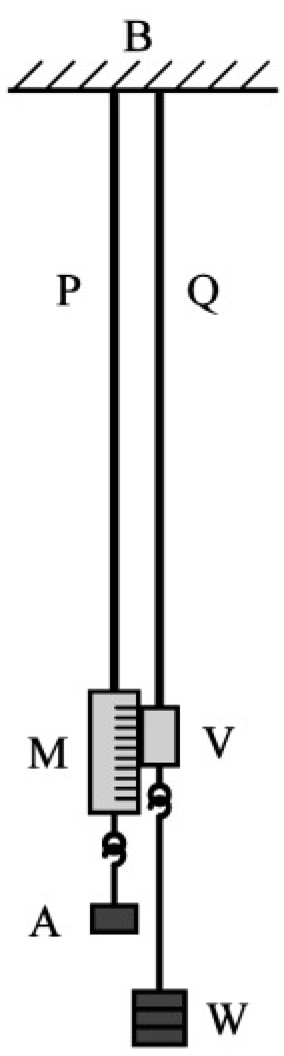


Figure 1.6

(1) When the wire is thin, even a smaller load of a few kilograms would provide a large tensile stress. Hence, by using a long wire, a measurable extension can be obtained.

(2) The following errors are eliminated by using two wires P and Q of the same metal of equal lengths.

i. Error due to dispalcement of the support downloads when the wire Q is being loaded.

ii. Error due to expansion of the wire during changes in temperature.

(3) A load should be hung from the end of the wire to remove any bends in the wire. Extensions cannot be measured accurately when bends exist in the wire.

(4) Since the extension is very small, a venier scale is needed for its measurement. A metre ruler is sufficient to measure the initial length. Comparing with an initial length of about 4 m (4000 mm), the fractional error 1 mm is negligible.

(5) Since the wire is thin, in order to obtain its radius, diameters must be measured in several places using a micrometer screw gauge and the mean value has to be found.

(6) When the loads are removed, the readings for extensions should be found using a vernier scale.

The wires P and Q are hung from a beam B in the ceiling. As shown in the set up in the figure 1.6, a scale *M* marked in mm is attached to the wire P and to keep the wire straight and vertical, a dead load (A) is hung from its lower end. A vernier scale *V* is attached to the wire Q adjacent to the scale *M* and a pan of weight equal to that of the dead load in P is hung from the lower end of Q. By placing loads of 0.5 kg each to this pan the tension in Q can be varied. Both scale *V* as well as scale *M* must be touching each other as shown in Figure 1.6.

When adding loads, obtain the vernier readings corresponding to the load. Obtain readings while unloading too and enter on the Table 1.2.

Measure the initial length '*l*' of the wire using a metre ruler and note it down. Using a micrometer screw gauge, measure two perpendicular diameters of the wire at three different places and note down the readings. Using these readings calculate the mean diameter of the wire and hence calculate the area of cross-section *A* of the wire.

Table 1.2

Load *m* (kg)	**Vernier reading**		**Extension *e* (m)**
	When adding loads	**When removing loads**	

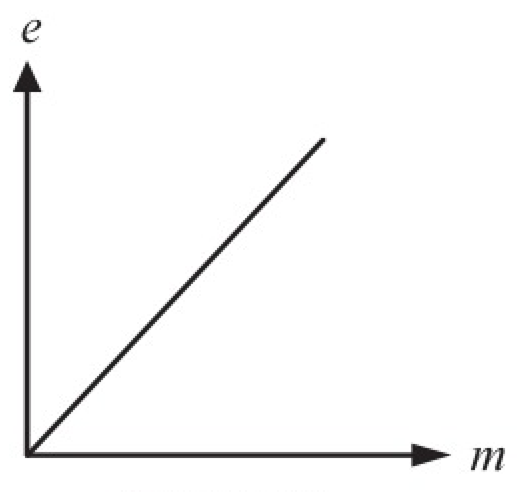


Figure 1.7

Let l be the initial length of the wire, A be the area of cross-section and e be the extension corresponding to a load of mass m. If Y is Young's modulus of the wire.

$$Y = \frac{mg/A}{e/l}$$

$$\frac{Ye}{l} = \frac{mg}{A}$$

$$e = \left(\frac{lg}{YA}\right) m$$

when e is plotted aganist m a graph will be obtained as shown in Figure 1.7.

$$\text{Gradient of the graph} = \frac{lg}{YA}$$

$$\therefore\ Y = \frac{lg}{A \times (\text{gradient})}$$

By substituting values for l, g, A and also the gradient the value of Y can be calculated.

[Details of this experiment are included in the G.C.E. (Advanced Level) Practical Physics Handbook prepared by the National Institute of Education(for the syllabus effective from 2017)]

1.9 Energy stored in an extended wire

Consider the work done in extending a wire within the elastic limit. Let 'e' be the extension in a thin wire clamped at one end, and a force F is applied to the other. If the elastic limit is not exceeded, the extension is directly proportional to the applied force (Figure 1.8).

The applied force has increased from zero to F.

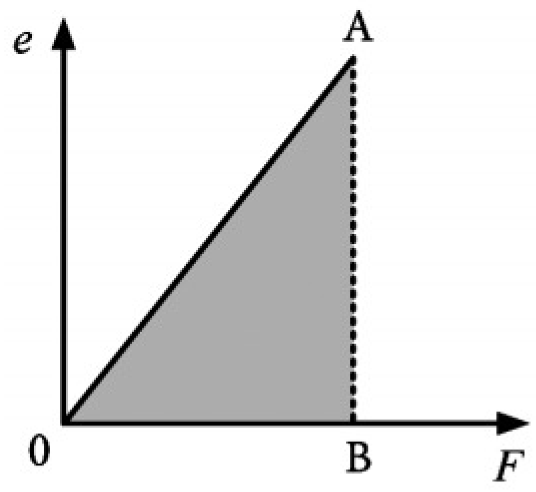


Figure 1.8

$$\text{Mean value of the force applied to the wire} = \frac{0+F}{2} = \frac{F}{2}$$

$$\text{Work done during this extension} = \text{mean force} \times \text{extension}$$

$$= \frac{1}{2} F \times e = \frac{1}{2} F e$$

This work gets stored as energy in the wire.

$$\therefore \ \text{Energy stored in the wire} = \frac{1}{2} F e$$

According to the graph of extension against the load (F) in the Figure 1.8,

$$\text{Area under the graph} = \Delta\text{OAB}$$

$$= \frac{1}{2} \text{OB} \times \text{AB} = \frac{1}{2} F e$$

$\therefore$ The energy stored in an extended wire is equal to the area of the graph of extension against the load.

© 2020 National Institute of Education. All Rights Reserved.

1.10 Forces produced in rods and wires clamped at both ends during temperature changes

Figure 1.9

When a rod clamped at both ends is heated, the rod attempts to expand. Since it is clamped at both ends, this expansion is prevented. The result is that the rod offers a thrust force on the clamps. In the absence of one clamp, let the rod expand by an amount '*e*' when the temperature is raised by θ °C. Hence it can be considered that, the thrust force F is built up, due to extension 'e' occurs in the rod [Figure 1.9 (a)].

When a wire clamped at both ends is cooled, the wire attempts to contract. Since the wire is clamped at both ends, it is prevented. The result is that, the wire offers a tensional force on the clamps. In the absence of one of the clamps, let the wire contract by an amount '*e*' when it is cooled by θ °C. Hence it can be considered that the tensional force F is built up extending the wire by an amount '*e*' [(Figure 1.9 (b)].

If Y is Young's modulus of the material of the wire, '*e*' is its linear expansion. A *is* its area of cross section and '*l*' its initial length.

$$Y = \frac{F/A}{e/l}$$

$$F = \frac{YAe}{l}$$

But $$e = \alpha l \theta$$

$$\therefore F = \frac{YA\alpha l\theta}{l}$$

$$F = YA\alpha\theta$$

A similar expression can be derived for the thrust exerted on the rod in which the expansion is prevented.

(3) A uniform iron rod of an initial length 500 mm and diameter 8.0 mm is heated uniformly until it expands by 0.4 mm. It is then rigidly clamped at its two ends and allowed to cool. Since the rod cannot contract, a tension is built up in it. Calculate this tension and the energy stored in it, after it cools to its original temperature.

Young's modulus of the material of the rod $= 1.8 \times 10^{11}$ N m^{-2} and $\pi = 3.14$.

Solution

$$\text{Extension } e_o = 0.4 \text{ mm} = 0.4 \times 10^{-3} \text{ m} = 4 \times 10^{-4} \text{ m}$$

$$\text{Initial length } l_0 = 500 \text{ mm} = 500 \times 10^{-3} \text{ m} = 0.5 \text{ m}$$

$$\text{Area of cross section } A = \pi \times \left(\frac{8.0}{2} \times 10^{-3}\right)^2$$
$$= \pi \times (4 \times 10^{-3})^2 \text{ m}^2$$
$$= 3.14 \times 16 \times 10^{-6} \text{ m}^2$$
$$= 50.24 \times 10^{-6} \text{ m}^2$$
$$= 5.024 \times 10^{-5} \text{ m}^2$$
$$\approx 5.02 \times 10^{-5} \text{ m}^2$$

If T_0 is the tension in the rod after cooling,

$$T_0 = \frac{A\,E}{l_0}\, e_0$$
$$= \frac{(5.02 \times 10^{-5} \text{ m}^2) \times (1.8 \times 10^{11} \text{ N m}^{-2}) \times (4 \times 10^{-4} \text{ m})}{(0.5 \text{ m})}$$
$$= \underline{\underline{7.24 \times 10^3 \text{ N}}}$$

$$\text{Energy stored} = \frac{1}{2} T_0\, e_0$$
$$= \frac{1}{2} \times (7.24 \times 10^3 \text{ N}) \times (4 \times 10^{-4} \text{ m})$$
$$= \underline{\underline{1.45 \text{ J}}}$$

1.11 Applications of elasticity in day-to-day life

The value of Young's modulus depends not on the dimensions of the sample but on the nature of its materials. If the Young's modulus has a high value, the material opposes strongly an elastic distortion and a large stress will be needed to produce a small strain.

In engineering science, Young's modulus is of immense importance. At the beginning, iron was used for the construction of railway bridges. But these bridges were found to collapse in a short period. Hence, attention was focussed on the essential necessity of making trustful energy calculations on the usage of materials thriftily and safely. Young's modulus is one quantity which should be known in this accurate calculation.

© 2020 National Institute of Education. All Rights Reserved.

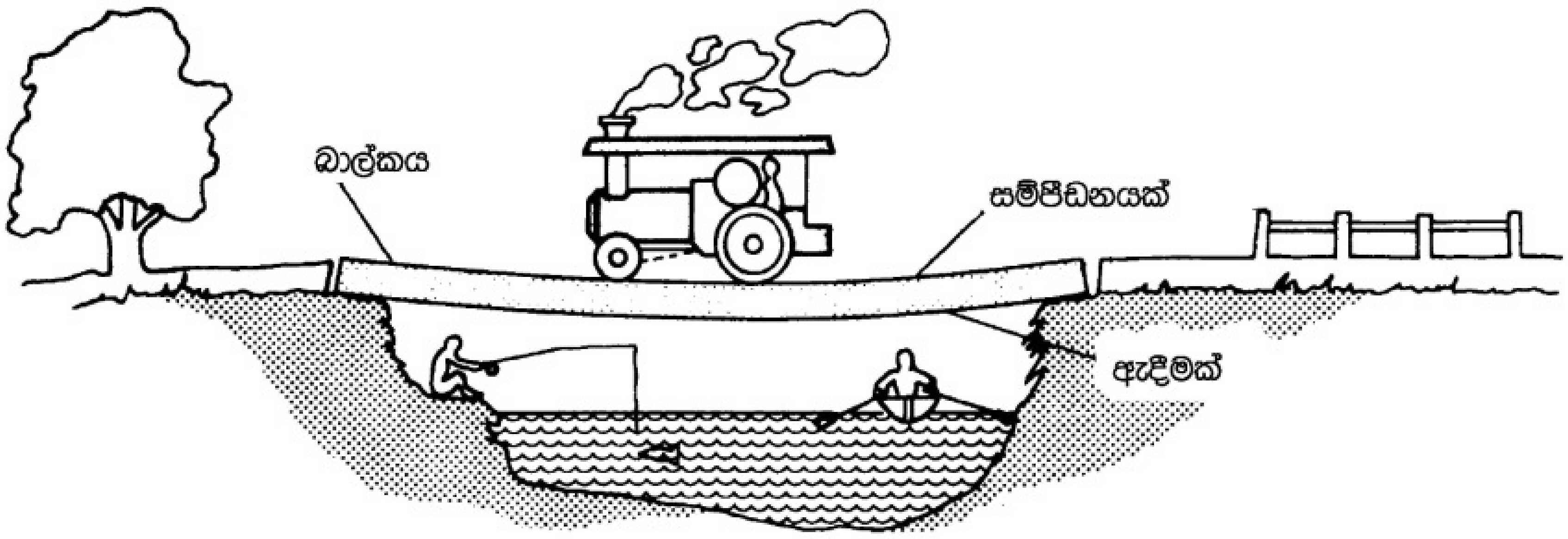


Figure 1.10

According to Figure 1.10, when a beam is subjected to a bending, one surface of the beam gets compressed while its other surface gets stretched. Young's modulus of the beam contributes to this change.

Elasticity becomes extremely useful in building construction work. The pillars and the walls of the lower storey should be strongly built in order to bear the weights of the upper storeys and the roof. Although the compressive stress in bricks is very high, the tensile stress is low. If bricks are inserted in the normal way to a place above a door frame or an arch, there is a possibility of the arch getting damaged under the tensile stresses. Hence a lintel (a concrete beam)is placed above the door frame or the arch and the bricks are kept on top of it. A compressive stress is formed here on the bricks.

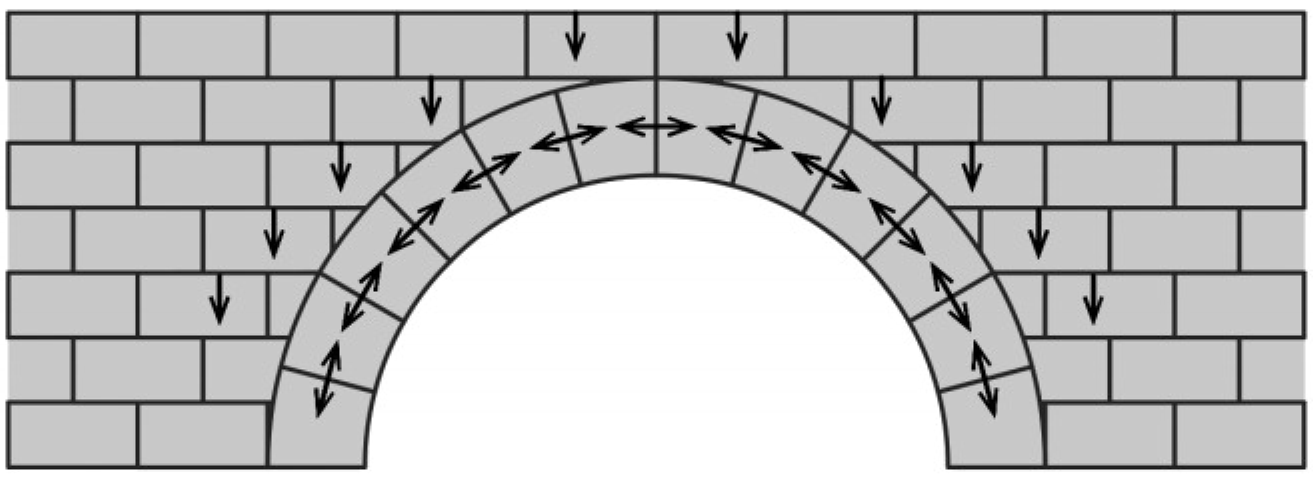


Figure 1.11

Also, as shown in Figure 1.11, when bricks are placed above the arch compressive forces act on the bricks and hence no breaking takes place.

This type of methodology had been used in the past in the construction of bridges and culverts. What is shown in Figure 1.12 is the AI bridge at Mawanella which has been built in this manner. This is built entirely of bricks, concrete and cement have not been used.

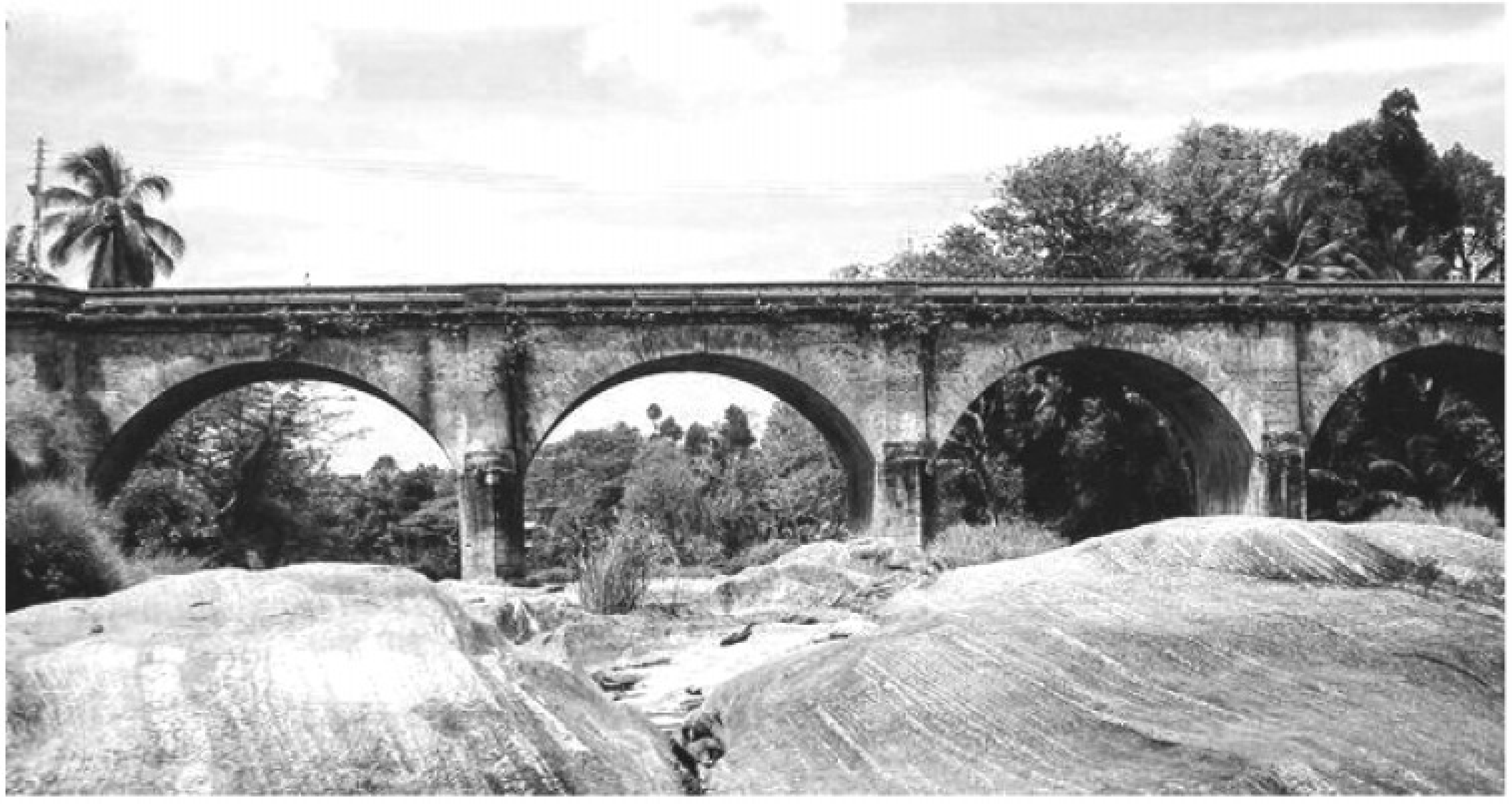


Figure 1.12

A knowledge of Young's modulus and its usage is essential in the preparation of house roofs. Compression forces act on certain beams placed in a roof while tensile forces act on the other beams. Accordingly, depending on the elasticity of the type of wood (e.g. Jak, Teak) used, its length and the area of cross-section is decided.

Exercises

(1) (a) A mass of 0.50 g is suspended from one end of a vertical wire of length 15 m and diameter 0.30 mm. If Young's modulus of the metal of the wire is 1.0×10^{11} Pa, find the extension of the wire.
($g = 10 \text{ m s}^{-2}$)

(b) A steel wire and a phosphor-bronz wire 1.5 m in length and 0.20 m in diameter respectively are connected to each other end to end to form a composite wire of length 3.0 m. What tension in the wire will cause a total extension of 0.064 m ?

Young's modulus of (1) steel $= 2.0 \times 10^{11}$ Pa

(II) Phosphor-bronze $= 1.2 \times 10^{11}$ Pa

(2) (1) Explain the experimental determination of stress, strain and Young's modulus.

From a vertical steel wire of length 350 cm and diameter 0.100 cm is hung a load of 20 kg.

Find (a) the extension in the wire

(b) the energy stored in the wire

Young's modulus of steel 2.00×10^{11} Pa and $g = 10 \text{ m s}^{-2}$

(3) Two vertical wires X and Y are hung from the same horizontal level. The lower ends of the two wires are connected by a light rod XY. Both wires have the same length l and the same area of cross / section A. A load of 30 N is hung from a point O on the wire such that PO : OQ = 1 : 2. Both wires keep stretched and the rod PQ remains horizontal. Young's modulus of the metal of wire X is 1.0×10^{11} Pa. Assuming that both wires have not exceeded the elastic limit calculate Young's modulus of wire Y.

(4) A wire of a uniform area of cross-section 10^{-6} m^2 has its two ends connected to two fixed points situated 1 m apart on the same horizontal level. Initially the wire remains straight and unextended. When a load of 0.5 kg is suspended from the mid point of the wire, the point descends by 10 cm and the load remains in equilibrium. Calculate Young's modulus of the material of the wire.

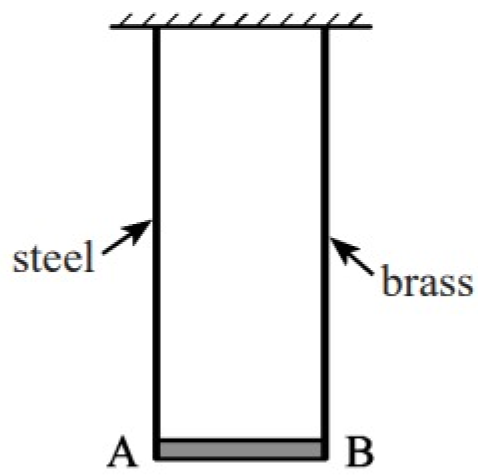
(5) A light rigid rod is suspended horizontally using two vertical wires, one steel and the other brass. Each wire is in length of 2.00 m. The diameter of the steel wire is 0.60 mm and the length of the rod (AB) is 0.20 m. When a load of 10.0 kg is hung from the mid point of AB the rod remains horizontal.

i. What is the extension in each wire?

ii. Calculate the extension of the steel wire and the energy stored in it.

iii. Calculate the diameter of the brass wire.

iv. If instead of the brass wire another brass wire of diameter 1.00 mm is used here, where should the load be hung in order to keep AB horizontal?



Young's modulus of (i) steel $= 2.0 \times 10^{11}$ Pa,
(ii) brass $= 1.0 \times 10^{11}$ Pa

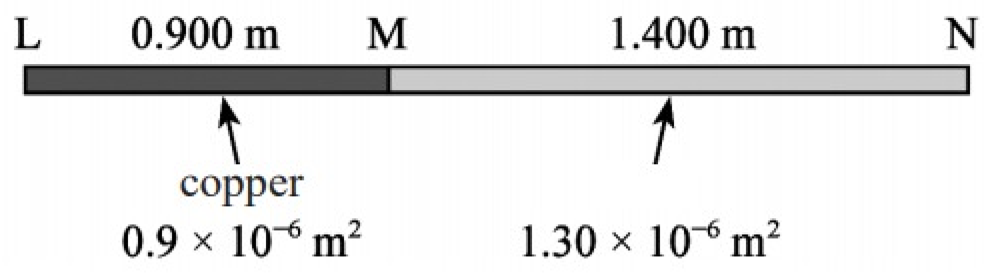
(6) Define tensile stress and Young's modulus.

Explain a laboratory experiment with the help of a clearly labelled diagram to determine Young's modulus of a steel wire.

A copper wire of length 1m and diameter 2 mm is connected at its upper end to a steel wire of identical dimensions and this composite wire is fixed at its upper end. At the lower end a load of 20 kg is hung. Find the extension of the composite wire.

Young's modulus of (i) copper $= 1.2 \times 10^{11}$ Pa
(ii) steel $= 2.0 \times 10^{11}$ Pa

(7) Copper wire LM has its end M fused to the end of steel wire MN as shown. The length of the copper wire is 0.900 m and its area of cross-section is 0.90×10^{-6} m^2 while the length of the steel wire is 1.400 m and has a cross-section area of 1.30×10^{-6} m^2. When the composite wire is stretched, its whole length increases by 0.0100 m.



Find (a) the ratio of the extensions in the two wires

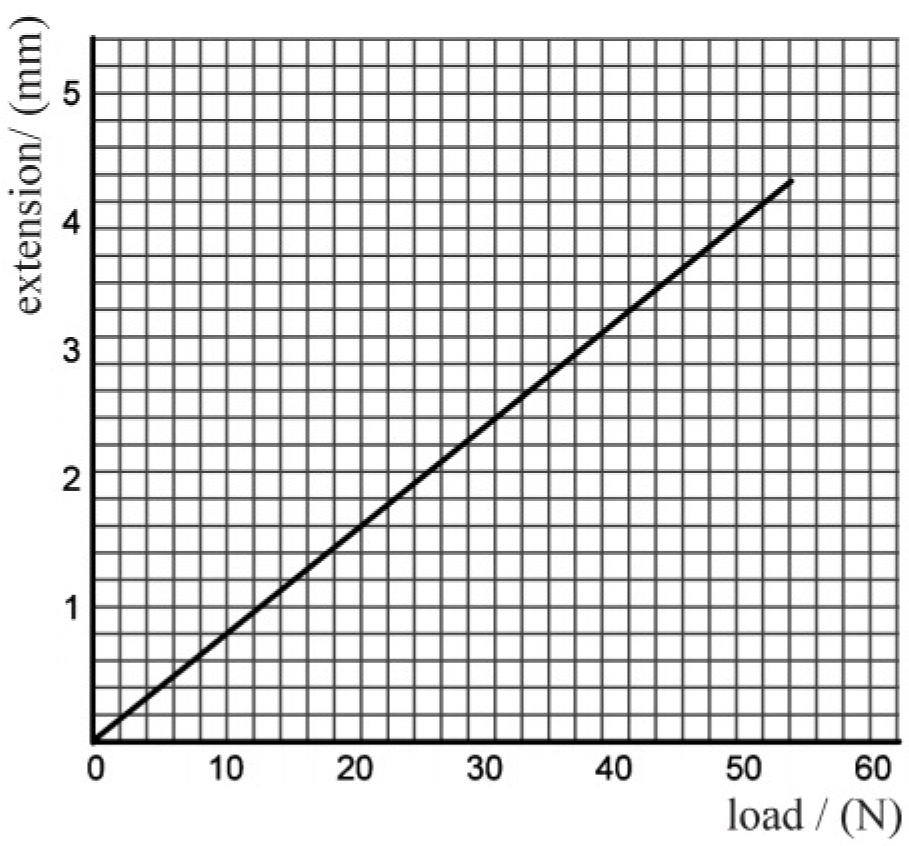
(b) the extension in each wire

(c) the extension applied to the composite wire

Young's modulus of (i) copper $= 1.30 \times 10^{11}$ Pa

(ii) steel $= 2.10 \times 10^{11}$ Pa

(8) The graph below shows the variation of the extension of a wire with the load applied to it.



A wire with the length 3.00 m and diameter 5.0×10^{-4} m is used for the experiment.

i. Calculate the tensile stress due to a load of 50 N.

ii. Find the energy stored in the wire when it is having the load.

iii. Calculate the decrease of the gravitational potential energy of a 5.0 kg mass when it is used as the load.

iv. Explain why the answers of (i) and (ii) above are different from each other.

v. Calculate the value of Young's modulus of the material of the wire.

(9) An iron rod having a circular cross section of diameter 240 mm is heated to a temperature of 600 K. Using a steel frame the rod is then clamped to it to have a gap of a 0.40 m between its ends. The temperature is now lowered to 300 K. Calculate the tension in the rod after the rod cools to 300 K.

Young's modulus of iron $= 2.0 \times 10^{11}$ Pa

Linear expansivity of iron $= 1.2 \times 10^{-5}\ K^{-1}$

Explain the terms stress, strain and elastic modulus.

Derive an expression for the energy stored in a rubber string which obeys Hooke's law, in terms of the tensile force and the extension.

The cross-section area of a rubber belt in a catapult is 1.0 mm^2 while its unstretched length is 10.0 cm. It is stretched to a length of 12.0 cm in order to project a stone of mass 5.0 g. Using concepts of energy, find the velocity of projection of the stone.

Young's modulus of rubber $= 5.0 \times 10^{8}$ Pa

State the assumptions you used in the calculation.

Chapter Two

Viscosity

2.1 Introduction

In the study of viscosity let us consider a few phenomena we observe in our day-to-day life (Figure 2.1).

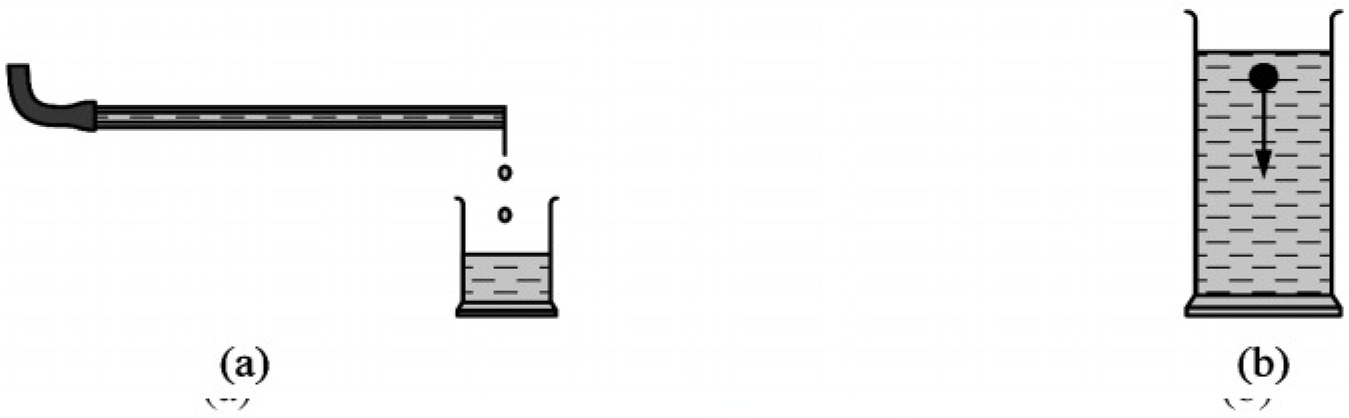


Figure 2.1

The flow of liquids such as coconut oil and glycerine through a narrow tube, appears to be slower than the flow of water through it. Also when a small spherical object such as a ball bearing is allowed to fall through a thick liquid it will be seen that its acceleration is decreasing, while it does not happen so when falling in free space.

This shows that in the flow of a liquid or the motion of an object in a liquid, the liquid layer offers a frictional force against this motion. This property of a liquid is called viscosity and the forces are viscous forces.

2.2 Streamline and turbulent motions

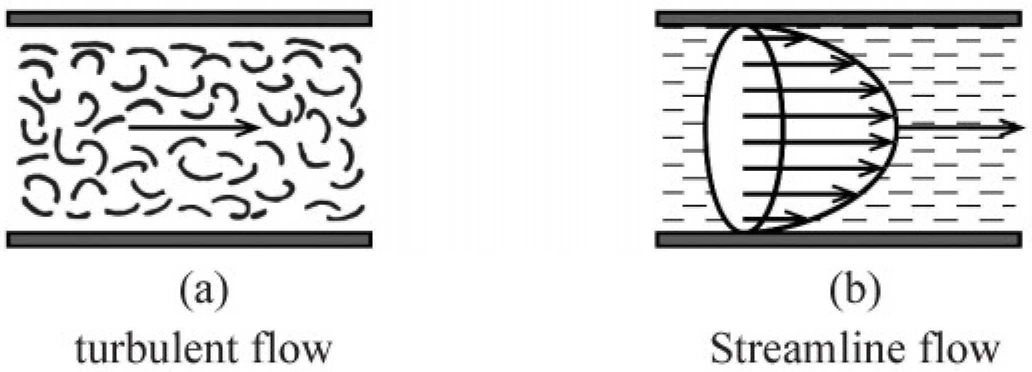


Figure 2.2

When the pressure difference between the ends of a tube exceeds a certain value, the flow becomes disorderly and is known as a turbulent flow [Figure 2.1 (a)]. When the pressure difference between the ends of the tube is not high, a uniform flow of the liquid takes place through the tube. This flow is known as a streamline (uniform, orderly, lamina) flow.

In a uniform or a lamina flow, all liquid particles passing through any given point move along the same path with the same speed. Such a flow can be illustrated by stream lines. Consider the relative motion between the layers of a liquid flowing through a tube. If small pieces of rigifoam are dropped at a certain moment along a line AB (Figure 2.3) on the surface of water flowing uniformly in a semi-circular gutter, after a while it would be observed that those particles arrange themselves along a curve such as ACB.

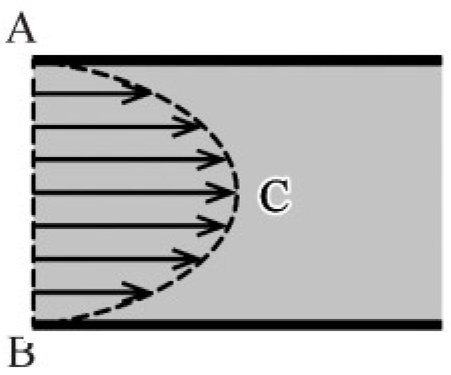


Figure 2.3

The liquid layer in the middle has the highest velocity. The speeds of layers gradually decrease when moving outwards towards the wall of the tube and the layer in contact with the wall can be considered to be at rest.

In this flow, the liquid layers slide on one another and hence frictional forces act against the relative motion of layers. Let us consider the factors on which these frictional forces depend on.

The frictional force between two solid surfaces does not depend upon the common area of the two surfaces and upon the relative velocity between the two surfaces. But the frictional force between two liquid layers depend upon the common area A between the two layers and also upon the relative velocity between the two surfaces.

The change of velocity according to the gap between two layers is called the velocity gradient.

As shown in figure 2.4 if v_1 and v_2 ($v_1 > v_2$) be the velocities of two layers which are at a gap 'd' between them.

$$\text{velocity gradient} = \frac{v_1 - v_2}{d} = \frac{\Delta v}{d}$$

If F is the frictional force between liquid layers,

$$F \propto \frac{\Delta v}{d}$$

$$F \propto A\frac{\Delta v}{d}$$

$$F = \eta A\frac{\Delta v}{d}$$

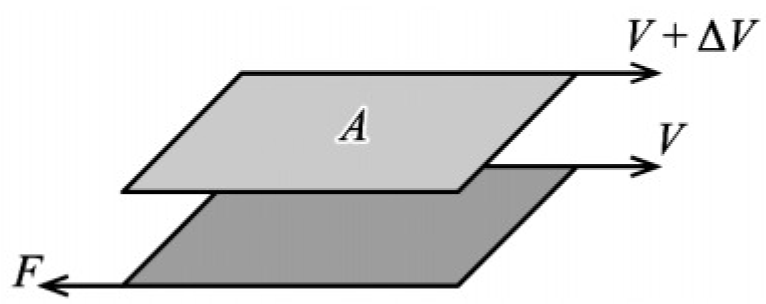


Figure 2.4

where η is a constant it is known as the coefficient of viscosity of the liquid and the above equation is referred to as Newton's equation. The liquids which obey this equation are referred to as Newtonian liquids. Many liquids are Newtonian but there could be liquids such as oil paints and gums which are not Newtonian.

2.3 Definition of the coefficient of viscosity

The coefficient of the viscosity of a fluid is defined as the tangential force on a unit area under a unit velocity gradient of the fluid in uniform stream line motion.

Units and dimensions of η,

$$\eta = \frac{F}{A.\dfrac{\Delta v}{d}}$$

$$\text{Units of } \eta = \frac{\text{N}}{\text{m}^2\,\dfrac{\text{m s}^{-1}}{\text{m}}} = \text{N s m}^{-2}$$

$$\text{Dimensions of } \eta = \frac{\text{M L T}^{-2}}{\text{L}^2\,\text{T}^{-1}} = \text{M L}^{-1}\,\text{T}^{-1}$$

2.4 Poiseuille's equation

Poiseuille's equation provides an expression for the rate of flow of a liquid in uniform flow through a capillary tube.

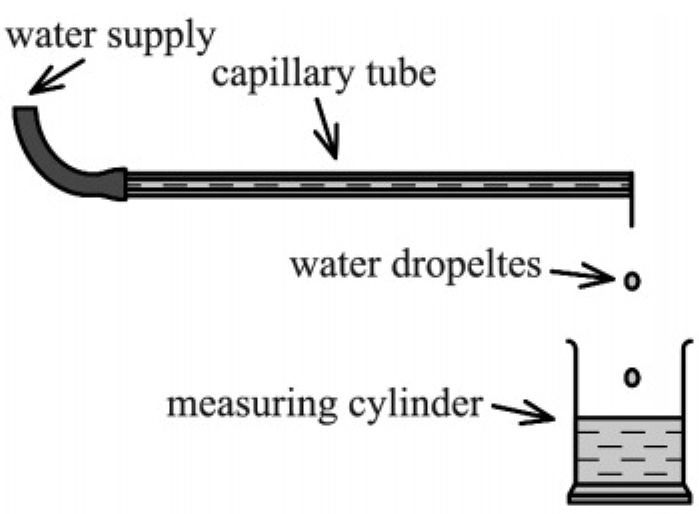


Figure 2.5

If the volume of a liquid is flowing through a tube having radius 'a' and length 'l' in time 't', the pressure difference between the two ends of that tube could be given as,

$\Delta p = p_1 - p_2$

$$\frac{V}{t} = \frac{\pi a^4}{8\,\eta}\left(\frac{\Delta p}{l}\right)$$

This equation is known as Poiseuille's equation after the name of the French scientist Leonard Poiseuille who first investigated this matter in 1944.

Confirmation of Poiseuille's equation dimensionally

1. Dimensions of $\dfrac{V}{t} = \dfrac{\text{L}^3}{\text{T}} = \text{L}^3\,\text{T}^{-1}$

$$\left(\frac{\Delta p}{l}\right) = \left(\frac{\text{M L T}^{-2}}{\text{L}^2}\right)\frac{1}{\text{L}} = \text{M L}^{-2}\,\text{T}^{-2}$$

$$= \text{M L}^{-1}\,\text{T}^{-1}$$

$\therefore$ Dimensions of η

$$\frac{\pi a^4}{8\eta}\left(\frac{\Delta p}{l}\right) = \frac{L^4}{M L^{-1} T^{-1}}$$

$$= L^3 T^{-1}$$

Hence dimensions on the left hand side = dimensions on the right hand side.

$\therefore$ Poiseuille's equation is dimensionally correct.

Conditions for the validity of Poisellie's equation

(a) The flow of the liquid should be a lamina (stream line) flow. For this purpose the liquid should flow under a low pressure difference.

(b) The liquid flow should take place under steady conditions.

(c) The liquid should be incompressible.

(d) The tube should be horizontal and narrow.

Worked example

Two tubes A and B of radius r and $2r$ respectively are connected end to end and a uniform flow of a liquid takes place through this composite tube. Tube B is eight times longer than tube A and the pressure difference between the ends of the composite tube is 9000 N m^{-2}. What is the pressure difference across the tube A?

Solution

Let p_1, p_2 and p_3 be the pressures at the free end of A, at the junction of the two tubes and at the free end of B respectively.

Since the flow of the liquid takes place uniformly through the tube, then by Poiseuille's equation.

$$\frac{V}{t} = \frac{\pi (p_1 - p_2) r^4}{8 \eta l} \quad \text{———} \quad (1)$$

$$= \frac{\pi (p_2 - p_3)(2r)^4}{8 \eta \times 8 l} \quad \text{———} \quad (2)$$

where η is the coefficient of viscosity

(1) = (2)

$$(p_1 - p_2) = 2(p_2 - p_3)$$

© 2020 National Institute of Education. All Rights Reserved.

$$\therefore \quad \frac{p_2 - p_3}{p_1 - p_2} = \frac{1}{2}$$

$$\frac{p_2 - p_3}{p_1 - p_2} + 1 = \frac{1}{2} + 1$$

$$\frac{p_2 - p_3 + p_1 - p_2}{p_1 - p_2} = \frac{1 + 2}{2}$$

$$\frac{p_1 - p_3}{p_1 - p_2} = \frac{3}{2}$$

But $p_1 - p_3 = 9000 \text{ N m}^{-2}$

$$\therefore \quad \frac{9000}{p_1 - p_2} = \frac{3}{2}$$

$$\therefore \quad p_1 - p_2 = 6000 \text{ N m}^{-2}$$

$\therefore$ Pressure difference across tube A $= 6000 \text{ N m}^{-2}$

2.5 Motion of a small spherical body falling freely through a viscous medium

The forces acting initially on the sphere would be its weight W and the upthrust U on it (Figure 2.6).

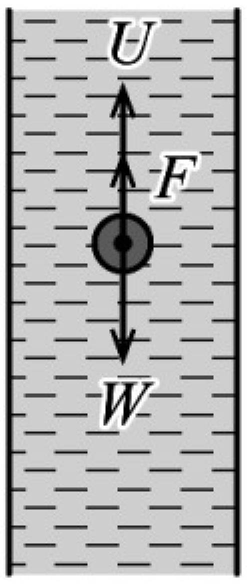


Figure 2.6

If $W > U$, an unbalanced force acts on the sphere and hence falls with an acceleration. When a body moves through a liquid in this manner, the liquid layer close to the body too moves with the speed of the body. The speeds of the layers decrease with increasing distance from the moving body until the layers become still. Due to this variation of velocities there exists a relative motion between the liquid layers. As a result, viscous frictional forces occur between two successive layers.The resultant of these frictional forces act on the moving body in the opposite direction of motion of the body. Hence the acceleration of the body gradually decreases. But, with the increase of the velocity of the body, this viscous frictional force F increases. A certain stage, when F together with the upthrust, $F + U$ equals the weight W. The resultant force on the sphere becomes zero. The sphere now begins to move with a uniform velocity.

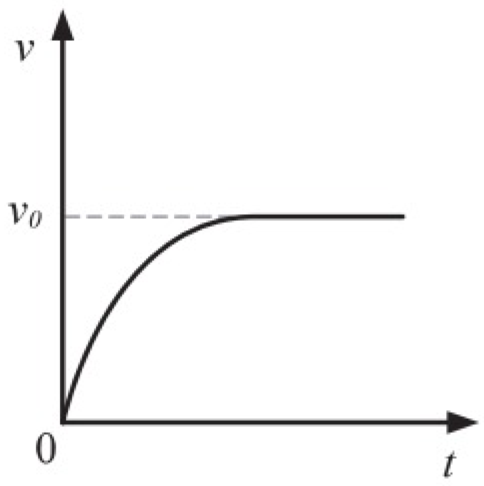


Figure 2.7

The Figure 2.7 illustrates this variation of velocity with time.

The relationship between the viscous frictional force F acting on a small sphere moving freely in a viscous liquid and the terminal velocity it attains is revealed by Stokes' law.

2.5.1 Stokes' law

The british scientist George Stokes (1819 – 1903 A. D), considering a spherical body moving in a liquid presented the relationship between the magnitude of the viscous frictional force 'F' and the terminal velocity 'v' attained by the body by the following equation.

$$F = 6\pi\eta a v$$

where η is the coefficient of viscosity of the liquid, 'a' the radius of the spherical body and 'v' its terminal velocity. This expression is referred to as Stokes' law.

Proof of Stokes' law as dimensionally correct.

$$F = 6\pi \eta a v$$

dimensions of η = $M L^{-1} T^{-1}$
dimensions of a = L
dimensions of v = $L T^{-1}$

dimension of left hand side (L.H.S.) = $M L T^{-2}$

dimensions of right hand side (R.H.S.) = $M L^{-1} T^{-1} \times L \times L T^{-1}$
= $M L T^{-2}$

$\therefore$ dimensions of L.H.S .= dimensions of R.H.S.

Stokes' law is dimensionally correct.

The following conditions should be satisfied by a body released in a vessel when Stokes' law is applied to it.

1. The fluid should be at rest.
2. The fluid should be infinite in extent (The fluid spreads over a large volume relative to the body).
3. If '*a*' is the radius of the sphere and '*R*' the radius of the vessel '*R*' must be substantially bigger than '*a*'.
4. The sphere should be released from the rest.
5. The sphere should be released along the axis of the vessel.
6. The region which is used in the measurement of the terminal velocity should be far from the bottom of the vessel.

2.5.2 Derivation of an expression for the terminal velocity of a small spherical body moving through a viscous liquid

When a small spherical light body (eg. a wax ball) is dropped into a viscous liquid, as the density ρ_1 of the material of the sphere is less than the density σ of the liquid ($\rho_1 < \sigma$), the sphere will move upwards [Figure 2.8 (a)].

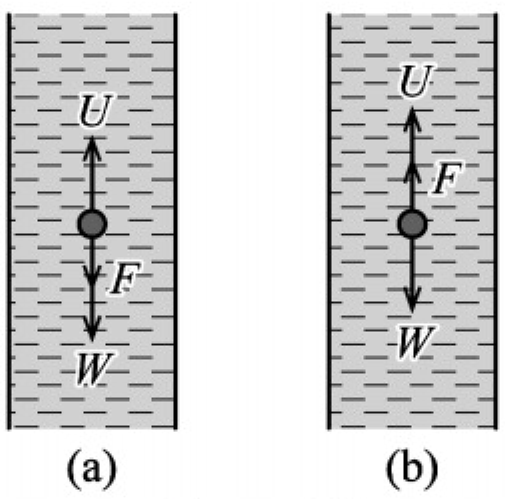


Figure 2.8

When a small spherical heavy body (eg. a bicycle ball bearing) is dropped into the liquid, as the density ρ_2 of the material of the sphere is greater than the density ρ_2 of the liquid ($\rho_2 > \sigma$), the sphere will move downwards [Figure 2.8 (b)].

© 2020 National Institute of Education. All Rights Reserved.

Let W be the weight of the sphere, U the upthrust on the sphere and F the viscous force in it.

(a) For the sphere moving upwards

$$W = \frac{4}{3}\pi a^3 \rho_1 g$$

$$U = \frac{4}{3}\pi a^3 \sigma g$$

If 'a' is the radius of the sphere, η the coefficient of viscosity of the liquid and v_1 the terminal velocity of sphere,

$$F = 6\pi\eta a v_1$$

$$U = F + W$$

$$F = U - W$$

$$6\pi\eta a v_1 = \frac{4}{3}\pi a^3 \sigma g - \frac{4}{3}\pi a^3 \rho_1 g$$

$$= \frac{4}{3}\pi a^3 (\sigma - \rho_1) g$$

$$v_1 = \frac{4}{3}\pi a^3 (\sigma - \rho_1) g \times \frac{1}{6\pi\eta a}$$

$$v_1 = \frac{2}{9}\frac{a^2}{\eta}(\sigma - \rho_1) g$$

(b) For the sphere moving downwards,

$$W = \frac{4}{3}\pi a^3 \rho_2 g$$

$$U = \frac{4}{3}\pi a^3 \sigma g$$

If 'a' is the radius of the sphere η the coefficient of viscosity of the liquid and v_2 the terminal velocity of the spere,

$$F = 6\pi\eta a v_2$$

$$F = 6\pi\eta a v_2$$

$$U + F = W$$

$$F = W - U$$

$$6\pi\eta a v_2 = \frac{4}{3}\pi a^3 \rho_2 g - \frac{4}{3}\pi a^3 \sigma g$$

$$= \frac{4}{3}\pi a^3 (\rho_2 - \sigma) g$$

$$v_2 = \frac{2}{9}\frac{a^2}{\eta}(\rho_2 - \sigma) g$$

Solved example:

Time is measured of a steel ball bearing a diameter of 8.0 mm falling through oil in a uniform velocity. It takes 0.56 s to fall through vertical distance of 0.2 m. Assuming the density of steel is 7800 kg m^{-3} and that of oil is 900 kg m^{-3}, calculate

(a) Weight of the ball

(b) Upthrust on the ball

(c) Viscosity of oil

Solution

(a) weight of a ball $W = \frac{4}{3} \pi a^3 \rho g$

where 'a' is the radius of the ball and ρ is the density of steel

$$\frac{4}{3} \times 3.14 \times [(4 \times 10^{-3})^3 \text{ m}^3] \times (7800 \text{ kg m}^{-3}) \times (10 \text{ m s}^{-2})$$

$$= \underline{\underline{0.02 \text{ N}}}$$

(b) Upthrust on the ball $U = \frac{4}{3} \pi a^3 \sigma g$

where σ is the density of oil

$$= \frac{4}{3} \times 3.14 \times [(4 \times 10^{-3})^3 \text{ m}^3] \times (900 \text{ kg m}^{-3}) \times (10 \text{ m s}^{-2})$$

$$= \frac{4}{3} \times 3.14 \times 64 \times 9 \times 10^{-6} \text{ N}$$

$$= \underline{\underline{2.41 \times 10^{-3} \text{ N}}}$$

(c) Viscus force on the ball $F = 6 \pi \eta a v$

where η is the coefficient of viscosity of oil and v is the termial velocity of the ball.

$$F = 6 \times 3.14 \times \eta \times 4 \times 10^{-3} \times \frac{0.20}{0.56}$$

$$= 2.691 \eta \times 10^{-2}$$

Since the ball bearing attains a uniform velocity

$$F = W - U$$

$$2.691 \eta \times 10^{-2} = 0.02 - 2.41 \times 10^{-3}$$

$$= \frac{0.01759}{2.691 \times 10^{-2}}$$

$$= \underline{\underline{0.65 \text{ N s m}^{-2}}}$$

© 2020 National Institute of Education. All Rights Reserved.

2.6 Ex-comparison of viscosities of different liquids

2.6.1 Using Stokes' law

Consider the following example.

A viscous liquid is placed in a tall vessel (Figure 2.9). Let us allow a small sphere to fall through this liquid and measure the time to fall through the range between two levels using a stop clock. Mark A and B close to the middle of the vessel. Assuming that the sphere has attained the terminal velocity, let us calculate it as v_1.

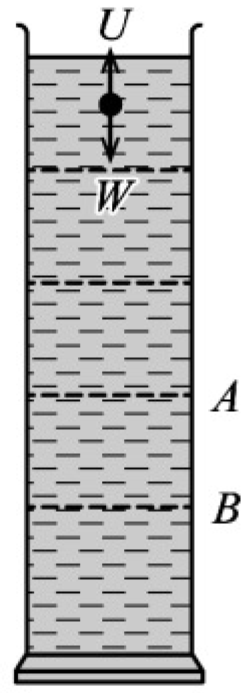


Figure 2.9

Then, if ρ is the density of the sphere, σ_1 the density of the liquid and η_1 its coefficient of viscosity,

according to the expression obtained earlier.

$$v_1 = \frac{2}{9}\frac{a^2}{\eta_1}(\rho - \sigma_1)\,g \qquad \text{---------} \; (1)$$

Next let us fill the vessel with another viscous liquid of density σ_2 and coefficient of viscosity η_2. Repeat the activity with the same sphere and find its terminal velocity v_2.

$$v_2 = \frac{2}{9}\frac{a^2}{\eta_2}(\rho - \sigma_2)\,g \qquad \text{---------} \; (2)$$

(1) ÷ (2) ;Dividing the above equations

$$\frac{\eta_1}{\eta_2} = \frac{(\rho - \sigma_1)}{(\rho - \sigma_2)} \times \frac{v_2}{v_1}$$

Thus it is possible to compare the viscosities of two liquids or find the coefficient of viscosity of one liquid if that of the other is known.

2.7 Application of viscosity

Viscosity of liquids decreases quickly with increasing temperature. Thick liquids like treacle are easy to pour when heated.

Viscous liquids are used to reduce the friction between moving metallic components of machines. Viscosity is one factor which decides whether lubricating oil is suitable for use in machines.

The society of Automotive Engineering has introduced a system of SAE numbers for the standard terminology of oils having different viscosities.

Viscosity increases with increasing SAE numbers.

© 2020 National Institute of Education. All Rights Reserved.

Exercises

(1) (a) Explain what is meant by lamina flow and define the coefficient of viscosity.

(b) A flat sheet of area 0.1 m^2 is placed on another flat surface trapping an oil layer of coefficient of viscosity 1.5 N s m^{-2} and thickness 10^{-5} m between them. Calculate the force required to slide the sheet over the surface at a speed of 1 mm s^{-1}.

(2) A capillary tube of length 20 cm and internal radius 1.0 mm is attached pointing outwards to a point on a side wall of an empty open vessel, close to its base. If water is allowed to flow at a uniform rate of 1.6 cm^3 s^{-1} into the vessel, at what depth will the water level in the vessel stop rising?

(Assume that the flow is uniform. Coefficient of viscosity of water = 1.0×10^{-3} N s m^{-2}, density of water 1.0×10^3 kg m^{-3} and acceleration due to gravity =10 m s^{-2}).

(3) (a) Show that Poiseulle's equation giving the rate of streamline flow of a viscous liquid through a capillary tube is dimensionally correct.

(b) A viscous liquid flows through a capillary tube of radius 4×10^{-4} m and under a pressure gradient of 4×10^{4} N m^{-2} under streamline conditions. If 60 cm^3 of water flows through the tube in 20 minutes. Calculate the coefficient of viscosity of water.

(4) A lamina flow of a viscous liquid is maintained on a horizontal sheet which is at rest. The top layer of the liquid is moving with a constant velocity 'v' while the bottom layer which is at rest is at a depth 'd'.

(i) If η is the coefficient of viscosity of the liquid, write down an expression for the force 'F' that would be applied on a surface area 'A' of the top layer.

(ii) Illustrate by means of a diagram using arrows, the variation of the velocities of the intermediate layers.

(iii) A person pushes a block of mass 0.5 kg on a horizontal surface. When a horizontal force of 0.25 N is applied on the block it attains a constant velocity of 0.01 m s^{-1}. When a thin oil layer is spread on the surface, the force required to push the block by the same velocity of 0.01 m s^{-1} horizontally decreases to 0.05 N. The contact surface area of the block is 100 m^2 and the thickness of the oil layer is 1 mm.

(a) Calculate the coefficient of viscosity of the oil.

(b) Find the coefficient of sliding friction between the block and the surface after spreading the oil layer.

(c) What is the amount of energy that can be retained in one second due to the introduction of the oil layer?

(5) The density of castor oil at 20 °C is 940 kg m^{-3} and the coefficient of viscosity is 2.42 N s m^{-2}. Considering the density of steel 7800 kg m^{-3}, calculate the terminal velocity of a steel ball of radius 2 mm falling under gravity in castor oil.

(6) Stokes' law for a sphere of radius 'a' falling with a velocity 'v' in a fluid expressed by the equation $F = 6\pi\eta a v$ where 'F' is the force acting on the sphere and η is the coefficient of viscosity of the fluid.

(a) Show that this equation is dimensionally correct and explain why this equation is valid only for low velocities.

(b) Explain why a sphere released to fall freely in a fluid falls with a decreasing acceleration until it attains its terminal velocity.

(c) Calculate the terminal velocity of an oil drop of radius 3.0×10^{-6} m falling in air of coefficient of viscosity 1.8×10^{-5} Pa s. Density of oil is given as 8.0×10^{2} kg m^{-3} and consider that the density of air is negligible.

(7) Two rain drops of equal sizes falling vertically attain terminal velocities of 0.150 m s^{-1}. If the two rain drops fuse together to form a larger oil drop, what will be its terminal velocity?

(Assume that Stokes' law can be applicable for water drop)

(8) Considering the forces acting on a small sphere falling down vertically in viscous medium.

(a) Explain why it finally attains a terminal velocity and why these forces are unable to bring the sphere to rest.

(b) An aluminum ball of mass 2.5×10^{-3} kg and external radius 1 cm has a cavity inside. When it is released from rest from the bottom of a deep glycerine tank it rises through the liquid. The density of glycerine is 1.26×10^{3} kg m^{-3} and its coefficient of viscosity is 0.03 kg m^{-1} s^{-1}.

(i) Find the viscous force on the ball and its acceleration when its velocity is 0.1 m s^{-1}.

(ii) Calculate its terminal velocity.

Chapter Three

Surface Tension

3.1 Introduction

Certain creatures are seen to walk on water. Drops of water hang in the form of bags from the tap end prior to dropping out. A dry steel needle can be placed smoothly and made to float on water. Mercury when spread on a clean glass surface, set in the form of small drops. These are phenomena we well witness in our day to day lives. From these observations, it is evident that the surface of a liquid behaves as a stretched elastic skin.

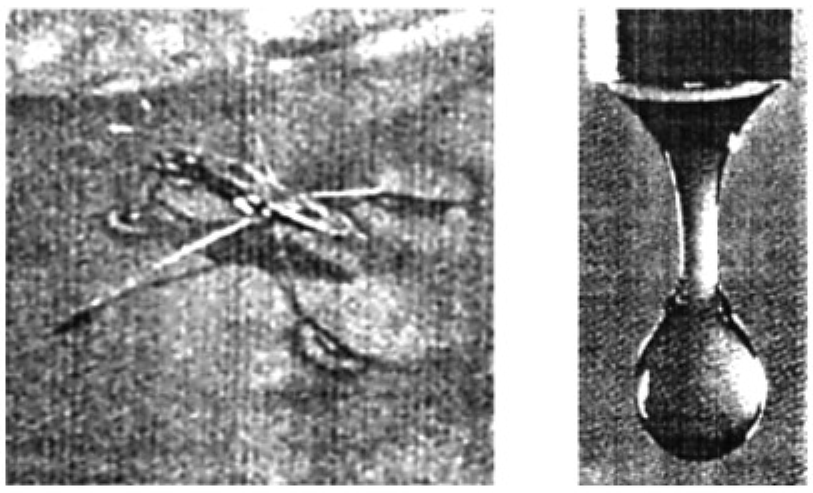

(a) (b)

Figure 3.1

3.2 Explanation of surface tension using molecular theory

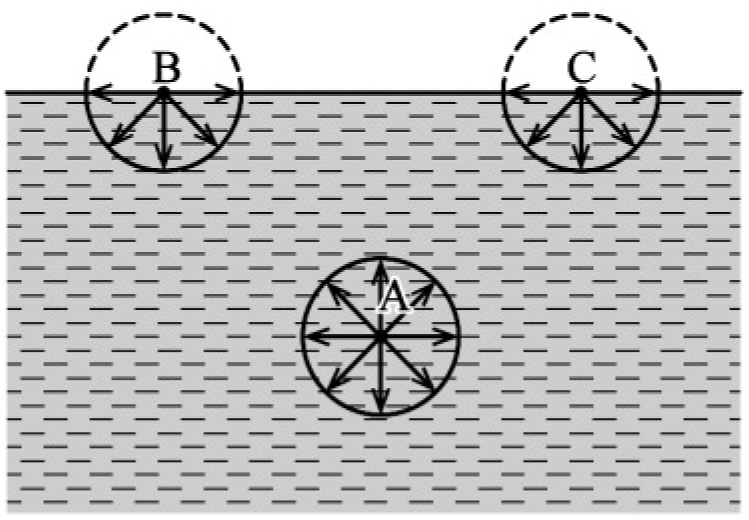


Figure 3.2

The tension of a liquid surface can be explained using intermolecular attraction. The molecules in a liquid are attracted by surrounding molecules. A sphere be constructed with a molecule at its centre so that it includes all other molecules attracting it. This sphere is referred to as the sphere of influence.

The sphere of influence of a molecule such as A (Figure 3.2) is situated completely inside the liquid. All molecules in the sphere attracts molecule A at the centre in all directions. Hence the resultant force on A becomes zero. However, when considering a molecule such as B or C on the liquid surface, only one half of the sphere of influence is in the liquid while its upper half is in air. Since the number of liquid molecules in the lower half (which is in the liquid) is very much larger than the number of liquid molecules in the upper half (which is in air). A resultant force acts vertically downwards on each of B and C. This tends the molecules on the liquid surface to move into the liquid thereby shrinking the surface. Hence the liquid surface behaves as a stretched skin causing tensional forces to act along it.

Due to the tendency of molecules to move into the liquid a small volume of a liquid takes the shape of a sphere.

3.3 Cohesive forces and adhesive forces

Figure 3.3

When a small quantity of water is placed on a clean glass surface, the water spreads over the glass surface (Figure 3.3a). But when some mercury is placed on the glass surface, the mercury settles in the form of drops and does not wet the glass [Figure 3.3. (b)]. Let us enquire into this difference in the behaviour of water and mercury.

Forces of attraction between similar molecules are known as cohesive forces while those forces between dissimilar molecules are known as adhesive forces. The adhesive force between a water molecule and a glass molecule is greater than the cohesive force between two water molecules. Hence water flows along the glass surface and thus water is said to wet glass. The cohesive force between two mercury molecules is greater than the adhesive force between a mercury molecule and a glass molecule. This causes mercury to set as drops on glass.

3.4 Definition of surface tension

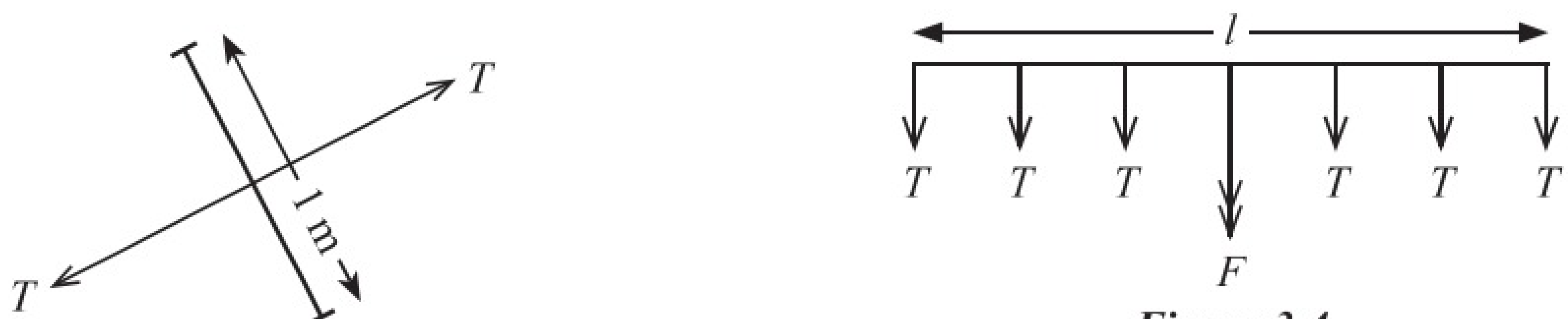


Figure 3.4

The surface tension of a liquid is defined as the force acting along the surface of the liquid on a unit length of an imaginary line drawn on the surface of the liquid, normal to and towards one side of the line.

If F is the total force acting normally on an imaginary line of length 'l' to one side of it,

Surface tension $T = \dfrac{F}{l}$

Units of surface tension $= \dfrac{\text{N}}{\text{m}} = \text{N m}^{-1}$

Dimensions of surface tension $= \dfrac{\text{M L T}^{-2}}{\text{L}} = \text{M T}^{-2}$

3.5 Shapes of liquid surfaces and angle of contact

The shape of a liquid surface whould arrange normally on the resultant force acting on it. If not, a component of this force will act parallel to the liquid surface to cause a motion. Usually a liquid surface is horizontal and normal to the gravitational force. But when it is in contact with a solid surface it generally becomes curved.

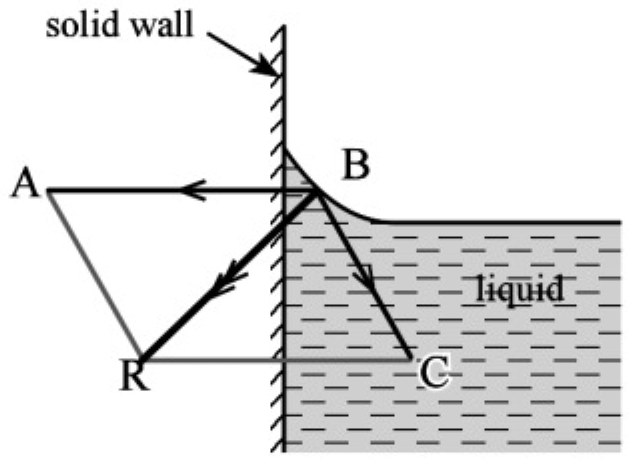


Figure 3.5

The shape of the surface depends on the cohesive forces between the liquid molecules and on the adhesive forces between liquid molecules and the solid molecules. Consider the liquid at B in contact with the solid wall as shown in Figure 3.5. Due to the cohesive forces of the liquid molecules in the vicinity, an attractive force F_1 acts on it. Also, due to the adhesive forces of the solid molecules close to it, another force of attracion F_2 acts on it. If the cohesive forces are stronger than the adhesive forces, the resultant forces F_R on B act in the direction BR as shown in the Figure 3.5. Since the liquid surface at B should be normal to F_R it gets curved downwards.

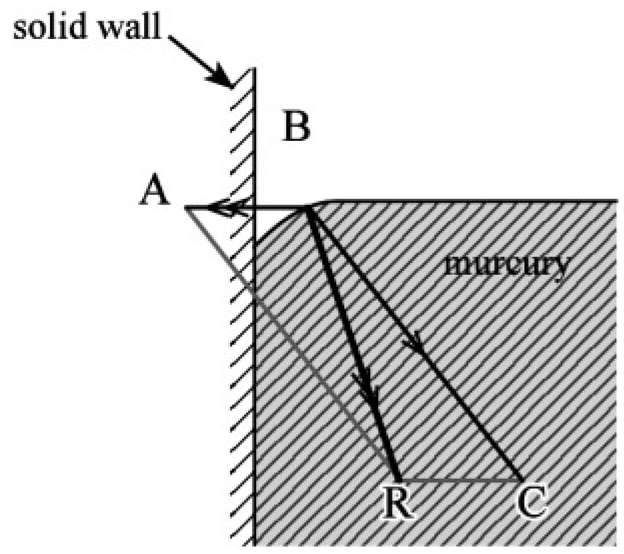
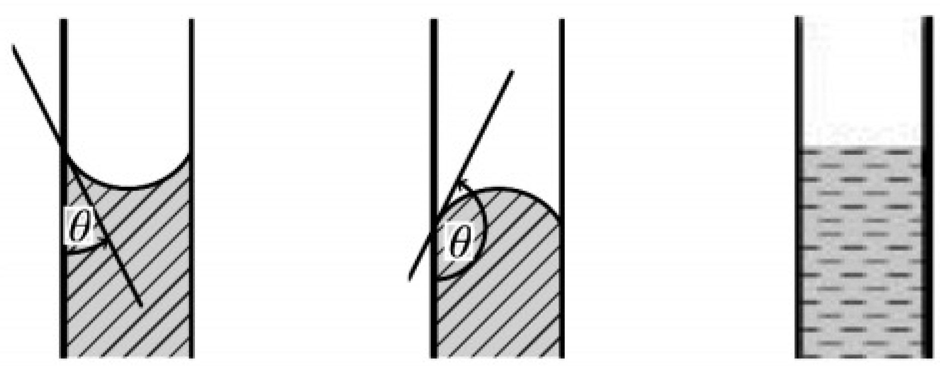


Figure 3.6

The cohesive force between liquid molocules is greater than the adhesive force between liquid molocules and solid molecules. The resultant force F_R gets inclined towards the cohesive force F_1 as shown in Figure 3.6. Since the liquid surface should set normally to F_R, it gets curved upwards. This occurs between mercury and glass.



$\theta < 90°$ (a) $\theta > 90°$ (b) $\theta = 0$ (c)

Figure 3.7

The angle measured through the liquid between the solid surface and the tangent drawn to the liquid surface at the point of contact is the **angle of contact** (θ) between the solid and the liquid.

In the Figure 3.7 (a), the angle of contact between the liquid and the solid is an acute angle ($\theta < 90°$) while in the Figure 3.7 (b) the angle of contact is an obtuse angle ($\theta > 90°$). In the Figure 3.7 (c) the water surface is parallel to a clean glass surface at the point of contact between the two and hence the angle of contact is zero ($\theta = 0°$).

3.5.1 Capillary rise and capillary fall

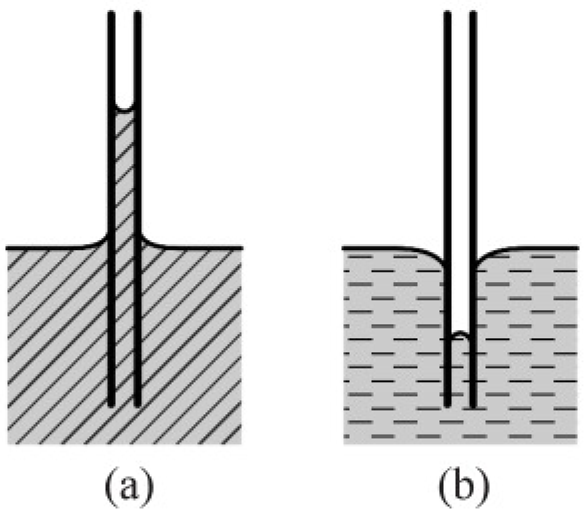


Figure 3.8

When a clean glass capillary tube is placed in a liquid of which the angle of contact is an acute angle, a liquid column rises in the glass tube and stands at a level higher than that of the outside liquid level. [Figure 3.8 (a)]. This is known as capillary rise. The capillary rise increases when the capillary gets thinner. This phenomenon is known as capillary attraction. When the capillary tube is placed in a liquid of which the angle of contact is an obtuse angle, the liquid level in the tube falls below the outside liquid level. This is known as capillary fall.

3.6 The work done in increasing the surface area of a soap film

Consider a soap liquid film made in the space between a three-sided wire frame and another wire AB placed on it. The surface area of this film can be increased by pulling AB by means of a string towards the right, as shown in the Figure 3.9.

If F is the force required to pull AB with a uniform speed, 'l' the length of the wire AB and T the surface tension of the soap liquid,

$$F = T \times 2l = 2Tl$$

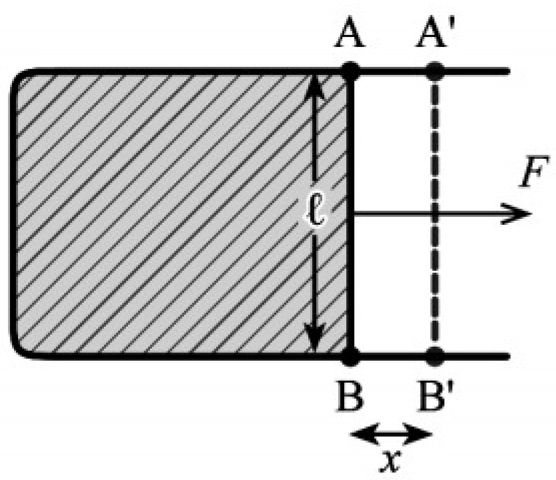


Figure 3.9

$2\,l$ is taken, as surface tension exists on both sides of the film.

When the wire AB is moved a distance 'x' to increase the area of the film isothermally,

the work done, $W = F \times x$

$= 2Tl \times x$

$= T \times 2lx$

$= T \times$ increase of area

The work done against surface tensional forces in increasing the area of the film by a unit area is known as its **free surface energy.**

If E is the free surface energy, the extra energy stored $= 2Elx$

If there is no loss of energy,

Extra energy stored = Work performed externally

$\therefore\ 2Elx = T \times 2lx$

$E = T$

Free surface energy = Surface tension

© 2020 National Institute of Education. All Rights Reserved.

3.7 Expression for the pressure difference across a spherical meniscus

Consider spherical air bubbles in a liquid and the equilibrium of one half of it.

Let 'r' be the radius of the bubble, 'T' the surface tension of the liquid, 'p_1' the pressure inside the bubble and 'p_2' the pressure outside it.

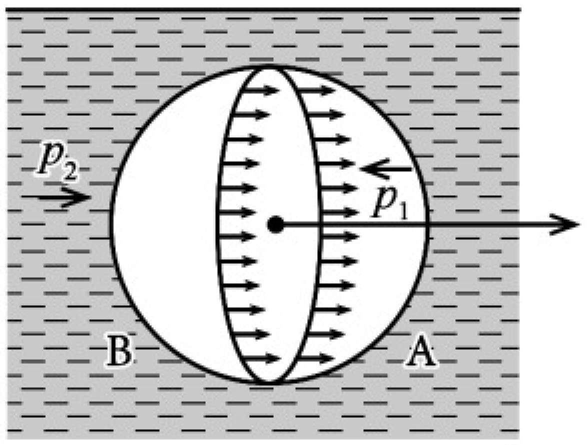


Figure 3.10

Taking the Figure 3.10,

The force acted upon the half B by surface tension	=	$2\pi r T \rightarrow$
The force acted upon the curved surface of the half B by the external pressure p_2	=	$\pi r^2 p_2 \rightarrow$
The force acted upon the curved surface of the half B by the internal pressure p_1	=	$\pi r^2 p_1 \leftarrow$

for equilibrum of the half bubble,

$$\pi r^2 p_1 = \pi r^2 p_2 + 2\pi r T$$

$$(p_1 - p_2)\pi r^2 = 2\pi r T$$

$$p_1 - p_2 = \frac{2T}{r}$$

The excess pressure inside a soap solution bubble.

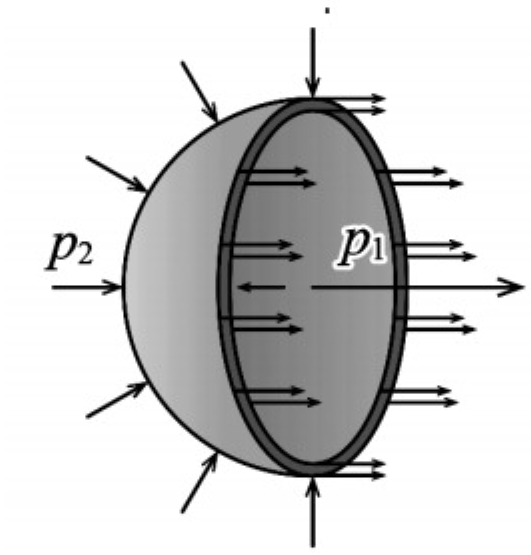


Figure 3.11

If 'r' is the radius of the bubble, 'T' the surface tension of the soap solution 'p_1' the pressure inside the bubble and 'p_2' the air pressure outside it.

Considering the equilibrium of one half of the bubble,

Force acted on the half of the bubble by surface tension	$\} = 4\pi r T \rightarrow$
Force acted on the curved surface of of the half of the bubble by external air pressure p_2	$\} = \pi r^2 p_2 \rightarrow$
Force acted on the curved surface of the half bubble by internal air pressure p_1	$\} = \pi r^2 p_1 \leftarrow$

for equilibrum of the half bubble,

$$\pi r^2 p_1 = \pi r^2 p_2 + 4 \pi r T$$

$$(p_1 - p_2) \pi r^2 = 4 \pi r T$$

$$p_1 - p_2 = \frac{4T}{r}$$

3.8 Derivation of the expression for capillary rise due to surface tension

3.8.1 Using the pressure difference

Let p_A be the pressure at point A just above the liquid meniscus, p_2 the pressure at point B just below the meniscus and p_C the pressure at point C at the tube at the same level as the outside liquid level.

Considering R as the radius of curvature of the liquid menisus, r as the radius of the capillary tube, and the surface tension of the liquid as T, its density as ρ the angle of contact as θ (Figure 3.12),

$$p_A - p_B = \frac{2T}{R} \quad \text{--------- (1)}$$

$$p_C = p_B + h \rho g \quad \text{--------- (2)}$$

If 'h' is the capillary rise,

$$p_C - p_B = h \rho g$$

But $p_A = p_C$

$$\frac{2T}{R} = h \rho g$$

Relation between the radius of curvature (R) of the liquid meniscus and the radius (r) of the tube,

According to Figure 3.12

$$\frac{r}{R} = \cos\theta$$

$$\therefore \quad R = \frac{r}{\cos\theta}$$

Substituting of R in the earlier expression,

$$\frac{2T}{r / \cos\theta} = h \rho g$$

$$\frac{2T\cos\theta}{r} = h \rho g$$

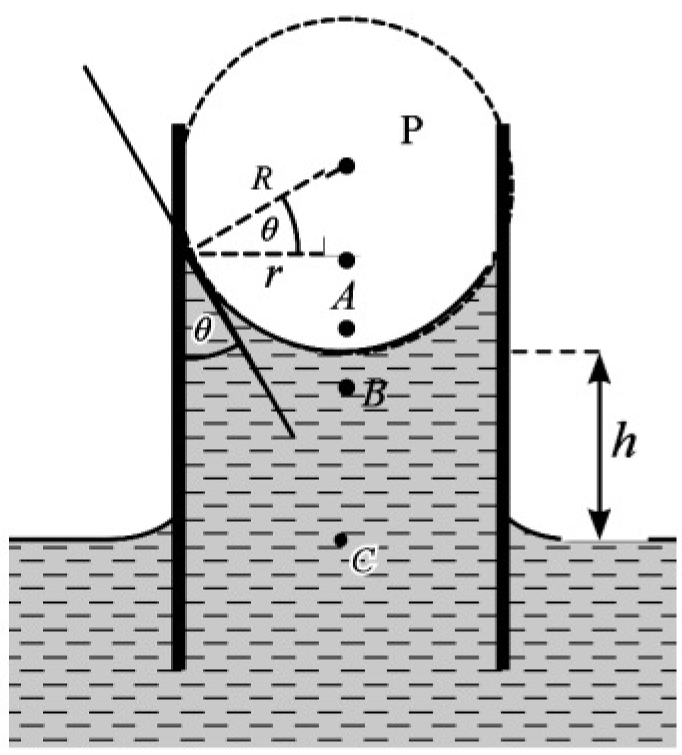


Figure 3.12

© 2020 National Institute of Education. All Rights Reserved.

3.8.2 Using the equilibrium of forces

Force by surface tension along the wall of the tube on a unit length of the circumference of the meniscus is $T \cos\theta$ (Figure 3.13).

Upward surface tensional force acting on the liquid column $= 2\pi r \times T\cos\theta$

$= 2\pi r T\cos\theta$

This force holds the liquid column of height h.

If the density of the liquid is ρ

Weight of the liquid column $= \pi r^2 h \rho g$

Considering the equilibrium of forces,

$$2\pi r T\cos\theta = \pi r^2 h \rho g$$

$$\frac{2T\cos\theta}{r} = h\rho g$$

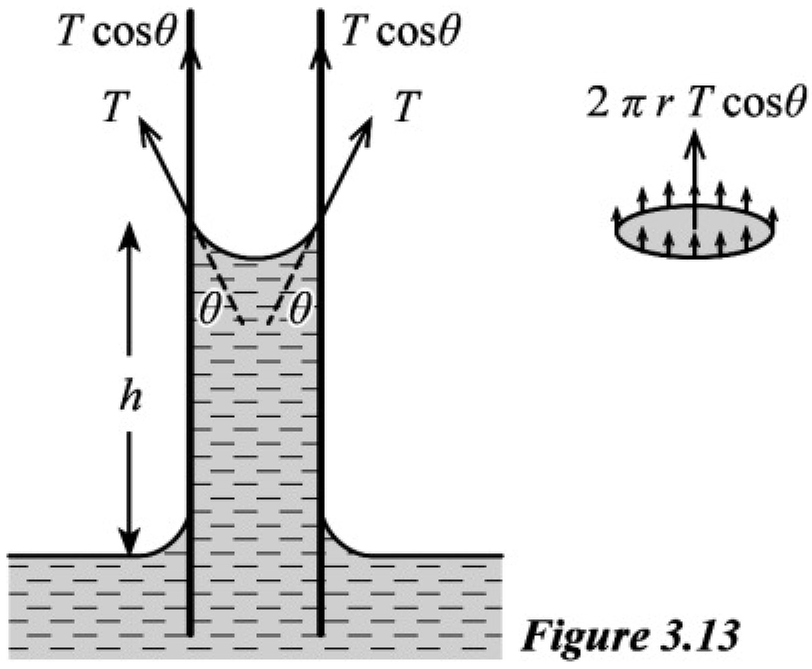


Figure 3.13

3.9 Methods of determining surface tension

3.9.1 Using a microscope slide

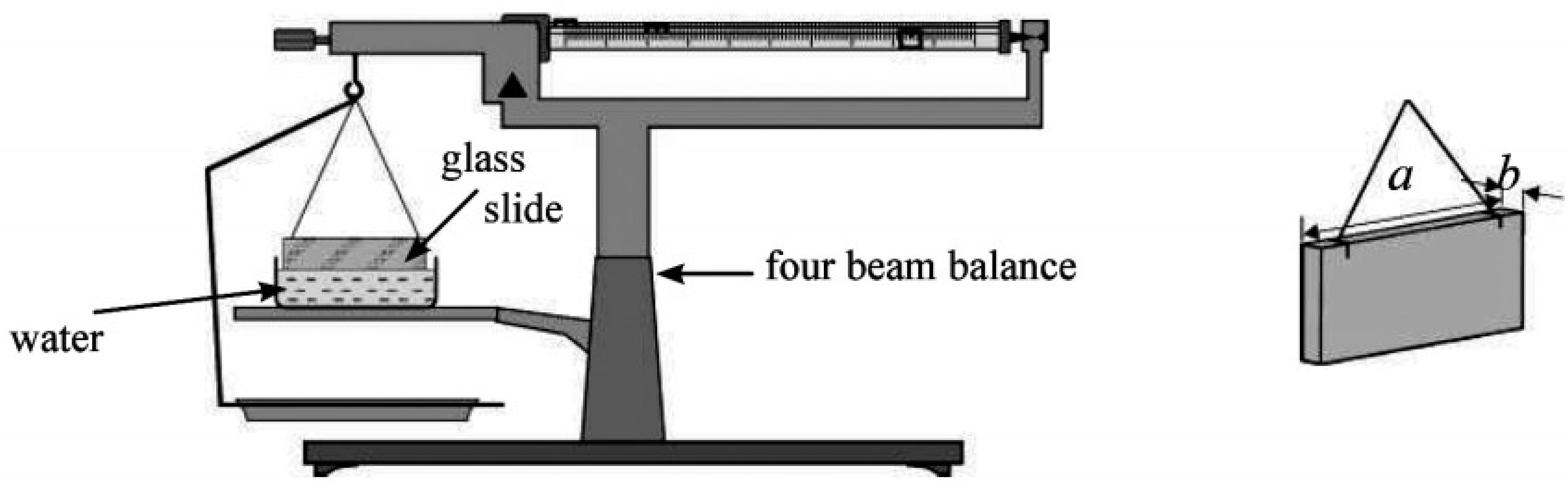


Figure 3.14

Wash the microscope slide first with a soap solution, then with a dilute acid and finally with water clean it.

Next, suspend the slide from the hook carrying the pan of a four-beam balance. Adjust the indicators on the beams to balance the instrument. Adjust the supporting pan attached to the balance to be a little above the balance pan.

Place a small vessel of water on the supporting pan and adjust till the slide just touches the water surface (Figure 3.14). Due to the surface tensional forces acting vertically downwards along the perimeter of the slide, a downward force acts on the slide. The balance of the instrument is now lost. Adjust the indicators on the beams to restore balance again.

If 'mg ' is the extra weight added to restore balance, 'a' the length and 'b' the thickness of the slide, and 'T' the surface tension of water.

Downward surface tensional force $= 2(a+b)T$

Force necessary to balance the surface $= mg$

Tensional force $2T(a+b) = mg$

$$T = \frac{mg}{2(a+b)}$$

3.9.2 Capillary rise method

Clean the capillary tube by washing it, first with a soap solution, next with a dilute acid and finally with water.

As shown in Figure 3.15, support the capillary tube vertically on a stand and dip its lower end in water carried in a beaker. Attach a squarely bent pin (P) to the tube and adjust its tip to just touch the water surface in the beaker.

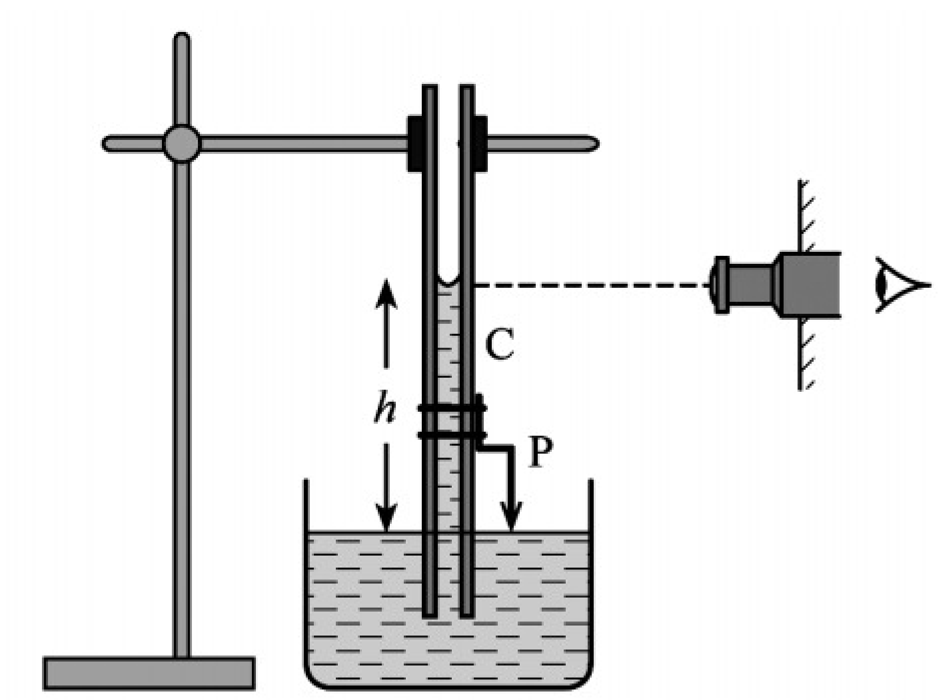


Figure 3.15

Set up the travelling microscope as shown in Figure 3.15 and adjust the microscope to focus its horizontal cross wire to the bottom of the water meniscus. Obtain the reading in the microscope scale using the vernier scale. Next remove the water breaker, focus the microscope on to the tip of the pin P and obtain the scale reading again. The difference between these two readings gives the capillary rise h.

To find the radius of the capillary tube, measure its two perpendicular diameters by focusing the microscope to the ends of two such diameters and obtain two parts of readings corresponding to those ends. Calculate two perpendicular diameters from the difference in readings of each pair and hence the mean diameter from which the radius 'r ' can be calculated.

If 'T' is the surface tension of water and ρ its density.

$$\frac{2T\cos\theta}{r} = h\rho g$$

Since the angle of contact between water and glass is zero, $\theta = 0^\circ$ $(\cos 0^\circ = 1)$

Solved exercise

A uniform glass tube of internal radius 12 mm and 0.4 mm thickness of walls is suspended vertically from a sensitive spring balance. A beaker carrying a liquid is now slowly brought until the liquid surface just touches the lower end of the hanging tube.

(i) What will then happen to the reading of the balance? Explain your answer.

(ii) The liquid beaker is now raised until the balance indicates its original reading. The depth to which the tube is immersed is 3.67 cm. Assuming that the angle of contact between the liquid and glass is zero, calculate the surface tension of water (density of liquid = 1000 kg m^{-3})

Solution

The reading balance increases due to the surface tensional forces acting downwards. Surface tensional force acting on the bottom of the tube.

$$\downarrow F_1 = 2\pi r T + 2\pi (r + d) T$$

where 'r' is the radious of the tube, 'd' the thickness of the wall of the tube and 'T' is the surface tension of liquid.

Upthrust acting on the tube

$$\uparrow F_2 = \pi [(r+d)^2 - r^2] h \rho g$$

$$= \pi (d^2 + 2rd) h \rho g,$$

where 'h' is the depth to which the tube is immersed in the liquid and ρ the density of the liquid.

When the reading of the balance equals its original reading,

$$F_1 = F_2$$

$$2\pi T (r + r + d) = \pi d (d + 2r) h \rho g$$

$$T = \frac{d h \rho g}{2}$$

$$= \frac{(0.4 \times 10^{-3}\ \text{m}) \times (3.67 \times 10^{-2}\ \text{m}) \times (1000\ \text{kg m}^{-3}) \times (10\ \text{m s}^{-2})}{2}$$

$$= 2 \times 3.67 \times 10^{-2}\ \text{N m}^{-1}$$

Surface tension $= \underline{\underline{7.34 \times 10^{-2}\ \text{N m}^{-1}}}$

© 2020 National Institute of Education. All Rights Reserved.

3.9.3 Jaeger's method

In this method surface tension is determined by measuring the excess pressure needed to emit an air bubble in a liquid.
Water is allowed to drop from the dropping funnel into the flask as shown in Figure 3.16. This gradually increases the pressure and the increasing pressure is indicated in the manometer.

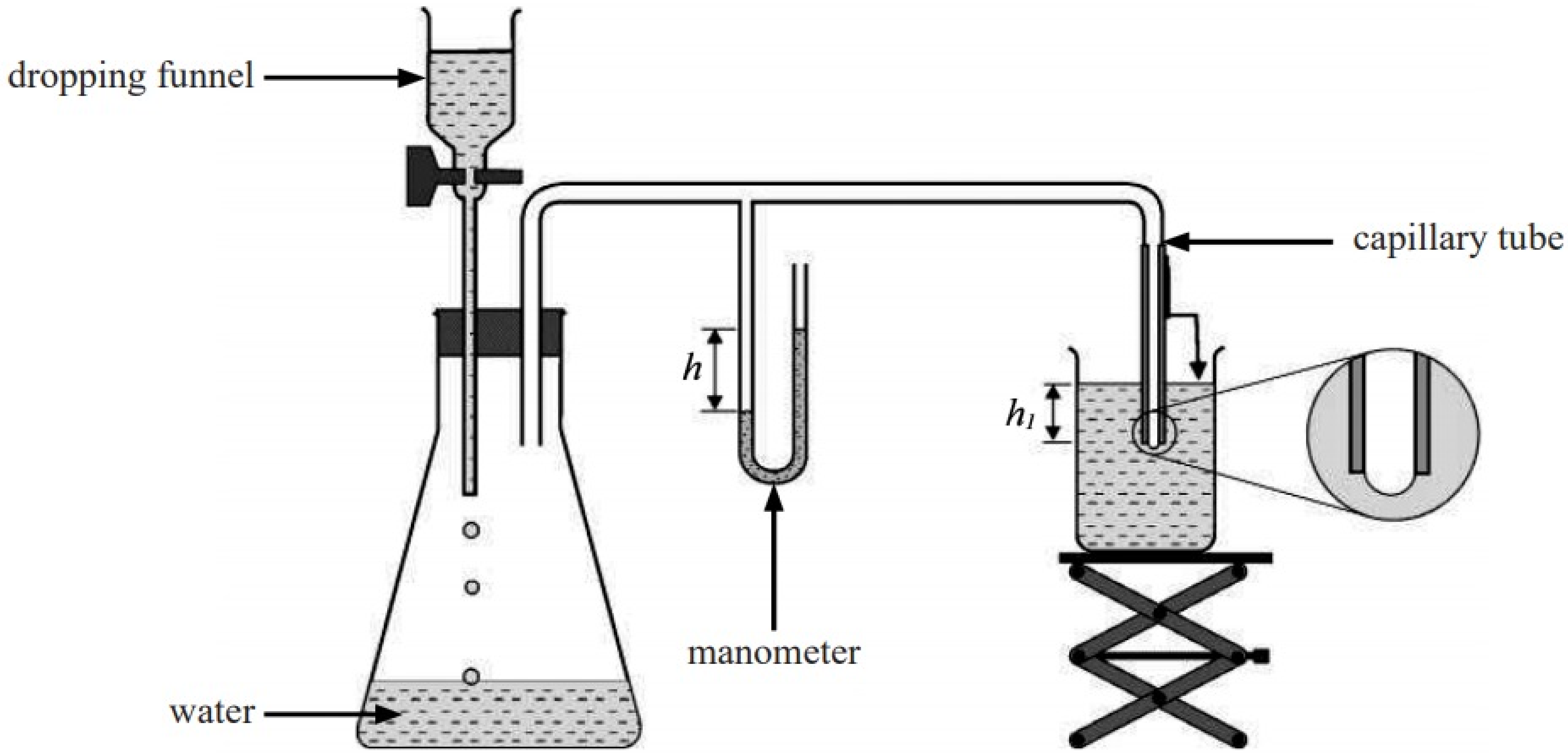


Figure 3.16

An air bubble grows at the end of the capillary tube which dips into the liquid of which surface tension is to be found and after the pressure becomes a maximum, the bubble gets released and the difference between the liquid levels in the manometer falls. The pressure was maximum when the radius of the bubble was minimum. This minimum radius is the radius of the capillary tube.

If p is the atmospheric pressure, 'h' the maximum value of the difference between the liquid levels of the manometer and p the density of the liquid levels.

Pressure inside the bubble $= p + h\rho g$

If p_1 is the density and T the surface tension of the liquid and h_1 the depth upto which the capillary tube is dipped in the liquid,

Pressure in the liquid outside the bubble $= p + h_1 \rho_1 g$

Excess pressure in the bubble $= (p + h\rho g) - (p + h_1 \rho_1 g)$

$= h\rho g - h_1 \rho_1 g$

But, excess pressure $= \dfrac{2T}{r}$

$\dfrac{2T}{r} = (h\rho - h_1\rho_1)\, g$

$\therefore \quad T = \dfrac{gr}{2}(h\rho - h_1\rho_1)$

© 2020 National Institute of Education. All Rights Reserved.

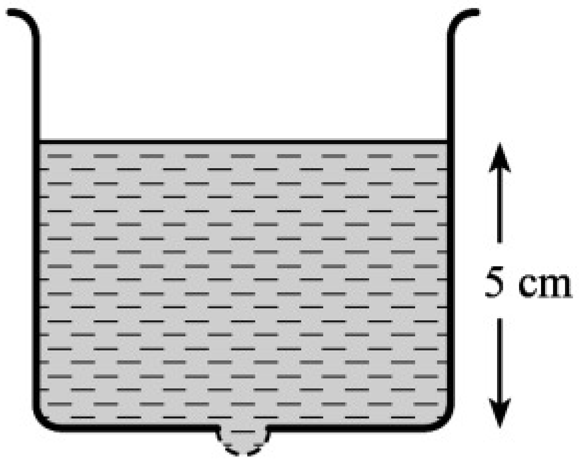
Hence 'T' can be calculated.

In this experimental method, the liquid can be heated to varius temperatures and the variation of surface tension with temperature can be investigated.

Solved exercises

(1) At the flat bottom of a bucket is a small circular hole of radius 0.1 mm and the bucket contains an oil of density 800 kg m^{-3} and surface tension 0.03 N m^{-1} to a depth of 5 cm. Show that the oil does not leak out through the hole.

If the bucket without any oil is forced vertically into water, to what depth will the bucket be immersed when the water begins to flow into the bucket? Surface tension of water is 0.075 N m^{-1} and the density is 10^3 kg m^{-3}.



Solution

When the height of the liquid increases the growth of a water drop can be seen. The drop begins to get released, when the radius of the drop ≤ radius of the hole.

Maximum excess pressure the drop can hold $= \dfrac{2T}{r_0}$ when r_0 is the radius of the hole.

$$= \frac{(2 \times 0.03 \text{ N m}^{-1})}{(0.1 \times 10^{-3} \text{ m})}$$

$$= 600 \text{ N m}^{-2}$$

Excess pressure exerted by liquid $= h_1 \rho_1 g$

$$= (5 \times 10^{-2} \text{ m}) \times (800 \text{ kg m}^{-3}) \times (10 \text{ m s}^{-2})$$

$$= 400 \text{ N m}^{-2}$$

$$\therefore\ h_1 \rho_1 g < \frac{2T}{r_0}$$

∴ The liquid does not leak out of the hole

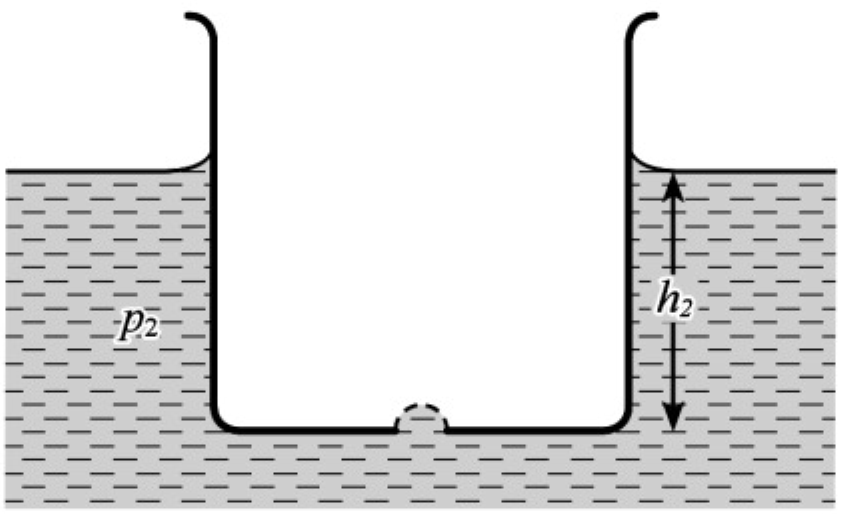
when height h_2,

$$h_2 \rho_2 g = \frac{2T_2}{r_0}$$

If h_2 satisfies the condition,

$$\therefore\ h_2 \rho_2 g < \frac{2T}{r_0}$$

water begins to flow into the bucket



$$h_2 = \frac{(2 \times 0.075 \text{ N m}^{-1})}{(10^3 \text{ kg m}^{-3}) \times (10 \text{ m s}^{-2}) \times (0.1 \times 10^{-3} \text{ m})}$$

$$= \underline{\underline{0.15 \text{ m}}}$$

(2) When a capillary tube of radius 2×10^{-4} m was dipped in a liquid contained in a beaker and was clamped vertically, it was observed that the liquid level rose by 3.26×10^{-2} m in the tube. The pressure of the air in the capillary tube was then increased and measured by a manometer. When the pressure was increased in this manner, an air bubble was produced at the lower end of the tube and when this bubble was about to break away, the difference in the liquid levels of the manometer was found to be 5.6×10^{-2} m. The depth to which the lower end of the capillary tube was immersed in the liquid was 2.5×10^{-2} m and the densities of the liquid in the beaker and of the manometer liquid are 800 kg m^{-3} and 1000 kg m^{-3} respectively. Calculate the surface tension of the liquid and the angle of contact between the liquid and glass.

Solution

Let T be the surface tension of the liquid, θ the angle of contact between glass and the liquid and p the atmospheric pressure.

For the capillary rise in the tube,

$$\frac{2\,T\cos\theta}{r} = h\,\rho\,g$$

$$\frac{2\,T\cos\theta}{2 \times 10^{-4}} = 3.26 \times 10^{-2} \times 800 \times 10$$

$$\therefore \quad T\cos\theta = 26.08 \times 10^{-3} \qquad \text{———— (1)}$$

Pressure in the bubble when it is about to break away $= p + (5.6 \times 10^{-2} \times 1000 \times 10)$

$= (p + 5.6 \times 10^{2})$ N m^{-2}

Pressure outside the bubble at the same instant $= p + 2.5 \times 10^{-2} \times 800 \times 10$

$= (p + 2 \times 10^{2})$ N m^{-2}

Pressure difference across the bubble when it is about to break away $= (5.6 - 2) \times 10^{2}$ N m^{-2}

$= 3.6 \times 10^{2}$ N m^{-2}

© 2020 National Institute of Education. All Rights Reserved.

When the bubble is about to break away its radius is equal to the radius of the capillary tube

$$\text{Pressure difference} = \frac{2T}{r}$$

$$= \frac{2T}{2 \times 10^{-4}\ \text{m}}$$

$$= T \times 10^{4}$$

$$\text{But} \quad T \times 10^{4} = 3.6 \times 10^{2}$$

$$\therefore \quad T = \underline{\underline{3.6 \times 10\ \text{N m}^{-1}}}$$

From equation (1)

$$\cos\theta = \frac{26.08 \times 10^{-3}}{3.6 \times 10^{-2}}$$

$$= 0.7244$$

$$\therefore \quad \theta = 43^\circ\ 35'$$

$$\approx \underline{\underline{44^\circ}}$$

3.10 Applications of surface tension

The drying action in blotting paper is due to the capillary effect which makes ink to flow upwards. In weldings surface tension helps the molten lead to penetrate into the cracks. In dyeing of clothes the success depends on the capillary effect on the dye to penetrate into the clothes.

Practically it is important to know as to how a liquid behaves when in contact with a solid. In welding work if the molten welding substance (an alloy of tin and lead) spreads on and wets the metal to be welded, a strong joint can be obtained. If the welding liquid has a low surface tension, the spreading takes place quickly. In welding, the metal surface is cleaned by applying substances like resin and it acts to wet the surface which helps spreading. In normal painting or using a sprayer for the purpose the spreading should be in the form of layers and not as drops. Agents used for wetting, play a main role in this respect.

An agent such as stearic acid helps lubricating oil to keep sticking to the axle or to the bearing.

In clothes, dirt such as oils are removed by using detergents. These should be spread over the clothes before removing dirt. Hence these substances should have a low surface tension and a low angle of contact. Weather proof clothes, which are not affected by rain and wind etc., can be produced by including Silicon.

© 2020 National Institute of Education. All Rights Reserved.

Exercise

(1) Define surface tension.

A rectangular sheet of length, breadth and thickness 6 cm, 4 cm and 2 mm respectively is placed horizontally with its largest surface lying on a water surface .Calculate the force on the sheet due to surface tension.

If the sheet is placed vertically with its long side just touching the water surface what is the force due to surface tension acting on it vertically downwards? (surface tension of water = 7.0×10^{-2} N m^{-2}).

(2) What are the dimensions of surface tension?

A capillary tube of diameter 0.4 mm is placed vertically.

i. A water surface with surface tension 6.5×10^{-2} N m^{-1} and zero angle of contact.

ii. A liquid surface with density 300 kg m^{-3} surface tension 5.0×10^{-2} N m^{-1} and angle of contact 30°.

Calculate the heights risen by each liquid in the capillary tube on the above situations.

(3) The diameters of the vertical limbs of a 'U' tube are 1 cm and 1 mm respectively. The 'U' tube is placed vertically and a liquid of surface tension 7.0×10^{-2} N m^{-1} is poured into it. Find the difference in the levels of the liquid in the two limbs.

The density of the liquid is 1000 kg m^{-3} and its angle of contact is zero.

(4) A capillary tube with internal diameter 50 μm is clamped vertically with its lower end dipping in a liquid of surface tension 5.5×10^{-2} N m^{-1}. The liquid has an angle of contact of 20° with glass. The pressure in the tube above the meniscus, is now adjusted until the meniscus reaches the level of the outside liquid surface. Calculate the difference between the pressure above the liquid meniscus in the tube and the pressure of the air on the free liquid surface outside the tube.

(5) A clean glass capillary tube of internal diameter 0.04 cm is fixed vertically with lower end immersed below a pure water surface and a length of 10 cm above the water surface. To what height will water rise in the tube?

If the tube is now lowered until only 5 cm of it is above the water surface, what will happen? (surface tension of water = 7.2×10^{-2} N m^{-1})

(6) Two soap solution bubbles of different sizes are formed at the two ends of a tube which is closed by a tap in its middle. What will be the result if this tap is opened? Show by means of a diagram the two bubbles after reaching equilibrium.

© 2020 National Institute of Education. All Rights Reserved.

(7) A capillary tube of internal diameter 0.7 mm is fixed vertically so that its lower end is situated below a water surface. Air is slowly forced from the upper end of the tube which is connected to a 'U' tube manometer containing a liquid if density 800 kg m^{-3}. It was observed that, the difference between the liquid levels in the manometer rose to 9.1 cm and then fell to 4.0 cm. This phenomenon was repeated.

Find (a) the depth of the lower end of the tube below the water surface.

(b) the surface tension of water

(8) Using the expression for the pressure difference across a spherical liquid surface, derive an expression for the capillary rise of a liquid in terms of the density of the liquid, its surface tension, the radius of curvature of the liquid surface and the acceleration due to gravity.

Water rises to a height of 4.8 cm in a glass capillary tube of internal radious 0.03 cm placed vertically.

(a) If the angle of contact between water and glass is zero and the density of water is 10^3 kg m^{-3}, calculate the surface tension of water.

(b) A column of water is now admitted to the capillary tube and is placed vertically with both its ends open to the air. When the length of the water column is,

(i) 3 cm

(ii) 1.5 cm

Calculate the radius of curvature of the lower meniscus in each case.

(9) A 'U' tube having limbs of diameters 0.05 mm and 1 mm respectively is inverted and placed with its open ends immersed under a water surface. The pressure in the tube is now increased so that the water mensicus in one of its limbs is at the same level as the outside water surface. Find the height of the water column in the other limb (surface tension of water = 7.2×10^{-2} N m^{-1}).

(10) A clean glass tube of internal diameter 2.0 mm and external diameter 8.0 mm is suspended vertically instead of the pan of an arm of a balance. The instrument is balanced by placing the required weight on the pan of the other arm of the balance. A beaker of water is now placed below so that 1.0 cm of the lower portion of the suspended tube is now under its water surface. Calculate the surface tension of water (density of water = 10^3 kg m^{-3})

© 2020 National Institute of Education. All Rights Reserved.

References

දිසානායක, ඩී. (1995). *පදාර්ථයේ යාන්ත්‍රික ගුණ*, දීපානි ප්‍රකාශන, නුගේගොඩ.

ජාතික අධ්‍යාපන ආයතනය (2019). ***අ.පො.ස. (උසස් පෙළ) භෞතික විද්‍යාව ප්‍රායෝගික අත්පොත***, ජාතික අධ්‍යාපන ආයතනය, මහරගම.

Edmonds Jr., D. S. (1993). *Cioffari's Experiments in College Physics - Ninth Edition.* D. C. Heath and Company, Massachusetts, USA.

Nelkon, M. & Ogborn, J. M. (1987). *Advanced Level Practical Physics - Fourth Edition*. Heinemann Educational Books, London, UK.

Nelkon, M. & Parker, P. (1995). *Advanced level physics. Seventh edition.* Heinemann publishers (Oxford). UK.

© 2020 National Institute of Education. All Rights Reserved.